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Generation of four-particle GHZ states in bimodal cavity QED*

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We propose a scheme for preparing four-particle Greenberger–Horne–Zeilinger states using two identical bimodal cavities, each supports two modes with different frequencies. This scheme is an alternative to another published work [Christopher C Gerry 1996 *Phys. Rev.* A **53** 4591]. Comparisons between them are discussed. The fidelity and the probability of success influenced by cavity decay for the generated states are also considered.

Keywords: four-particle GHZ states, bimodal cavity QED
PACC: 4250

1. Introduction

Entanglement, a fundamental feature of quantum mechanics, has been recently seen as a useful resource for many researches in quantum information processing.[1] In particular, certain entangled states have been identified and thus can be directly used for many such researches. Foremost among these are the so-called Bell states (EPR states) and Greenberger–Horne–Zeilinger states (GHZ states),[2,3] which are not only relevant to the study of foundations of quantum mechanics, but also the fundamental ingredient of various quantum information protocols.[1] As shown by Bell,[2] two particles in Bell states possess correlations that are incompatible with a local realistic theory,[4] no matter what is the separation between them. Greenberger–Horne–Zeilinger showed that three or N-particle ($N > 3$) GHZ states possess correlations that violate the local realistic theory much stronger than Bell states.[3] Thus, how to demonstrate such correlations becomes important, since it would have profound significance for our understanding of the physics world. Therefore, it is important to be able to generate such states, especially N-particle GHZ states in practical experiment. Three-photon,[5] four photon,[6] and five-photon[7] GHZ states have been realized and used to verify quantum nonlocality. Experimental realization of three-particle GHZ entangled states in ion traps has also been reported.[8]

Cavity quantum electrodynamics (QED), with Rydberg atoms crossing superconducting microwave cavities, provides one of the candidates for generating entangled states and quantum information processing.[9] Experimental realizations of two-atom Einstein–Podolsky–Rosen (EPR) states through the resonant atom–cavity interaction[10] and two-atom collision[11] have been reported. Based on the cavity QED technique, a variety of schemes have been proposed for generating GHZ entangled states involving more particles.[12−15] Gerry has discussed a method for producing such entangled states, through the dispersive atom–cavity interaction.[12] In the scheme, the cavity is initially prepared in a coherent state. In order to generate a three-atom GHZ state one should collapse the cavity field into one of three coherent states, of which the amplitudes should be large enough such that they are orthogonal and can be distinguished. Zheng has proposed an alternative scheme for realizing multi-atom GHZ states based on the dispersive atom–cavity interaction.[13] In the scheme, the cavity must be also prepared in a coherent state, of which the amplitude can be arbitrary; then an additional atom can be sent to probe the cavity field, and then detected directly to obtain the GHZ state. Cirac and Zoller have proposed a scheme for generating multi-atom GHZ states based on the resonant atom–cavity interaction.[14] The method requires that the cavity

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field must be firstly prepared in the superposition of $|0\rangle$ and $|n\rangle$ states, which might be a great challenge as it requires to send a sequence of atoms through the cavity to prepare such states. Based on the Raman atom–cavity field interaction, Zheng has proposed another scheme for generating such GHZ states.[15] The scheme requires that the cavity field must be initially prepared in the superposition of zero- and one-photon states, and it requires an additional resonant atom to absorb the photon in the cavity and finally to be detected. Other cavity QED schemes have been proposed for the generation of multi-atom or multi-cavity GHZ states.[16−21]

Based on the protocol in Ref.[22], an experiment has been performed by Haroche's group to prepare a three-particle GHZ states (composed of two atoms and zero- or one-photon of a single-mode cavity).[23] However, the fidelity of the engineered state needs to be largely improved in order to be useful for testing quantum nonlocality and quantum information processing.

