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Simultaneous negative permittivity and permeability in a coherent atomic vapour*

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A new quantum optical mechanism to realize simultaneously negative electric permittivity and magnetic permeability is suggested. In order to obtain a negative permeability, we choose a proper atomic configuration that can dramatically enhance the contribution of the magnetic-dipole allowed transition via the atomic phase coherence. It is shown that the atomic system chosen with proper optical parameters can give rise to striking electromagnetic responses (leading to a negative refractive index) and that the atomic vapour becomes a left-handed medium in an optical frequency band. Differing from the previous schemes of artificial composite metamaterials (based on classical electromagnetic theory) to achieve the left-handed materials, which consist of anisotropic millimetre-scale composite structure units, the left-handed atomic vapour presented here is isotropic and homogeneous at the atomic-scale level. Such an advantage may be valuable in realizing the superlens (and hence perfect image) with left-handed atomic vapour.

Keywords: negative permittivity, negative permeability, coherent atomic vapour, electric- and magnetic-dipole allowed transitions

PACC: 4250, 4225, 7820

1. Introduction

Historically, new intriguing matter and materials could often be predicted by purely mathematical considerations.[1] For example, in the 1920s Dirac found that in order to keep the completeness condition of the electron wavefunctions, the negative-energy solutions (corresponding to positron) of Dirac's relativistic wave equation should not be excluded.[2] The anti-matter was thus predicted within the framework of relativistic quantum mechanics. The same thing took place in electrodynamics.[3] In the 1960s Veselago considered a possibility of negative refractive index $n = -\sqrt{\mu\epsilon}$, and studied the electromagnetic wave behaviour inside a hypothetical material with a negative index of refraction.[3] Veselago found that the hypothetical material (if it exists) has simultaneously negative permittivity and permeability, and that a new phenomenon known as negative refraction would take place for an incident light from air to the hypothetical material.[3] It follows from the Maxwellian curl equations that the phase velocity of the light-wave propagating inside the material with a negative refractive index is in opposite direction of energy flow, that is, the Poynting vector and the wave vector of electromagnetic wave is antiparallel, in others words, the vector $\boldsymbol{k}$, electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$ form a left-handed system. Thus Veselago referred to such a kind of materials as 'left-handed' media, and correspondingly, the ordinary materials in which $\boldsymbol{k}$, $\boldsymbol{E}$ and $\boldsymbol{H}$ form a right-handed system may be termed the 'right-handed' media. Veselago's discursive work on negative refractive index have received less attention for nearly three decades. But with the development of materials technology (particularly the fabrication techniques of artificial composite materials), the novel properties and phenomena in negative index materials have now attracted attention of many researchers in various areas such as materials science, applied electromagnetism and classical optics.[4−11] In the literature, some authors called such materials 'negative-index media',[4] 'backward wave materials',[5] 'double-negative media'[6] or 'negative phase-velocity media'.[7] As many dramatically different propagation characteristics would stem from the sign change of the optical 'constants',[9] the

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left-handed media can exhibit a number of peculiar electromagnetic and optical effects, including reversals of both Doppler shift and Cerenkov radiation, intrinsic optical bistability,[12] amplification of evanescent waves,[13] unusual quantum optical effects,[14] negative Goos–Hänchen shift[15] and reversed circular Bragg phenomenon.[16,17] Recently, more attention was paid to the potential applications of left-handed media. For example, the left-handed media can be utilized to fabricate so-called superlens (perfect lens) because a left-handed slab may have a power to focus all Fourier components of a two-dimensional image, including those that do not propagate in a radiative manner.[13] Another interesting application of left-handed media is the compact thin subwavelength resonator.[18,19]

During the past few years, the design and fabrication techniques of left-handed materials have achieved great success. In general, the artificial negative electric permittivity can be realized with the structure of *array of long metallic wires* (ALMWs),[20] which simulates the plasma behaviour at microwave frequencies, and the artificial negative magnetic permeability can be achieved via small resonant metallic particles, e.g., the *split ring resonators* (SRRs) with very high magnetic polarizability.[21−23] A combination of these two structures yields a left-handed medium. Recently, Shelby *et al* reported their first experimental realization of such artificial composite materials, the permittivity and permeability of which have negative real parts.[24] Apart from the mechanism of artificial metamaterials, some anisotropic materials can also exhibit a negative refraction effect. For example, in the literature Mackay and Lakhtakia first suggested the possibility of the arbitrary-direction negative phase-velocity propagations in gyrotropic (Faraday) chiral media and obtained general and complete results,[25] and Sridhar *et al* considered the negative refraction and left-handed electromagnetism in metallic microwave photonic crystals.[26] These materials can exhibit the equivalent negative refractive indices.

In the present paper, an alternative method (quantum optical scheme) is suggested to realize the left-handed medium: specifically, under certain conditions, the electric-dipole and magnetic-dipole transitions in a multilevel atomic system with proper configuration and atomic parameters will exhibit simultaneously negative permittivity and negative permeability of the atomic vapour. The left-handed vapour produced is isotropic and homogeneous at the atomic-scale level (i.e., with atomic-scale microscopic structure units).

Apparently, the scheme of negative refraction achieved inside the atomic vapour is built based on the atomic phase coherence. In the literature, quantum coherence has many novel applications, including the coherent population trapping, inversionless laser,[27] electromagnetically induced transparency,[28−30] quantum computation[31] and some interesting quantum optical effects.[32,33] To realize negative optical 'constants' can be viewed as a new application of quantum coherence.

2. Quantum optical mechanism to realize negative indices

Though both Veselago's original paper[3] and most of the recent theoretical works considered mainly the characteristics of electromagnetic wave propagation inside the isotropic left-handed media, up to now, the left-handed media that have been prepared successfully experimentally are in fact anisotropic in nature, and it would be very difficult to fabricate an isotropic left-handed medium.[34−36] More recently, some authors suggested new ideas to realize negative refractive index, including photonic crystal structure,[37,38] transmission line simulation[39] as well as chiral nihility.[25,40] In all these previous studies, investigators used classical optical schemes (classical electromagnetic theory) to design and fabricate negatively refracting materials. In order to achieve an isotropic left-handed medium with negative refractive index in an optical frequency band, we study the possibility of realizing simultaneously negative permittivity and permeability in an atomic vapour with quantum optical mechanism (quantum coherence effect).

