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Atom–photon entanglement in the system with competing k-photon and l-photon transitions*

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We have investigated the evolution of the atomic quantum entropy and the entanglement of atom–photon in the system with competing k-photon and l-photon transitions by means of fully quantum theory, and examined the effects of competing photon numbers (k and l), the relative coupling strength between the atom and the two-mode field (λ/g), and the initial photon number of the field on the atomic quantum entropy and the entanglement of atom–photon. The results show that the multiphoton competing transitions or the large relative coupling strength can lead to the strong entanglement between atoms and photons. The maximal atom–photon entanglement can be prepared via the appropriate selection of system parameters and interaction time.

Keywords: mode–mode competition system, quantum entropy, entanglement
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1. Introduction

Quantum entanglement (QE), which is considered nowadays as a main physical resource of quantum information processing, has been discovered as a physical phenomenon representing 'the characteristic trait of quantum mechanics' (Schrödinger 1935). Quantum entangled states are extensively used in quantum computation and quantum communication,[1−3] quantum cryptography,[4] quantum teleportation,[5−8] and quantum dense coding,[9] etc. For future applications like quantum networks or the quantum repeater[10] it is mandatory to achieve entanglement between different quantum objects like atoms and photons, which forms the interface between atomic quantum memories and photonic quantum communication channels.[11,12]

One of the key problems in QE study is the generation[13−19] and quantification[20,21] of quantum entangled states. For a pure state of many qubits, QE, which is quantified by the von Neumann entropy of the reduced density matrix, is a measure of how a subsystem is correlated with the rest of the system. Recently, the considerable interest has been aroused in the properties of the evolution of the field (atomic) quantum entropy and the entanglement between atom and field in various generalized Jaynes–Cummings models.[22−36] It is noticeable that in such models it is assumed that a given mode acts only on one optical transition, i.e. the two modes do not act simultaneously on one transition. Semiclassically it is well known that if an atom interacts with different fields, several nonlinear processes can take place at the same time, and a vast literature has dealt with the interaction of a bichromatic field with a two-level atom. The bichromatic fields give rise to many new dynamical aspects as a result of mode–mode competition,[37−39] such as Stokes and anti-Stokes processes.[40] Jyotsna and Agarwal[41] showed that the quantum effects were rather dominant in the mode–mode competition systems. The other authors have studied the atomic dynamical properties and the statistical properties of the photon in the mode–mode competition model.[42−45] However, the quantum entropy property and entanglement in this system is little reported. In this paper, we shall derive a general calculation formalism of the atomic quantum entropy in a system with competing k-photon and l-photon processes ($k \neq l, k > 1, l \geq 1$), and investigate the influences of competing photon number (k and l), the relative competing strength between the atom and the two-mode field (λ/g), and the initial photon number of the field on the atomic quantum entropy and the atom–photon entanglement.

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2. Model and the reduced density matrix of the atom

We consider the interaction between a two-level atom with states $|+\rangle$ and $|-\rangle$ separated by a frequency ω_0, and a two-mode field with frequencies ω_a and ω_b. We assume that one of the modes, a mode, interacts with the atom through a k-photon process and the other mode, b mode, interacts via an l-photon process. For simplicity, we further assume that two modes are in perfect resonance with the atomic transition, i.e. $\omega_0 = k\omega_a = l\omega_b$. In the interaction picture and in the rotating wave approximation, Hamiltonian of the system can be written as[41]

$$H_{\rm I} = \lambda(S_+ a^k + S_- a^{\dagger k}) + g(S_+ b^l + S_- b^{\dagger l}), \qquad (1)$$

where the field modes a and b are described by the annihilation and creation operators a, $a^\dagger$ and b, $b^\dagger$, respectively. $S_\pm$ are the atomic pseudospin operators. λ and g are the coupling constants for the k- and l-photon processes, respectively. We assume that the a mode is in a Fock state $|kn\rangle$, the b mode in vacuum state $|0\rangle$ and the atom in the ground state $|-\rangle$ at time $t = 0$. Thus the initial state of the system is

$$|\,\Psi(0)\rangle = |kn, 0, -\rangle. \qquad (2)$$

The state vector at any time for each value of n is a linear combination of all the possible states which the system can exist in. The number of such possible states is $(2n + 1)$. The state vector of the system at time t for a particular n can be expressed in terms of a complete set of possible states in the interaction picture as

$$|\,\Psi_{fa}(t)\rangle = \sum_{m=0}^{n} A_m(t)|k(n-m), lm, -\rangle + \sum_{m=1}^{n} B_m(t)|k(n-m), l(m-1), +\rangle. \qquad (3)$$