All the above cavity QED schemes use the cavities that support only one mode. Gerry has proposed a distinctive scheme for preparation of four-particle GHZ states,[24] by employing two cavities both supporting two modes with different frequencies (also known in the literature as *bimodal cavity*). In the scheme, a three-state atom is first sent through two identical bimodal cavities sequentially, the frequencies for the atomic transition and the cavity modes are chosen in such a way that the atom can make two-photon transition in each of the two cavities (i.e. the middle state of the three-state atom can be adiabatically eliminated as the transition frequencies for two pairs of the neighbouring states are respectively large-detuned from the frequencies for the corresponding cavity modes.). On the assumption that the velocity of the atom is large enough, the detection of this atom in a definite state can approximately collapse the four cavity modes into a GHZ state. Then four resonant two-level atoms, each having a pair of levels resonant with only one mode of the cavity, can probe the cavity field directly to obtain the relevant measurements needed to demonstrate the nonlocality. The advantage of the scheme is that careful velocity controlling in order to achieve the precise atom–cavity interaction time is not required. However, the success probability of the scheme is very slight.

In this paper, we propose an alternative work for preparing four-particle GHZ states by using two bimodal cavities. We replace the three-state atom in Ref.[24] with a four-state atom, of which one state is only auxiliary and not affected by the atom–cavity interaction. The frequencies for the atomic transition and the cavity modes are chosen in a way that the atom can make resonant single-photon transition between two pairs of the neighbouring states (except the auxiliary state). Then we can obtain the required states without any approximation by the preset time control for the atom–cavity interaction, with the success probability being $\frac{1}{2}$ in principle. We also consider the cavity decay and discuss the generation process over again.

The rest of the paper is organized in the following way. In Section 2, we propose a scheme for the preparation of four-particle GHZ states. In Section 3, we discuss the proposed scheme by taking into account the cavity decay. Section 4 presents the conclusion.

2. Four-particle entanglement without decay

Figure 1 displays the setup for preparing four-particle GHZ entangled states. The Rydberg atoms are produced in Zone $B_i(i = 0, 1, 2, 3, 4)$ by the excitation of velocity-selected atomic beam effusing from oven O_i. Atom A_0 produced in Zone B_0 has a four-level configuration as sketched in Fig.2, the four states are $|e\rangle$, $|g\rangle$, $|i\rangle$, and $|g'\rangle$. The other four atoms $A_k(k = 1, 2, 3, 4)$ respectively produced in Zones $B_i(i = 1, 2, 3, 4)$ are of the type with a pair of states $|e\rangle_{A_k}$ and $|g\rangle_{A_k}$. F is a classical field tuned to the transition $|g\rangle \leftrightarrow |g'\rangle$. C_1 and C_2 are two identical millimetre-wave superconducting microwave bimodal cavities. D stands for the atomic ionization detector.

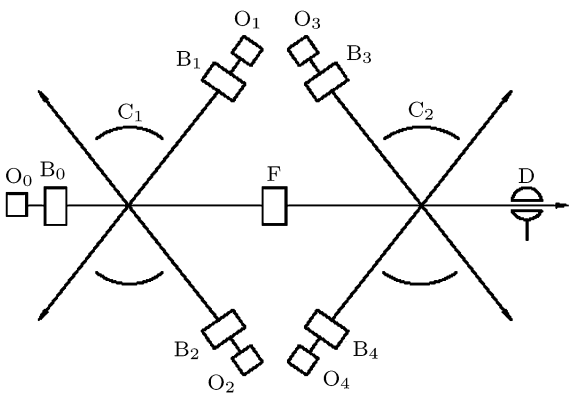


Fig.1. Sketch of the setup of the proposed scheme for generating four-particle GHZ states.