The quantum optical mechanism to realize negative indices is to choose an atomic system with proper configuration (see, for example, Fig.1), the electric- and magnetic-dipole allowed transitions of which can lead to the nontrivial electromagnetic responses that can dramatically modify the optical properties of the atomic vapour, even make the electric permittivity and magnetic permeability simultaneously negative in certain frequency bands. Very recently, some authors suggested that the electromagnetically induced transparency (EIT) atomic system may exhibit simultaneously negative permittivity and permeability.[41] Though such a kind of schemes sounds reasonable, it requires sufficiently large magnetic-dipole transition

moment to achieve significant magnetic responses. But in general, the dimensionless ratio of the magnetic dipole moment to the electric dipole moment is small in an atomic system such a ratio is approximately equal to the fine structure constant, e.g., $|m_{13}/(\wp_{23}c)| \simeq 1/137$, where m_{13}, $\wp_{23}$ and c denote the magnetic- and electric-transition dipole matrix elements and the speed of light in vacuum, respectively.

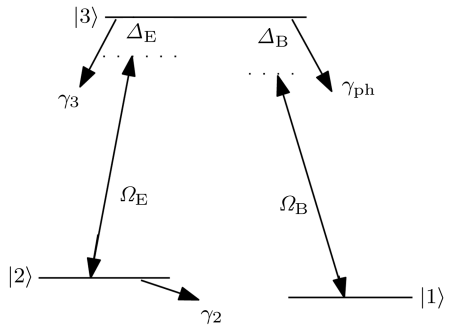


Fig.1. The schematic diagram of a three-level atomic system interacting with the electric and magnetic fields of a probe light. The electric-dipole allowed transition $|2\rangle$–$|3\rangle$ and the magnetic-dipole allowed transition $|1\rangle$–$|3\rangle$ would give rise to negative permittivity and permeability, respectively.

For this reason, the contribution of the magnetic-dipole allowed transition to the vapour is often negligibly small as compared with that of the electric-dipole allowed transition, and therefore the magnetic-dipole transition should not be taken into account in the treatment for a wave propagation in an ordinary atomic vapour. In the atomic configuration presented here, however, the density matrix element ρ_{31} (related to the magnetic-dipole allowed transition) can be dramatically enhanced via the phase coherence (quantum coherence) between the two lower levels, and the atomic magnetic polarizability $\beta_{\rm m}$ would greatly increase and hence have the same order of magnitude as the atomic electric polarizability $\beta_{\rm e}$. In other words, in order to achieve the negative index of refraction in atomic vapour, one should choose an atomic system with a configuration that can enhance the contribution of the magnetic-dipole allowed transition. Further analysis shows that such an atomic vapour may exhibit negative permittivity and permeability, and therefore will become an ideal candidate for realizing an isotropic left-handed medium. Apart from this, the present atomic vapour could exhibit optical nonlinearity that can lead to controllable manipulation of optical properties of the left-handed medium. This may be valuable for the development of new techniques in quantum optics and would have potential applications in information technology.

In the sections that follow, we first consider the steady solutions of the equation of motion for the density matrix elements of the atomic system and obtain the explicit expressions for the atomic electric and magnetic polarizabilities, and then discuss the local field effects of the atomic vapour. Based on this, an illustrative example will be presented with numerical results of the negative permittivity and permeability (and hence the negative refractive index) in some certain narrow frequency bands. We conclude with some remarks on the characteristic features of the scheme of negative refraction in the quantum coherent atomic vapour.

3. Atomic electric and magnetic polarizabilities

Consider a Λ-type three-level atomic ensemble with one upper level $|3\rangle$ and two lower levels $|1\rangle$ and $|2\rangle$ (see Fig.1. Such an atomic system interacts with a weak probe light: specifically, the electric and magnetic fields of the probe light couple level pairs $|2\rangle$–$|3\rangle$ and $|1\rangle$–$|3\rangle$, respectively. Thus the parity of level $|2\rangle$ is different from those of both $|1\rangle$ and $|3\rangle$. If, for example, level $|2\rangle$ possess an odd parity, then levels $|1\rangle$ and $|3\rangle$ will have even parity. Obviously, in order that the resonances in both electric permittivity and magnetic permeability could possibly occur, the transition frequencies of $|1\rangle$–$|3\rangle$ and $|2\rangle$–$|3\rangle$ transitions should be close to each other. The equation of motion of off-diagonal density matrix elements of this atomic system coupled to the applied probe light field is given by

$$\begin{aligned}
\dot{\rho}_{31} &= -\left(\mathrm{i}\Delta_{\rm B} + \frac{\Gamma}{2}\right)\rho_{31} + \frac{\mathrm{i}}{2}\Omega_{\rm B}\left(\rho_{11} - \rho_{33}\right) + \frac{\mathrm{i}}{2}\Omega_{\rm E}\rho_{21}, \\
\dot{\rho}_{32} &= -\left(\mathrm{i}\Delta_{\rm E} + \frac{\gamma}{2}\right)\rho_{32} + \frac{\mathrm{i}}{2}\Omega_{\rm E}\left(\rho_{22} - \rho_{33}\right) + \frac{\mathrm{i}}{2}\Omega_{\rm B}\rho_{12}, \\
\dot{\rho}_{21} &= -\left[\mathrm{i}\left(\Delta_{\rm B} - \Delta_{\rm E}\right) + \frac{\gamma_2}{2}\right]\rho_{21} + \frac{\mathrm{i}}{2}\Omega_{\rm E}^{*}\rho_{31} - \frac{\mathrm{i}}{2}\Omega_{\rm B}\rho_{23},
\end{aligned} \tag{1}$$