Through solving the Schrödinger equation

$$\mathrm{i}\frac{\partial|\,\Psi_{fa}(t)\rangle}{\partial t} = H_{\rm I}|\,\Psi_{fa}(t)\rangle, \qquad (4)$$

we can obtain the equations for the complex amplitudes

$$\begin{aligned}\dot{A}_p(t) = \; & -\mathrm{i}\lambda B_{p+1}(t)\Big\{k(n-p)\big[k(n-p)-1\big]\big[k(n-p)-2\big]\cdots\big[k(n-p)-(k-1)\big]\Big\}^{1/2} \\ & -\mathrm{i}gB_p(t)\Big\{lp(lp-1)(lp-2)\cdots\big[lp-(l-1)\big]\Big\}^{1/2}, \end{aligned} \qquad (5)$$

$$\begin{aligned}\dot{B}_p(t) = \; & -\mathrm{i}\lambda A_{p-1}(t)\Big\{\big[k(n-p)+1\big]\big[k(n-p)+2\big]\cdots\big[k(n-p)+k\big]\Big\}^{1/2} \\ & -\mathrm{i}gA_p(t)\Big\{lp(lp-1)(lp-2)\cdots\big[lp-(l-1)\big]\Big\}^{1/2}. \end{aligned} \qquad (6)$$

Here p varies from 0 to n. At time $t = 0$, $A_0 = 1$ and $A_p = 0$, $B_p = 0$ where $p = 1, 2, \cdots, n$. It is difficult to obtain the analytical solutions to the coupled equations (5) and (6), so we use the fourth-order Runge–Kutta method to solve these equations with the initial conditions.

3. Quantum entropy and atom–photon entanglement

Since we have assumed that the atom and the two-mode field are initially in a disentangled pure state, the considered system is a bicomponent quantum system in a pure state. For such a system, Phoenix and Knight[46,47] have shown that entropy is a very sensitive operational measure of the purity of the quantum state because it automatically includes all moments of the density operator. The time evolution of the field (atomic) quantum entropy reflects the time evolution of the degree of atom–photon entanglement. The higher the entropy, the greater the entanglement.

The reduced density operator of the atom for the

system can be written as

$$\rho_a(t) = \mathrm{Tr}_f|\Psi_{fa}(t)\rangle\langle\Psi_{fa}(t)| = \begin{pmatrix} \sum_{m=1}^{n}|B_m(t)|^2 & 0 \\ 0 & \sum_{m=0}^{n}|A_m(t)|^2 \end{pmatrix}. \quad (7)$$

The atomic quantum entropy $S_a(t)$ can be written as

$$S_a(t) = -\sum_{m=1}^{n}|B_m(t)|^2 ln\left(\sum_{m=1}^{n}|B_m(t)|^2\right) - \sum_{m=0}^{n}|A_m(t)|^2 ln\left(\sum_{m=0}^{n}|A_m(t)|^2\right). \quad (8)$$

In what follows, we shall explore the properties of the atomic quantum entropy and atom–photon entanglement.

Firstly, we study the influence of the photon number of the two competing transitions on the atomic quantum entropy. Figure 1 shows the quantum entropy of the atom as a function of time for several values of k and l with $n = 4$ and $\lambda/g = 0.01$. It is obvious that $S_a(t)$ evolves periodically when the k-photon transition is much weaker than the l-photon transition. Figure 1(a) shows that in the case of competing two-photon and one-photon transitions system ($k = 2, l = 1$), the atomic entropy evolves in periods of π/g; when $t = n\pi/g \quad (n = 0, 1, 2, \cdots)$, $S_a(t)$ achieves zero values, that is, the atom and the two-mode field are completely disentangled, while when $t = (n + 0.5)\pi/g \quad (n = 0, 1, 2, \cdots)$, $S_a(t)$ evolves to their maximum values that are all 0.04, which is small all the same. In the competing three-photon and one-photon transitions system, i.e. $k = 3, \quad l = 1$, the behaviour of $S_a(t)$ is very similar to that of the two-photon and one-photon transitions system as shown in Fig.1(b), but the amplitude of $S_a(t)$ is extremely larger than that in the case $k = 2, \quad l = 1$, namely, the entanglement of atom–photon becomes stronger. The time evolution of $S_a(t)$ in the competing four-photon and two-photon transitions ($k = 4, \quad l = 2$) system is displayed in Fig.1(c).