The principal quantum numbers for the states $|e\rangle$, $|g\rangle$, $|i\rangle$, and $|g'\rangle$ of atom A_0 are 51, 50, 49, and 48, respectively, and the energies for these states are ω_e, ω_g, ω_i and $\omega_{g'}$, respectively ($\omega_e > \omega_g > \omega_i > \omega_{g'}$). Let us first consider such a system coupling to two modes of a cavity, as sketched in Fig.2. The transitions $|e\rangle \longleftrightarrow |g\rangle$ and $|g\rangle \longleftrightarrow |i\rangle$ are coupled resonantly to modes 1 and 2, respectively. The state $|g'\rangle$, which is unaffected by the atom–cavity field interaction, is used to perform the local manipulation with the state $|g\rangle$ by the classical field F.[25] In the interaction picture, the interaction Hamiltonian for the atom–cavity system under dipole and rotating wave appropriate can be written as[26–28]

$$H_{\rm I}^{{\rm A}_0-{\rm C}_l} = g(a_1^+|g\rangle_{{\rm A}_0}\langle e| + a_1|e\rangle_{{\rm A}_0}\langle g| + a_2^+|i\rangle_{{\rm A}_0}\langle g| + a_2|g\rangle_{{\rm A}_0}\langle i|), \quad (l=1,2), \quad (1)$$

where $a_1^+(a_2^+)$ and $a_1(a_2)$ are the creation and annihilation operators for the cavity mode 1(2), respectively. Here we have assumed the same atom–field coupling constants for both transitions. This condition can be satisfied by choosing approximate transitions in atomic system, frequencies, etc.[29]

In the following part of this section, we only consider the generation of four-particle GHZ states without cavity decay.

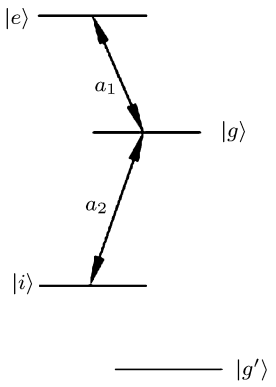


Fig.2. Sketch map of a four-level atom interacting with a two-mode cavity field.

We assume both cavities C_1 and C_2 are initially in the two-mode vacuum state $|0,0\rangle_{{\rm C}_l}$. Atom A_0, initially excited to the state $|e\rangle_{{\rm A}_0}$, is first sent through cavity C_1, after an interaction time τ_1, the atom–cavity system evolves into the state

$$|\Psi^1(\tau_1)\rangle_{{\rm A}_0-{\rm C}_1} = \frac{1}{2}[\cos(\sqrt{2}g\tau_1)+1]|e\rangle_{{\rm A}_0}|0,0\rangle_{{\rm C}_1} + \frac{1}{2}[\cos(\sqrt{2}g\tau_1)-1] \times |i\rangle_{{\rm A}_0}|1,1\rangle_{{\rm C}_1} - \frac{\rm i}{\sqrt{2}}\sin(\sqrt{2}g\tau_1)|g\rangle_{{\rm A}_0}|1,0\rangle_{{\rm C}_1}. \quad (2)$$

If the interaction time is chosen carefully so that $\sqrt{2}g\tau_1 = \pi/2$ is fulfilled, we have

$$|\Psi^1\rangle_{{\rm A}_0-{\rm C}_1} = \frac{1}{2}|e\rangle_{{\rm A}_0}|0,0\rangle_{{\rm C}_1} - \frac{1}{2}|i\rangle_{{\rm A}_0}|1,1\rangle_{{\rm C}_1} - \frac{\rm i}{\sqrt{2}}|g\rangle_{{\rm A}_0}|1,0\rangle_{{\rm C}_1}. \quad (3)$$

After exiting from cavity C_1, atom A_0 crosses the classical field F, leading to the transformation $|g\rangle \rightarrow |g'\rangle$,[25] then we have

$$|\Psi^1\rangle_{{\rm A}_0-{\rm C}_1} = \frac{1}{2}|e\rangle_{{\rm A}_0}|0,0\rangle_{{\rm C}_1} - \frac{1}{2}|i\rangle_{{\rm A}_0}|1,1\rangle_{{\rm C}_1} - \frac{\rm i}{\sqrt{2}}|g'\rangle_{{\rm A}_0}|1,0\rangle_{{\rm C}_1}. \quad (4)$$