where the decay rates are defined as $\Gamma = \gamma_{\rm ph} + \gamma_3$ and $\gamma = \gamma_2 + \gamma_3 + \gamma_{\rm ph}$. Here, γ_2 and γ_3 denote the spontaneous decay rates of levels $|2\rangle$ and $|3\rangle$, respectively, and $\gamma_{\rm ph}$ stands for the dephasing rate (non-radiative decay rate) of level $|3\rangle$. The electric and magnetic frequency detunings are $\Delta_{\rm E} = \omega_{32} - \omega_{\rm p}$ and $\Delta_{\rm B} = \omega_{31} - \omega_{\rm p}$, where $\omega_{\rm p}$ denotes the angular frequency of the probe light. The electric and magnetic Rabi frequencies $\Omega_{\rm E}$ and $\Omega_{\rm B}$ are defined through $\Omega_{\rm E} = \wp_{32}\mathcal{E}_{\rm p}/\hbar$ and $\Omega_{\rm B} = m_{31}\mathcal{B}_{\rm p}/\hbar$, where $\mathcal{E}_{\rm p}$ and $\mathcal{B}_{\rm p}$ represent the electric and magnetic field envelopes (slowly varying amplitudes) of the probe light. The intensity of the probe light is assumed to be sufficiently weak so that nearly all the atoms remain in the ground state, *i.e.*, the atomic population in level $|1\rangle$ is close to unity. It should be noted that here the atoms are assumed to be nearly stationary and hence any Doppler shifts are neglected. Under these conditions, the steady values of the off-diagonal density matrix elements that have close relation to the magnetic and electric responses are given as follows:

$$\begin{aligned}
\rho_{31} &= \frac{-(\Delta_{\rm B} - \Delta_{\rm E}) + {\rm i}\dfrac{\gamma_2}{2}}{\left({\rm i}\Delta_{\rm B} + \dfrac{\Gamma}{2}\right)\left[{\rm i}(\Delta_{\rm B} - \Delta_{\rm E}) + \dfrac{\gamma_2}{2}\right] + \dfrac{1}{4}\Omega_{\rm E}^*\Omega_{\rm E}} \times \frac{\Omega_{\rm B}}{2}, \\
\rho_{32} &= -\frac{{\rm i}\Omega_{\rm B}^*\Omega_{\rm B}}{\left(-{\rm i}\Delta_{\rm B} + \dfrac{\Gamma}{2}\right)\left[-{\rm i}(\Delta_{\rm B} - \Delta_{\rm E}) + \dfrac{\gamma_2}{2}\right] + \dfrac{1}{4}\Omega_{\rm E}^*\Omega_{\rm E}} \times \frac{1}{{\rm i}\Delta_{\rm E} + \dfrac{\gamma}{2}} \times \frac{\Omega_{\rm E}}{8}.
\end{aligned} \tag{2}$$

The density matrix element ρ_{12} that can characterize the atomic phase coherence of the present system is

$$\rho_{12} = -\frac{\rm i}{2} \times \frac{\Omega_{\rm E}\rho_{13} - \Omega_{\rm B}^*\rho_{32}}{-{\rm i}(\Delta_{\rm B} - \Delta_{\rm E}) + \dfrac{\gamma_2}{2}}. \tag{3}$$

The selection rule for the present electric-dipole allowed transition ($|2\rangle$-$|3\rangle$) is $\Delta l = \pm 1, \Delta m = 0, \pm 1$, and the selection rule for the magnetic-dipole allowed transition ($|1\rangle$–$|3\rangle$) is $\Delta l = 0, \Delta m = 0, \pm 1$. By using the definitions of atomic microscopic electric and magnetic polarizabilities

$$\beta_{\rm e} = \frac{2\wp_{23}\rho_{32}}{\epsilon_0\mathcal{E}_{\rm p}}, \qquad \beta_{\rm m} = \frac{2\mu_0 m_{13}\rho_{31}}{\mathcal{B}_{\rm p}}, \tag{4}$$

one can obtain the explicit expressions for the electric and magnetic polarizabilities of the above coherent atomic vapour as follows:

$$\begin{aligned}
\beta_{\rm e} &= \frac{|\wp_{23}|^2}{4\epsilon_0\hbar} \frac{-{\rm i}\Omega_{\rm B}^*\Omega_{\rm B}\left({\rm i}\Delta_{\rm E} + \dfrac{\gamma}{2}\right)^{-1}}{\left(-{\rm i}\Delta_{\rm B} + \dfrac{\Gamma}{2}\right)\left[-{\rm i}(\Delta_{\rm B} - \Delta_{\rm E}) + \dfrac{\gamma_2}{2}\right] + \dfrac{1}{4}\Omega_{\rm E}^*\Omega_{\rm E}}, \\
\beta_{\rm m} &= \frac{\mu_0|m_{13}|^2}{\hbar} \frac{-(\Delta_{\rm B} - \Delta_{\rm E}) + {\rm i}\dfrac{\gamma_2}{2}}{\left({\rm i}\Delta_{\rm B} + \dfrac{\Gamma}{2}\right)\left[{\rm i}(\Delta_{\rm B} - \Delta_{\rm E}) + \dfrac{\gamma_2}{2}\right] + \dfrac{1}{4}\Omega_{\rm E}^*\Omega_{\rm E}}.
\end{aligned} \tag{5}$$

It should be noted that the above formulation is also applicable to the case where both amplitudes of electric and magnetic fields are complex. In this case, the electric and magnetic induction field strengths can be written as

$$\boldsymbol{B}_{\rm p} = \frac{\boldsymbol{\mathcal{B}}_{\rm p}{\rm e}^{-{\rm i}\omega t} + \boldsymbol{\mathcal{B}}_{\rm p}^*{\rm e}^{{\rm i}\omega t}}{2}, \tag{6}$$

where $\boldsymbol{\mathcal{E}}_{\rm p}$ and $\boldsymbol{\mathcal{B}}_{\rm p}$ denote the slowly-varying amplitudes of the electric and magnetic induction fields, respectively. Assume the wavefunction of the three-level atomic system is $|\Psi\rangle$, which is a linear combination of $|1\rangle$, $|2\rangle$ and $|3\rangle$. Thus one can verify that the ex-

pectation values of the electric and magnetic dipoles induced by the applied light field are

$$\begin{aligned} \langle \Psi|\mathrm{e}\boldsymbol{r}|\Psi\rangle &= \wp_{23}\rho_{32}\mathrm{e}^{-\mathrm{i}\omega t} + \wp_{32}\rho_{23}\mathrm{e}^{\mathrm{i}\omega t} \\ &\equiv \boldsymbol{p} + \boldsymbol{p}^*, \\ \langle \Psi|(e/2m_\mathrm{e})(\boldsymbol{L}+2\boldsymbol{S})|\Psi\rangle &= \boldsymbol{m}_{13}\rho_{31}\mathrm{e}^{-\mathrm{i}\omega t} + \boldsymbol{m}_{31}\rho_{13}\mathrm{e}^{\mathrm{i}\omega t} \\ &\equiv \boldsymbol{m} + \boldsymbol{m}^*. \end{aligned} \tag{7}$$