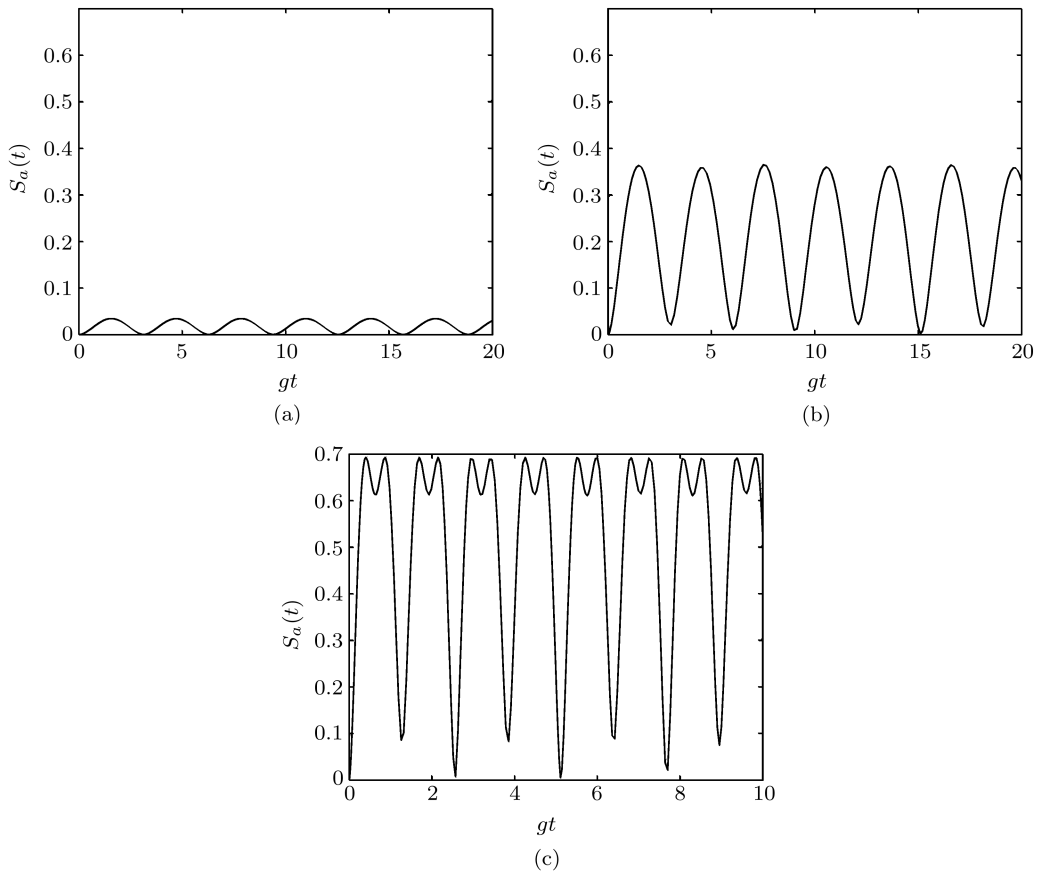


Fig.1. The time evolution of the atomic quantum entropy $S_a(t)$ for $\lambda/g = 0.01, n = 4$, (a) $k = 2, \quad l = 1$, (b) $k = 3, \quad l = 1$, (c) $k = 4, \quad l = 2$.

We can see that there exhibits the maximal entanglement of atom–photon periodically, which is crucial for the many applications of long-range quantum communication. Comparing Figs.1(a) and 1(b) with 1(c), one can find that the multiphoton competing transitions give rise to a larger amplitude on the evolution of the atomic quantum entropy, which means the stronger entanglement of atom–photon. The amplitude modulation is dependent on the competing photon numbers k and l.

Next, we shall examine the influence of the relative competing strength between the atom and the two-mode field on the atomic quantum entropy, which is shown in Figs.1(a), 2(a) and 2(b) for $\lambda/g = 0.01,\ 0.1,\ 1$ with $k = 2$, and $l = 1$. A comparison between Fig.1(a) and Fig.2(a) shows that the increase of the relative competing strength λ/g results in an increase in the amplitude of the atomic quantum entropy, i.e. the larger the λ/g, the stronger the entanglement of atom–photon. When the relative competing strength between the atom and the two-mode field is equivalent, say, $\lambda/g = 1$ (in Fig.2(b)), we can see that $S_a(t)$ exhibits high-frequency periodical oscillations with higher amplitudes.