Then it traverses cavity C_2, and after another interaction time τ_2, the whole system evolves into the state

$$|\Psi^1(\tau_2)\rangle_{{\rm A}_0-{\rm C}_1-{\rm C}_2} = \frac{1}{2}|0,0\rangle_{{\rm C}_1}\Big\{\frac{1}{2}[\cos(\sqrt{2}g\tau_2)+1]|e\rangle_{{\rm A}_0}|0,0\rangle_{{\rm C}_2} + \frac{1}{2}[\cos(\sqrt{2}g\tau_2)-1]|i\rangle_{{\rm A}_0}|1,1\rangle_{{\rm C}_2} - \frac{\rm i}{\sqrt{2}}\sin(\sqrt{2}g\tau_2)|g\rangle_{{\rm A}_0}|1,0\rangle_{{\rm C}_2}\Big\} - \frac{1}{2}|i\rangle_{{\rm A}_0}|1,1\rangle_{{\rm C}_1}|0,0\rangle_{{\rm C}_2} - \frac{\rm i}{\sqrt{2}}|g'\rangle_{{\rm A}_0}|1,0\rangle_{{\rm C}_1}|0,0\rangle_{{\rm C}_2}. \quad (5)$$

If the interaction time is chosen to fulfil $\sqrt{2}g\tau_2 = \pi$, the state of the whole system composed of the atom and two cavities reads

$$|\Psi^1\rangle_{{\rm A}_0-{\rm C}_1-{\rm C}_2} = -\frac{1}{2}|{\rm i}\rangle_{{\rm A}_0}(|0,0\rangle_{{\rm C}_1}|1,1\rangle_{{\rm C}_2} + |1,1\rangle_{{\rm C}_1}|0,0\rangle_{{\rm C}_2}) - \frac{\rm i}{\sqrt{2}}|g'\rangle_{{\rm A}_0}|1,0\rangle_{{\rm C}_1}|0,0\rangle_{{\rm C}_2}. \quad (6)$$

Now a measurement is performed on atom A_0. If this atom is detected in the state $|i\rangle_{A_0}$, two cavities are projected into the state

$$|\Psi^1\rangle_{C_1-C_2} = \frac{1}{\sqrt{2}}(|0,0\rangle_{C_1}|1,1\rangle_{C_2} + |1,1\rangle_{C_1}|0,0\rangle_{C_2}), \quad (7)$$

where the phase factor -1 is discarded. Thus we obtain a two-photon four-mode GHZ entangled state. The probability of success is $\frac{1}{2}$ in principle.

In order to probe the cavity field, we may employ four identical two-level atoms $A_1 - A_4$, each has a pair of states and should be made resonant with only one mode of the bimodal cavities. The problem remains how to make two identical atoms interact with two modes at different frequencies in a bimodal cavity. Indeed, this is a crucial point, since it requires that, when the atom is resonant with one mode, the interaction with the second one cannot happen, otherwise the competition between the two modes will occur. Fortunately, an experiment reported in Ref.[30] showed that an atom can be controlled by the method of Stark effect, thus it can be coupled sequentially with two modes at different frequencies in a bimodal cavity. So one can control these atoms through the Stark effect, which can be achieved by applying a static field between two cavity mirrors,[31] to make them resonant with the corresponding modes at different frequencies to probe the cavity field. Here we assume atoms A_1 and A_3 are resonant with mode 1, while A_2 and A_4 are resonant with mode 2.