According to the definition of the electric polarizability (microscopic quantity) β_e, we have

$$\boldsymbol{p} = \beta_\mathrm{e}\epsilon_0\left(\frac{\boldsymbol{\mathcal{E}}_\mathrm{p}\mathrm{e}^{-\mathrm{i}\omega t}}{2}\right). \tag{8}$$

It should be noted that here the electric field strength $\boldsymbol{E}_\mathrm{p}$ is expressed by $\boldsymbol{\mathcal{E}}_\mathrm{p}\mathrm{e}^{-\mathrm{i}\omega t}/2$ rather than by $(\boldsymbol{\mathcal{E}}_\mathrm{p}\mathrm{e}^{-\mathrm{i}\omega t} + \boldsymbol{\mathcal{E}}_\mathrm{p}^*\mathrm{e}^{\mathrm{i}\omega t})/2$. The reason for this choice is just the *rotation wave approximation* (the effect related to the high-frequency oscillating factor such as $\mathrm{e}^{\pm\mathrm{i}\omega t}$ could be neglected). In the similar fashion, according to the definition of the magnetic polarizability (microscopic quantity) β_m,[42] we have

$$\boldsymbol{m} = \frac{\beta_\mathrm{m}}{\mu_0}\left(\frac{\boldsymbol{\mathcal{B}}_\mathrm{p}\mathrm{e}^{-\mathrm{i}\omega t}}{2}\right). \tag{9}$$

Thus, with the help of Eqs.(7), (8) and (9), one can obtain the atomic electric and magnetic polarizabilities (4).

To achieve the negative permittivity ϵ_r and negative permeability μ_r, the chosen vapour should be dense, and so the local field effects arising from the dipole–dipole interaction between neighbouring atoms need to be taken into consideration. This means that one should use both electric and magnetic Clausius–Mossotti relations, which can show the connection between the macroscopic quantities (relative electric permittivity ϵ_r and magnetic permeability μ_r) and the microscopic quantities (atomic electric polarizability β_e and magnetic polarizability β_m). These Clausius–Mossotti relations are of the form[42,43]

$$\epsilon_\mathrm{r} = \frac{1+\frac{2}{3}N\beta_\mathrm{e}}{1-\frac{1}{3}N\beta_\mathrm{e}}, \qquad \mu_\mathrm{r} = \frac{1+\frac{2}{3}N\beta_\mathrm{m}}{1-\frac{1}{3}N\beta_\mathrm{m}}, \tag{10}$$

where N denotes the atomic concentration of the vapour. Thus we have the explicit expressions for the electric permittivity and magnetic permeability of the atomic vapour. Note that sometimes the Clausius–Mosotti relations would be invalid (or approximately valid) for the cases of large dielectric constants, especially in the polar liquids and solids with large $N\beta_\mathrm{e}$ (and $N\beta_\mathrm{m}$).[44] But for the media consisting of isotropic molecules or atoms, the Clausius–Mosotti relations may be applicable to the case of large $N\beta_\mathrm{e}$ (and $N\beta_\mathrm{m}$), at least these relations can act as tentative tools to treat the isotropic solid and atomic vapour that can exhibit the linear material response. For example, some researchers demonstrated that the Clausius–Mosotti approach works well for crystalline fullerenes where $N\beta_\mathrm{e} \simeq 1.5$,[45] and in the literature some authors also used the Clausius–Mosotti relation for the atomic vapour with large $N\beta_\mathrm{e}$.[46] In the formulation of Clausius–Mosotti relation, one first considers the linear material response (where the local field effect is not involved in the Maxwell–Bloch equation), and then takes account of the local field correction to the dielectric constant via the Clausius–Mosotti relation). But the Clausius–Mosotti relation will be invalid for very dense vapour ($N \gg 10^{26}\,\mathrm{m}^{-3}$). In this case, the generalized Maxwell–Bloch equation with the local field effect involved in itself should be used if the vapour is a very dense medium having many atoms within a cubic resonance wavelength.[47] In this paper, though the atomic vapour is dense, the formulation of the linear material response is still valid, and the Clausius–Mosotti relation holds true for the present isotropic atomic vapour.

An illustrative example with numerical results will be given in the section that follows. In order that the atomic vapour can become a left-handed medium, the permittivity and permeability should simultaneously have negative real parts in the same frequency band. In the following discussion, let us see the problem of possibility of resonance in permittivity and permeability in the same narrow probe frequency band. If the applied electric field can now drive the $|2\rangle$–$|3\rangle$ transition, then in order to simultaneously achieve a sufficiently strong transition between two magnetic-dipole transition-allowed levels, one should need to tune the transition frequency between levels $|3\rangle$ and $|1\rangle$ to (nearly) equal the mode frequency of the applied light. This can be realized by making use of a dc magnetic field to tune the magnetic-dipole transition into resonance.[48] Thus for a given three-level atomic system, we can in principle obtain the condition $\Delta_\mathrm{B} = \Delta_\mathrm{E}$ with a proper dc magnetic field. In the following illustrative example, for convenience, the relation $\Delta_\mathrm{B} - \Delta_\mathrm{E} = 0$ is satisfied. But we should emphasize that such a relation is not necessarily fulfilled in a general treatment (i.e., the sufficiently negative permittivity and permeability could also be realized

in the case of $\Delta_{\rm B} - \Delta_{\rm E} \simeq 0$).