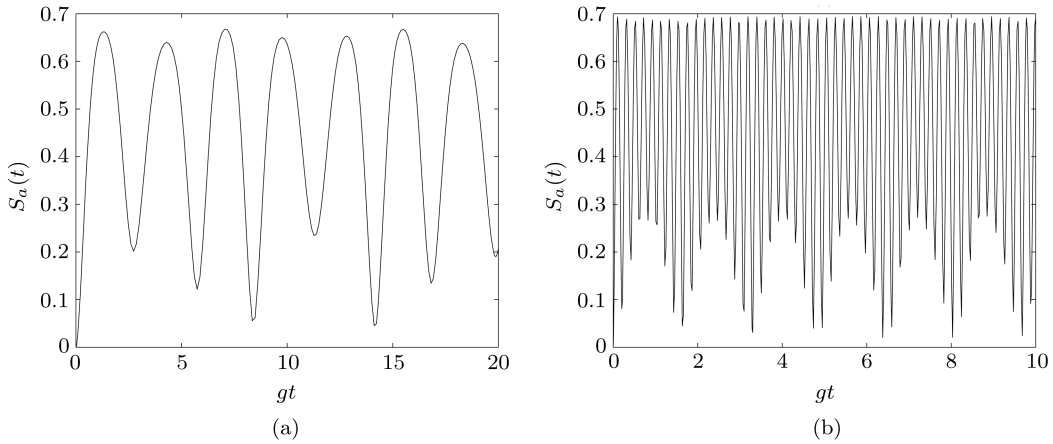


Fig.2. The time evolution of the atomic quantum entropy $S_a(t)$ for $k = 2,\ \ l = 1,\ \ n = 4$, (a) $\lambda/g = 0.1$, (b) $\lambda/g = 1$.

Finally, we investigate $S_a(t)$ with different initial photon numbers of a mode field and plot the time evolution of $S_a(t)$ for $n = 4$, $n = 22$ and $n = 33$ with $k = 3,\ l = 1$ and $\lambda/g = 0.01$ in Figs.1(b), 3(a) and 3(b). When the initial photon number of the a-mode field is small, take $n = 4$ for example (in Fig.1(b)), the atom and the two-mode field are almost disentangled at time $t \approx n\pi$ $(n = 0, 1, 2\cdots)$. When n is increased to 22, $S_a(t)$ reaches its maximum value in a very short while, and then remains maximum as time increases(see Fig.3(a)). In this case, atom and photon almost retain a strong entanglement in the time evolution process. In Fig.3(b), the representative case, $n = 33$, is considered.

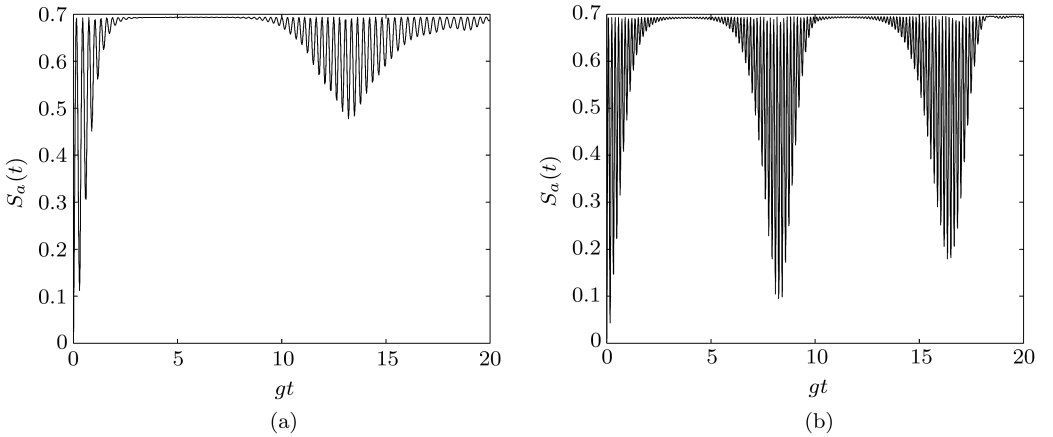


Fig.3. The time evolution of the atomic quantum entropy $S_a(t)$ for $k = 3,\ \ l = 1,\ \ \lambda/g = 0.01$, (a) $n = 22$, (b) $n = 33$.

It is observed that the maximum atomic entropy is achieved for the evolution of $S_a(t)$ during the first stage of the time evolution, then $S_a(t)$ evolves to minimum values, the atom and field state reach the almost pure state in half of the atomic inversion revival time, which is consistent with the conclusion given by Jyotsna and Agarwal.[41] As time goes on, the values of the minimum atomic entropy increase.

4. Conclusion

We have studied the atomic quantum entropy and the entanglement of atom–photon in a mode–mode competing system. We find that the quantum entropy of the atom and the entanglement of atom–photon are highly dependent on the competing photon number, the relative coupling strength between the atom and the two-mode field and the initial state of the system. Our results are as follows. (i)If the relative coupling strength λ/g is small, that is, the k-photon transition is much weaker than the l-photon transition, $S_a(t)$ evolves periodically. (ii)The larger the number of the competing photons or the relative competing strength is, the stronger the entanglement of atom–photon is. (iii)The time of maximum entropy is shortened with the increase of the initial photon number of the a-mode field. (iv)The maximal atom–photon entanglement can be prepared via the appropriate selection of system parameters and interaction time.

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