Then we send atoms A_1 and A_2 through cavity C_1 and simultaneously A_3 and A_4 through cavity C_2. The evolution of each atom with the corresponding cavity mode is dominated by the interaction Hamiltonian[32,33]

$$H_I^2 = g(a_j^+|g\rangle_{A_k}\langle e| + a_j|e\rangle_{A_k}\langle g|), \quad (k = 1,2,3,4, \quad j = 1,2). \quad (8)$$

Provided that these atoms are all in their ground states $|g\rangle_{A_k}$, the atoms make resonant single-photon transition by coupling with the corresponding modes, i.e.

$$|g\rangle_{A_k}|0\rangle_j \to |g\rangle_{A_k}|0\rangle_j, \quad (9)$$

$$|g\rangle_{A_k}|1\rangle_j \to \cos(gt_k)|g\rangle_{A_k}|1\rangle_j - \mathrm{i}\sin(gt_k)|e\rangle_{A_k}|0\rangle_j. \quad (10)$$

For the respective interaction time $t_k = \pi/2g$, the four atoms are engineered in the following GHZ state:

$$|\Psi^2\rangle_{A_1\sim A_4} = \frac{1}{\sqrt{2}}(|g\rangle_{A_1}|g\rangle_{A_2}|e\rangle_{A_3}|e\rangle_{A_4} + |e\rangle_{A_1}|e\rangle_{A_2}|g\rangle_{A_3}|g\rangle_{A_4}), \quad (11)$$

leaving all the cavity modes in vacuum states.

3. Four-particle entanglement including cavity decay

In this section, taking into account the cavity decay, we discuss the above-mentioned scheme over again and show how such decay will affect the fidelity and the probability of success of the scheme.

We first describe the preparation of the two-photon four-mode GHZ entangled state. Provided that, during the whole preparation process, there is no photon detected due to the leakage of a photon by the cavity mirrors. The evolution of the state of the system composed of atom A_0 and each cavity is dominated by the non-Hermitian Hamiltonian as follows:

$$H_{non}^{A_0-C_j} = g(a_1^+|g\rangle_{A_0}\langle e| + a_1|e\rangle_{A_0}\langle g| + a_2^+|i\rangle_{A_0}\langle g| + a_2|g\rangle_{A_0}\langle i|) - \mathrm{i}\frac{\kappa}{2}\sum_{i=1}^{2} a_i^+ a_i, \quad (j = 1,2), \quad (12)$$

where κ is the cavity decay rate. Here we have assumed the same decay rate for the two cavity modes. Once atom A_0 exits the cavity, the evolution of each cavity field state is dominated by the non-Hermitian Hamiltonian

$$H_{non}^{C_j} = -\mathrm{i}\frac{\kappa}{2}\sum_{i=1}^{2} a_i^+ a_i, \quad (j = 1,2). \quad (13)$$

We first divide the preparation process into four parts: (i) Time τ_1^c for atom A_0 coupling with cavity C_1; (ii) Time τ_2^c ($\tau_1^c \sim \tau_1^c + \tau_2^c$) for atom A_0 travelling between two cavities; (iii) Time τ_3^c ($\tau_2^c \sim \tau_2^c + \tau_3^c$) for atom A_0 coupling with cavity C_2; (iv) Time τ_4^c($\tau_3^c \sim \tau_3^c + \tau_4^c$) for the process of atomic state detection. We now describe these four periods as follows:

(i) Atom A_0 is sent into cavity C_1. If the initial state of the system composed of atom A_0 and two cavities reads $|e\rangle_{A_0}\ |0,0\rangle_{C_1}|0,0\rangle_{C_2}$, the evolution of the system can be written as