4. A numerical example

Here we present a numerical example to demonstrate that the simultaneously negative permittivity and permeability can be truly achieved by the present quantum optical scheme. The typical parameters in expression (5) for the electric and magnetic polarizabilities can be chosen as: electric and magnetic transition dipole moments $\wp_{32} = 3.0 \times 10^{-29}\,{\rm C{\cdot}m}$,[29] $m_{31} = 9.4 \times 10^{-24}\,{\rm C{\cdot}m^2{\cdot}s^{-1}}$, decay rates $\gamma_2 = 4.0 \times 10^6\,{\rm s^{-1}}$, $\gamma_3 = 1.3 \times 10^7\,{\rm s^{-1}}$,[30,49] dephasing rate $\gamma_{\rm ph} = 3.0 \times 10^6\,{\rm s^{-1}}$, electric and magnetic Rabi frequencies of the applied light fields $\Omega_{\rm B} = 3.3 \times 10^4\,{\rm s^{-1}}$, $\Omega_{\rm E} = 9.0 \times 10^6\,{\rm s^{-1}}$,[50] and atomic concentration $N = 7.0 \times 10^{25}\,{\rm m^{-3}}$. As assumed above, the electric and magnetic frequency detunings $\Delta_{\rm B} = \Delta_{\rm E}$ (expressed by Δ in the numerical example). The dispersive behaviours of the electric permittivity and magnetic permeability of the present atomic vapour are plotted in Figs.2 and 3.

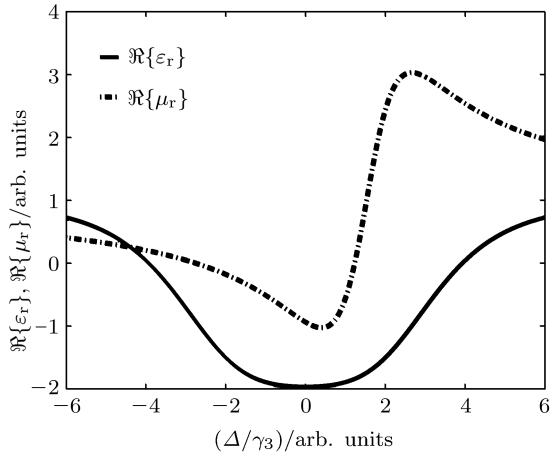


Fig.2. The real parts of the relative permittivity and permeability of the atomic vapour as the probe frequency detuning varies. The solid and dash-dot lines represent the electric permittivity and magnetic permeability, respectively.

It follows from Fig.2 that the relative permittivity has a negative real part $[-2.0, 0]$ in the probe frequency detuning range $[-4.0\gamma_3, +4.0\gamma_3]$, and the relative permeability has a negative real part $[-1.0, 0]$ in the probe frequency detuning range $[-2.5\gamma_3, +1.0\gamma_3]$. Therefore, the proper frequency detuning range, where the electric permittivity and the magnetic permeability simultaneously have negative real parts, is $[-2.5\gamma_3, +1.0\gamma_3]$. In this frequency band (i.e., negatively-refracting frequency range), the above atomic vapour would become an isotropic left-handed medium.

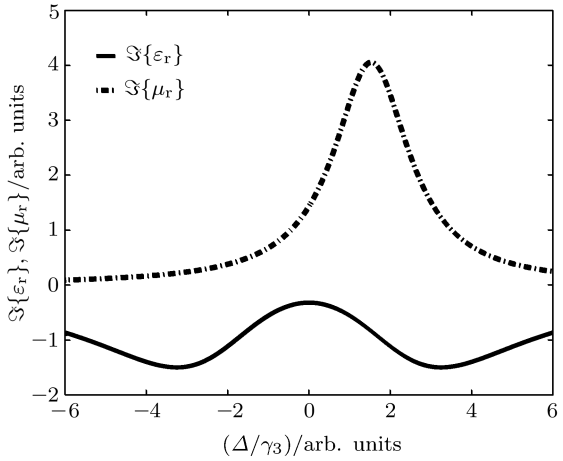


Fig.3. The imaginary parts of the relative permittivity and permeability of the atomic vapour as the probe frequency detuning varies. The solid and dash-dot lines represent the electric permittivity and magnetic permeability, respectively. The typical parameters for the atomic system are the same as in Fig.2.

Since the atomic phase coherence (between levels $|1\rangle$ and $|2\rangle$) plays a key role in achieving simultaneously negative permittivity and permeability of the atomic vapour, we plot the evolution of the real and imaginary parts of the off-diagonal density matrix elements ρ_{12} versus the frequency detuning (see Fig.4). It follows from Fig.4 that the real part of ρ_{12} at the resonant frequency is taken to be a minimum value and the corresponding imaginary part is zero.

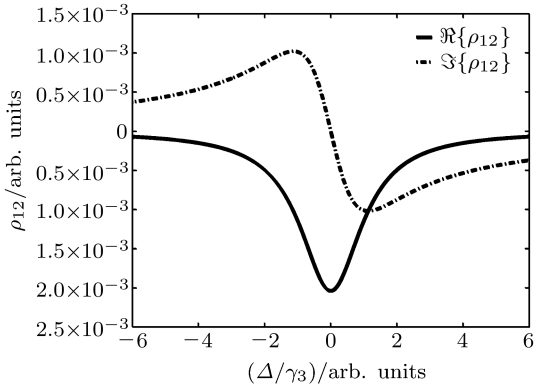


Fig.4. The real and imaginary parts of the off-diagonal density matrix element ρ_{12} versus the frequency detuning. The solid and dash-dot lines represent the real and imaginary parts of ρ_{12}, respectively.

The dispersive behaviour of the real and imaginary parts of the relative refractive index, $n_{\rm r}$, of

the coherent atomic vapour in a narrow negatively-refracting frequency detuning band is plotted in Fig.5. It can be seen that the relative refractive index has a negative real part $[-2.2, -0.7]$ in this frequency detuning band. Though the electric permittivity has a negative imaginary part (see the solid line in Fig.3), the imaginary part of the refractive index of the left-handed atomic vapour is positive (see Fig.5). It can thus be concluded that the present left-handed atomic vapour that exhibits a negative refractive index is an absorptive medium. The dimensionless absorption coefficient α (defined as $2\pi\Im\{n_{\rm r}\}/|\Re\{n_{\rm r}\}|$, i.e., the loss in the medium per wavelength) is also plotted in Fig.5 as a function of the frequency detuning of the probe light. From this curve one sees that the absorption increases as the frequency detuning increases in the present frequency range.

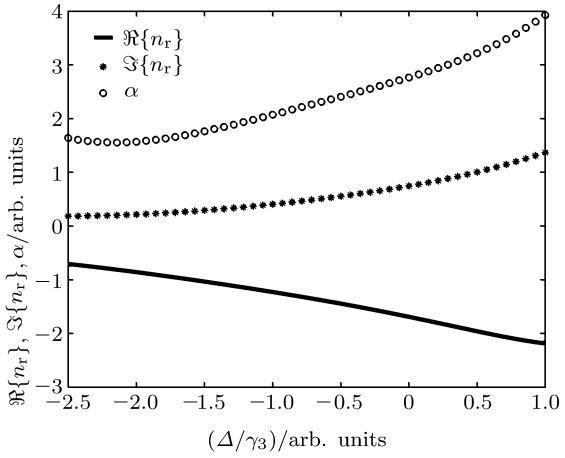


Fig.5. The real and imaginary parts of the relative refractive index and the dimensionless absorption coefficient of the left-handed atomic vapour as the probe frequency detuning varies in a narrow frequency band, where the refractive index of the atomic vapour has a negative real part. The solid and asterisk lines represent the real and imaginary parts of the refractive index, respectively, and the circle line stands for the absorption coefficient. The typical parameters for the atomic system are the same as in Fig.2.

The electric- and magnetic-dipole moments as well as the decay rates in the numerical example are the typical parameters for the multi-level metallic atomic system. At present, these parameters do not correspond to an existing atom. But the general features of the required atomic systems can be involved in the chosen typical parameters. The most principal feature of the present three-level atomic system is such that the energy separation ω_{21} should be much less than the transition frequencies ω_{31} and ω_{32} in order that the electric- and magnetic-dipole allowed transitions can simultaneously be driven by the applied light field. As an illustrative example, the atomic system $\{|1\rangle, |2\rangle, |3\rangle\}$ that has such a configuration could be $\{3d^8\ ^3F_4, 3d^9\ ^3D_3, 3d^8\ ^5F_5\}$ in neutral Nickel atom and $\{4f(5d)^1G_4, 4f(5d)^3F_2, 4f(5d^2)^5G_2\}$ in neutral Cerium atom, where ω_{21} is about 0.8% and 2.5% of ω_{31} (and ω_{32}), respectively.[51,52] The selection rules in Nickel atom for the electric-dipole transition between levels $3d^9\ ^3D_3$ and $3d^8\ ^5F_5$ and the magnetic-dipole transition between levels $3d^8\ ^3F_4$ and $3d^8\ ^5F_5$ are $\Delta l = 1, \Delta m = 1$ and $\Delta l = 0, \Delta m = 1$, respectively, and the selection rules in Cerium atom for the electric-dipole transition between levels $4f(5d)^3F_2$ and $4f(5d^2)^5G_2$ and the magnetic-dipole transition between levels $4f(5d)^1G_4$ and $4f(5d^2)^5G_2$ are $\Delta l = 1, \Delta m = 0$ and $\Delta l = 0, \Delta m = -1$, respectively.

It should be noted that the left-handed vapour medium produced is both isotropic and homogeneous due to the atomic-scale microscopic structure units, and the negative refraction could occur in the visible and infrared frequency bands since the typical transition frequency of atoms is in the frequency range 10^{13}–$10^{15}\,{\rm s}^{-1}$.

5. Discussions

It was shown in the preceding section that the level of absorption in the present illustrative example is high (e.g., see the absorption coefficient in Fig.5). Generally speaking, the absorption coefficient (loss) of the negative index media designed and fabricated by various methods such as metamaterials[21] and chiral nihility[40] is often very high. As the negative permittivity and permeability often occur near resonance, the realization of the negative permittivity and permeability with zero imaginary parts near resonance is in general impossible (partly because of the Kramers–Kronig causality relation). Though the negative refraction with a very small absorption coefficient in a broad frequency range does not exist, nearly lossless left-handed medium at certain frequencies (or very narrow frequency regions) really exists under certain conditions. If, for example, we can choose a typical three-level atomic system with the proper spontaneous decay rates such as $\gamma_2 = 1.0 \times 10^7\,{\rm s}^{-1}$, $\gamma_3 = 5.0 \times 10^7\,{\rm s}^{-1}$ (the spontaneous decay rates with such orders of magnitude can be found in Rb atoms), then we can achieve a left-handed vapour with a neg-

ligibly small absorption near the detuning frequency $-0.5\gamma_3$ (see Fig.6). It can be seen that in the detuning frequency range $[-0.5\gamma_3, +0.2\gamma_3]$, the electric permittivity and the magnetic permeability simultaneously have negative real parts, but only in a very narrow band (near the detuning frequency $-0.5\gamma_3$) can the vapour medium exhibit a very small (nearly zero) loss. Though the satisfactory negatively refracting frequency band is very narrow, the produced left-handed vapour medium is still of physical interest, since the left-handed vapour medium at such a working frequency can exhibit all the intriguing optical and electromagnetic properties and effects.

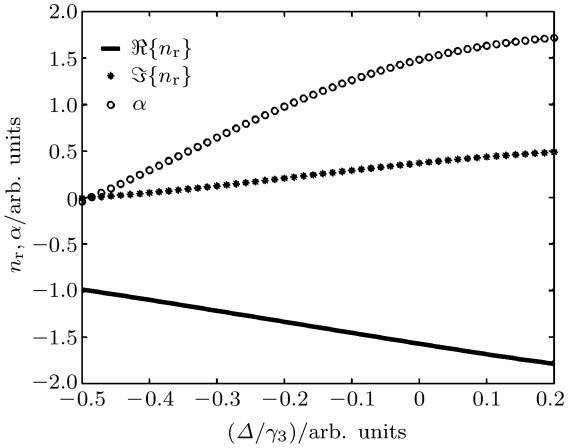


Fig.6. The real and imaginary parts of the relative refractive index and the dimensionless absorption coefficient of the left-handed atomic vapour with a small loss in a very narrow frequency band (close to the detuning frequency $-0.5\gamma_3$). The solid and asterisk lines represent the real and imaginary parts of the refractive index, respectively, and the circle line stands for the absorption coefficient.

Note that the produced left-handed vapour exhibits a negative absorption in the electric field (e.g., see Fig.3). However, the imaginary part of the negative refractive index is still positive (e.g., see Figs.5 and 6). It is the refractive index that is most crucial to the behaviour of the wave propagation. This means that the left-handed medium in the negative-index detuning frequency regions exhibits positive absorption (rather than the negative absorption) at the applied light field.