$$
\begin{aligned}
|\Psi_{\rm non}^{1-1}(\tau_1^{\rm c})\rangle_{\rm A_0-C_1-C_2} &= {\rm e}^{-{\rm i}H_{\rm non}^{\rm A_0-C_1}\tau_1^{\rm c}}{\rm e}^{-{\rm i}H_{\rm non}^{\rm C_2}\tau_1^{\rm c}}|e\rangle_{\rm A_0}|0,0\rangle_{\rm C_1}|0,0\rangle_{\rm C_2}\\
&= \frac{{\rm e}^{-\kappa\tau_1^{\rm c}/2}}{2}\Big\{\Big[\cos(\alpha\tau_1^{\rm c})\Big(\frac{-\kappa^2}{4\alpha^2}+1\Big)+\sin(\alpha\tau_1^{\rm c})\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big]|e\rangle_{\rm A_0}\\
&\quad\times|0,0\rangle_{\rm C_1}+\frac{{\rm i}g}{\alpha}\Big[\frac{\kappa}{\alpha}\cos(\alpha\tau_1^{\rm c})-2\sin(\alpha\tau_1^{\rm c})-\frac{\kappa}{\alpha}\Big]|g\rangle_{\rm A_0}|1,0\rangle_{\rm C_1}\\
&\quad+\Big[\Big(1+\frac{\kappa^2}{4\alpha^2}\Big)\cos(\alpha\tau_1^{\rm c})-\frac{2g^2}{\alpha^2}\Big]|i\rangle_{\rm A_0}|1,1\rangle_{\rm C_1}\Big\}|0,0\rangle_{\rm C_2},
\end{aligned}\tag{14}
$$

where

$$
\alpha=\Big(2g^2-\frac{\kappa^2}{4}\Big)^{1/2}.\tag{15}
$$

If the interaction time is chosen to satisfy $\tau_1^{\rm c}=\frac{\pi}{2\alpha}$, the state of the system reads

$$
\begin{aligned}
|\Psi_{\rm con}^{1-1}\rangle_{\rm A_0-C_1-C_2} &= \frac{1}{2}{\rm e}^{-\kappa\pi/(4\alpha)}\Big[\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big)|e\rangle_{\rm A_0}|0,0\rangle_{\rm C_1}-{\rm i}\Big(\frac{2g}{\alpha}+\frac{g\kappa}{\alpha^2}\Big)\\
&\quad\times|g\rangle_{\rm A_0}|1,0\rangle_{\rm C_1}-\frac{2g^2}{\alpha^2}|i\rangle_{\rm A_0}|1,1\rangle_{\rm C_1}\Big]|0,0\rangle_{\rm C_2}.
\end{aligned}\tag{16}
$$

(ii) During the second period time $\tau_2^{\rm c}$, the evolution of the system can be written as

$$
\begin{aligned}
|\Psi_{\rm non}^{1-2}(\tau_2^{\rm c})\rangle_{\rm A_0-C_1-C_2} &= {\rm e}^{-{\rm i}H_{\rm non}^{\rm C_1}\tau_2^{\rm c}}{\rm e}^{-{\rm i}H_{\rm non}^{\rm C_2}\tau_2^{\rm c}}|\Psi_{\rm non}^{1-1}\rangle_{\rm A_0-C_1-C_2}\\
&= \frac{1}{2}{\rm e}^{-\kappa\pi/(4\alpha)}\Big[\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big)|e\rangle_{\rm A_0}|0,0\rangle_{\rm C_1}-{\rm i}\Big(\frac{2g}{\alpha}+\frac{g\kappa}{\alpha^2}\Big)\\
&\quad\times{\rm e}^{(-\kappa/2)\tau_2^{\rm c}}|g'\rangle_{\rm A_0}|1,0\rangle_{\rm C_1}-\frac{2g^2}{\alpha^2}{\rm e}^{-\kappa\tau_2^{\rm c}}|i\rangle_{\rm A_0}|1,1\rangle_{\rm C_1}\Big]|0,0\rangle_{\rm C_2}.
\end{aligned}\tag{17}
$$

During this period, the atomic state $|g\rangle_{\rm A_0}$ is manipulated to $|g'\rangle_{\rm A_0}$ by the classical field F.

(iii) During the third period of atom $\rm A_0$ coupling with cavity $\rm C_2$, the state of the system, after the chosen time $\tau_3^{\rm c}=\pi/\alpha$, can be written as

$$
\begin{aligned}
|\Psi_{\rm non}^{1-3}\rangle_{\rm A_0-C_1-C_2} &= {\rm e}^{-{\rm i}H_{\rm non}^{\rm C_1}\tau_3^{\rm c}}{\rm e}^{-{\rm i}H_{\rm non}^{\rm A_0-C_2}\tau_3^{\rm c}}|\Psi_{\rm non}^{1-2}(\tau_2^{\rm c})\rangle_{\rm A_0-C_1-C_2}\\
&= \frac{1}{2}{\rm e}^{-\kappa\pi/(4\alpha)}\Big\{\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big)|0,0\rangle_{\rm C_1}\frac{{\rm e}^{-\kappa\pi/(2\alpha)}}{2}\Big(\frac{\kappa^2}{2\alpha^2}|e\rangle_{\rm A_0}|0,0\rangle_{\rm C_2}-\frac{2{\rm i}\kappa g}{\alpha^2}|g\rangle_{\rm A_0}|1,0\rangle_{\rm C_2}\\
&\quad-\frac{4g^2}{\alpha^2}|i\rangle_{\rm A_0}|1,1\rangle_{\rm C_2}\Big)-\Big[{\rm i}\Big(\frac{2g}{\alpha}+\frac{g\kappa}{\alpha^2}\Big)\,{\rm e}^{-(\kappa/2)\tau_2^{\rm c}}{\rm e}^{-\pi\kappa/(2\alpha)}|g'\rangle_{\rm A_0}\\
&\quad\times|1,0\rangle_{\rm C_1}+\frac{2g^2}{\alpha^2}{\rm e}^{-\kappa\tau_2^{\rm c}}{\rm e}^{-\pi\kappa/\alpha}|i\rangle_{\rm A_0}|1,1\rangle_{\rm C_1}\Big]|0,0\rangle_{\rm C_2}\Big\}.
\end{aligned}\tag{18}
$$

(iv) Then atom $\rm A_0$ exits from cavity $\rm C_2$. After time $\tau_4^{\rm c}$, the state of the system evolves into

$$
\begin{aligned}
|\Psi_{\rm non}^{1-4}\rangle_{\rm A_0-C_1-C_2} &= {\rm e}^{-{\rm i}H_{\rm non}^{\rm C_1}\tau_4^{\rm c}}{\rm e}^{-{\rm i}H_{\rm non}^{\rm C_2}\tau_4^{\rm c}}|\Psi_{\rm non}^{1-3}\rangle_{\rm A_0-C_1-C_2}\\
&= \frac{1}{2}{\rm e}^{-\kappa\pi/(4\alpha)}\Big\{\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big)|0,0\rangle_{\rm C_1}\frac{{\rm e}^{-\kappa\pi/(2\alpha)}}{2}\Big(\frac{\kappa^2}{2\alpha^2}|e\rangle_{\rm A_0}|0,0\rangle_{\rm C_2}-\frac{2{\rm i}\kappa g}{\alpha^2}{\rm e}^{-\kappa\tau_4^{\rm c}/2}|g\rangle_{\rm A_0}|1,0\rangle_{\rm C_2}\\
&\quad-\frac{4g^2}{\alpha^2}{\rm e}^{-\kappa\tau_4^{\rm c}}|i\rangle_{\rm A_0}|1,1\rangle_{\rm C_2}\Big)-\Big[{\rm i}\Big(\frac{2g}{\alpha}+\frac{g\kappa}{\alpha^2}\Big){\rm e}^{-(\kappa/2)\tau_2^{\rm c}}{\rm e}^{-\pi\kappa/(2\alpha)}{\rm e}^{-(\kappa/2)\tau_4^{\rm c}}|g'\rangle_{\rm A_0}\\
&\quad\times|1,0\rangle_{\rm C_1}+\frac{2g^2}{\alpha^2}{\rm e}^{-\kappa\tau_2^{\rm c}}{\rm e}^{-\pi\kappa/\alpha}{\rm e}^{-\kappa\tau_4^{\rm c}}|i\rangle_{\rm A_0}|1,1\rangle_{\rm C_1}\Big]|0,0\rangle_{\rm C_2}\Big\}.
\end{aligned}\tag{19}
$$