The left-handed medium that has negative permittivity and permeability (and hence the negative index of refraction) can cause strong anti-shielding effects, which enable the probe light in the left-handed medium to exhibit many physically interesting optical and electromagnetic properties and effects. In what follows, we discuss the spatial properties (including the polarization and helicity of the probe light) in the left-handed atomic vapour from the following two aspects:

i) **Orientation of electric and magnetic dipole moments matrix elements**. Here we consider the problem whether the orthogonal electric and magnetic dipoles of the atom could be induced by the light field with any given orientations of $\boldsymbol{E}$ and $\boldsymbol{B}$. If this is true, the optical 'constants' such as the atomic microscopic polarizabilities (and hence the permittivity, permeability and refractive index) are independent of the polarizations of the applied light field, and so the vapour produced could be viewed as an isotropic left-handed medium. As is known, the magnetic field of a plane electromagnetic wave propagating inside an isotropic and homogeneous medium is perpendicular to its electric field. If the induced magnetic dipole of the atomic system is also perpendicular to the induced electric dipole, then we can conclude that such an atomic system is an isotropic one for the propagating wave. Indeed, the angular momentum $(\boldsymbol{r} \times \boldsymbol{p})$ of the electron is always orthogonal to the electric dipole $(e\boldsymbol{r})$, i.e., $e\boldsymbol{r}\cdot(\boldsymbol{r}\times\boldsymbol{p}) \equiv 0$. This means that the induced electric dipole $\wp_{23}$ and the induced magnetic dipole $\boldsymbol{m}_{13}$ are truly perpendicular to each other. Then the produced medium via the quantum optical mechanism (atomic phase coherence) suggested in the present paper can be isotropically left-handed for both linear and circular polarizations.

ii) **Photon helicity reversal** Based on the scenario of controllable manipulation of negative refractive index, we suggest a new interesting application, i.e., the coherently controllable photon helicity reversal. Here we discuss briefly such an effect: the photon helicity is defined as the projection of the wave vector on photon's spin, and so the photon helicity would be inverted in a negatively refracting medium because of the reversal of the wave vector. Note that the energy level shift of the atomic system can be caused by an external static magnetic field via the Zeeman effect.[53] Then the refractive index of the atomic vapour at the probe frequency can therefore be manipulated by the external field, and the real part of the refractive index could be negative or positive by choosing some proper strengths of the external magnetic field. This, therefore, means that the photon helicity reversal as well as the transition between photon polarization states could possibly be controlled by the

external magnetic field. We believe the scheme presented here may have potential significance in both pure and applied physics. For example, the photon helicity reversal that accompanies the transition between the polarization states, which is exactly analogous to the transition operation between bits (0 and 1) in digital circuit, might be employed to the areas of information science and technology.

6. Concluding remarks

It was shown that the simultaneously negative permittivity and permeability could be realized in a coherent atomic vapour. In general, the strong electric response and negative permittivity can be easily realized in atomic vapours. But the large contribution of the magnetic response to the atomic vapour is rarely seen in the atomic systems. Since the dimensionless ratio of the magnetic dipole moment to the electric dipole moment is simply an order of magnitude of the small fine structure constant (1/137), the magnetic-dipole transition can often be neglected for the atomic vapours. In order to obtain a negative permeability of an atomic vapour, we choose an atomic configuration that can dramatically enhance the off-diagonal density matrix element (related closely to the contribution of magnetic-dipole allowed transition) via atomic phase coherence. Thus the contribution of the magnetic response to the atomic vapour should also be taken into account, and the enhanced magnetic response could be promising for realizing a negative permeability under certain conditions (related to the electric and magnetic dipole matrix elements, spontaneous decay rates, atomic concentration of the vapour as well as Rabi frequencies of the applied light fields).

The present scheme (within the framework of quantum optics) can obtain the visible and infrared frequency bands, in which the permittivity and the permeability of the atomic vapour are both negative. Because of the optical nonlinearity shown by the coherent atomic vapour, one can achieve as broad a frequency band as possible, where both the permittivity and the permeability have negative real parts, by choosing a proper probe field intensity. For the present, physicists' control over the behaviour of photons has spread to include the photon wave vector,[54] phase[55] and intensity of the light wave.[56] Our technology would benefit much if we could engineer all the degrees of freedom of photons.[55] By taking advantage of the optical nonlinearity in the left-handed atomic vapour, the controllable helicity reversal could play a role analogous to the digital-circuit operation process (the transition between bits 0 and 1), and would be potentially applicable to the area of information technology.

The scheme presented in this paper was treated within the framework of semiclassical theory. If the applied probe light is very weak, the full quantum treatment for the wave propagation in the left-handed atomic vapour would be of physical interest (partly because new peculiar quantum modification to the optical properties of the coherent atomic vapour would arise). As the left-handed atomic vapour has its characteristic features in optical and electromagnetic properties and potential applications in various areas, we think the present scheme for realizing simultaneously negative permittivity and permeability deserves further investigation both theoretical and experimental.

References

[1] Shen J Q 2005 *Phys. Lett.* A **344** 144

[2] Bjorken J D and Drell S D 1965 *Relativistic Quantum Fields* (New York: McGraw-Hill)

[3] Veselago V G 1968 *Sov. Phys. Usp.* **10** 509

[4] Gerardin J and Lakhtakia A 2002 *Phys. Lett.* A **301** 377

[5] Lindell I V, Tretyakov S A, Nikoskinen K I and Ilvonen S 2001 *Ser. Electromagn. Lab. Rept. Ser.* (Finland: Helsinki University of Technology) p366

[6] Ziolkowski R W 2001 *Phys. Rev.* E **64** 056625

[7] Depine R A and Lakhtakia A 2004 *Phys. Rev.* B **69** 057602

[8] Depine R A, Lakhtakia A and Smith D R 2005 *Phys. Lett.* A **337** 155

[9] Klimov V V 2002 *Opt. Commun.* **211** 183

[10] Shen J Q 2006 *Phys. Rev.* B **73** 045113
Shen J Q and Zhuang F 2004 *Acta Phys. Sin.* **53** 2000 (in Chinese)

[11] Zhang G M, Peng J C, Jian Z J and Huang X Y 2006 *Acta Phys. Sin.* **55** 1846 (in Chinese)

[12] Shi H Y, Jiang Y Y, Sun X D, Guo R H and Zhao Y P 2005 *Chin. Phys.* **14** 1571

[13] Pendry J B 2001 *Phys. Rev. Lett.* **85** 3966

[14] Shen J Q 2004 *Ann. Phys.* (Leipzig) **13** 335

[15] Lakhtakia A 2004 *Int. J. Electron. Commun.* (AEÜ) **58** 229

[16] Lakhtakia A 2002 *Adv. Matter.* **14** 447

[17] Lakhtakia A 2003 *Opt. Express* **11** 716

[18] Engheta N 2002 *IEEE Antennas Wireless Propagat. Lett.* **1** 10
Shen L, He S and Xiao S 2004 *Phys. Rev.* B **69** 115111

[19] Jiang T, Chen Y and Feng Y J 2006 *Chin. Phys.* **15** 1154
[20] Pendry J B, Holden A J, Robbins D J and Stewart W J 1998 *J. Phys. Condens. Matter* **10** 4785
[21] Pendry J B, Holden A J, Stewart W J and Youngs I 1996 *Phys. Rev. Lett.* **76** 4773
Pendry J B, Holden A J, Robbins D J and Stewart W J 1999 *IEEE Trans. Microwave Theory Tech.* **47** 2075
[22] Maslovski S I, Tretyakov S A and Belov P A 2001 *Inc. Microwave Opt. Tech. Lett.* **35** 47
[23] Simovski C R and He S 2003 *Phys. Lett.* A **311** 254
[24] Shelby R A, Smith D R and Schultz S 2001 *Science* **292** 77
[25] Mackay T G and Lakhtakia A 2004 *Phys. Rev.* E **69** 026602
[26] Parimi P, Lu W T, Vodo P, Sokoloff J and Sridhar S 2004 *Phys. Rev. Lett.* **92** 127401
[27] Zhu S Y and Scully M O 1996 *Phys. Rev. Lett.* **76** 388
Zhu S Y, Scully M O, Fearn H and Narducci L M 1992 *Z. Phys.* D **22** 483
Yao J Q, Wu H B and Wang H 2003 *Acta Sin. Quantum Opt.* **9** 121 (in Chinese)
[28] Harris S E 1997 *Phys. Today* **50**(7) 36
[29] Hong T 2003 *Phys. Rev. Lett.* **90** 183901
Imamoğlu A, Schmidt H, Woods G and Deutsch M 1997 *Phys. Rev. Lett.* **79** 1467
[30] Li Y Q and Xiao M 1995 *Opt. Lett.* **20** 1489
[31] Shor P W 1995 *Phys. Rev.* A **52** R2493
Zhang D Y 2002 *Acta Phys. Sin.* **51** 532 (in Chinese)
[32] Su X M, Zhuo Z C, Wang L J and Gao J Y 2002 *Chin. Phys.* **11** 35
[33] Liu X and Fang M F 2002 *Chin. Phys.* **11** 926
[34] Smith D R, Padilla W J, Vier D C, Nemat-Nasser S C and Schultz S 2000 *Phys. Rev. Lett.* **84** 4184
[35] Shelby R A, Smith D R, Nemat-Nasser S C and Schultz S 2001 *Appl. Phys. Lett.* **78** 489
[36] Hu L B and Chui S T 2002 *Phys. Rev.* B **66** 085108
[37] He S, Ruan Z C, Chen L and Shen J Q 2004 *Phys. Rev.* B **70** 115113
[38] Berrier A, Mulot M, Swillo M, Qiu M, Thylén L, Talneau A and Anand S 2004 *Phys. Rev. Lett.* **93** 073902
[39] Eleftheriades G V, Iyer A K and Kremer P C 2002 *IEEE Trans. Microwave Theory Tech.* **50** 2702
[40] Pendry J 2004 *Science* **306** 1353
[41] Oktel M Ö and Müstecaphoğlu Ö E 2004 *Phys. Rev.* A **70** 053806
Shen J Q, Ruan Z C and He S 2004 *J. Zhejiang Univ. Science* **5** 1322
[42] Cook D M 1975 *The Theory of the Electromagnetic Field* (Englewood Cliffs, NJ: Prentice-Hall) Chap. 11
[43] Jackson J D 1998 *Classical Electrodynamics* 3rd Edn (New York: Wiley) Chap. 4
[44] Guidice E D, Preparata G and Fleischmann M 2000 *J. Electro. Chem.* **482** 110
Sivasubramanian S, Widom A and Srivastava Y N 2002 *Mod. Phys. Lett.* B **16** 1201
[45] Ballard A, Bonin K and Louderback J 2000 *J. Chem. Phys.* **113** 5732
Eklund P C, Rao A M, Wang Y, Zhou P, Wang K A, Holden J M, Dresselhaus M S and Dresselhaus G 1995 *Thin Solid Films* **257** 211
Allen P B 2003 cond-mat/0307209
[46] Agarwal G S and Boyd R W 1999 *Phys. Rev.* A **60** 2681
[47] Bowden C M and Dowling J P 1993 *Phys. Rev.* A **47** 1247
[48] Siegman A E 1986 *Lasers* (California: University Science, Milly Valley), Sec. 31
[49] Yan M, Rickey E G and Zhu Y 2001 *Phys. Rev.* A **64** 041801
[50] Schmidt H and Imamoğlu A 1996 *Opt. Lett.* **21** 1936
[51] Litzén U, Brault J W and Thorne A P 1993 *Phys. Scr.* **47** 628
[52] Martin W C, Zalubas R and Hagan L 1978 *Atomic Energy Levels—The Rare Earth Elements*, Natl. Stand. Ref. Data Ser., Natl. Bur. Stand. (U.S.) **60**
[53] Fleischhauer M and Scully M O 1994 *Phys. Rev.* A **49** 1973
[54] Painter O, Srinivasan K and O'Brien J D 2001 *J. Opt.* A: *Pure Appl. Opt.* **3** 162
[55] Duan L M, Ciac J I and Zoller P 2001 *Science* **292** 1695
[56] Zhuang F and Shen J Q 2005 *Acta Phys. Sin.* **54** 955 (in Chinese)