Then a measurement is performed on atom $\rm A_0$. If this atom is detected in the state $|i\rangle_{\rm A_0}$, the two cavities are projected into the following state:

$$
|\Psi_{\rm non}^{1}\rangle_{\rm C_1-C_2}=-\frac{g^2}{\alpha^2}{\rm e}^{-\kappa\pi/(4\alpha)}\Big[\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big){\rm e}^{-\kappa\pi/(2\alpha)}{\rm e}^{-\kappa\tau_4^{\rm c}}|0,0\rangle_{\rm C_1}|1,1\rangle_{\rm C_2}+{\rm e}^{-\kappa(\tau_2^{\rm c}+\tau_4^{\rm c})}{\rm e}^{-\pi\kappa/\alpha}|1,1\rangle_{\rm C_1}|0,0\rangle_{\rm C_2}\Big].\tag{20}
$$

After normalization, we have

$$|\Psi_{\rm non}^{1'}\rangle_{\rm C_1-C_2}=\frac{1}{N_{\rm c}}\Big[\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big)|0,0\rangle_{\rm C_1}|1,1\rangle_{\rm C_2}+{\rm e}^{-\kappa\tau_2^{\rm c}}{\rm e}^{-\pi\kappa/(2\alpha)}|1,1\rangle_{\rm C_1}|0,0\rangle_{\rm C_2}\Big], \tag{21}$$

where $N_{\rm c}$ is a normalized factor given by

$$N_{\rm c}=\Big[\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big)^2+{\rm e}^{-2\kappa\tau_2^{\rm c}}{\rm e}^{-\pi\kappa/\alpha}\Big]^{1/2}. \tag{22}$$

The fidelity is

$$F_{\rm c}=|\langle\Psi_{\rm non}^{1'}|\Psi^1\rangle_{\rm C_1-C_2}|^2=\frac{[(\kappa/\alpha+2g^2/\alpha^2)+{\rm e}^{-\kappa\tau_2^{\rm c}}{\rm e}^{-\pi\kappa/(2\alpha)}]^2}{2N_{\rm c}^2}. \tag{23}$$

The probability of success is

$$P_{\rm c}=\frac{g^4{\rm e}^{-3\kappa\pi/(2\alpha)}{\rm e}^{-2\kappa\tau_4^{\rm c}}}{\alpha^4}\Big[\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big)^2+{\rm e}^{-2\kappa\tau_2^{\rm c}}{\rm e}^{-\kappa\pi/\alpha}\Big]. \tag{24}$$

Then four resonant two-level atoms ($\rm A_k$, $k=1,2,3,4$) are sent to probe the cavity field. Under the condition that no photon is detected due to the leakage of a photon by the cavity mirrors, the evolution for the state of the system composed of one of these atoms and a single cavity mode is dominated by the non-Hermitian Hamiltonian

$$H_{\rm non}^2=g(a_i^+|g\rangle_{\rm A_k}\langle e|+a_i|e\rangle_{\rm A_k}\langle g|)-{\rm i}\frac{\kappa}{2}a_i^+a_i,\quad (k=1,2,3,4;\ i=1,2). \tag{25}$$

According to the Hamiltonian of Eq.(25), we have the following evolution:

$$|g\rangle_{\rm A_k}|1\rangle_i\to{\rm e}^{-\kappa t_k^{\rm c}/4}\Big\{\Big[\cos(\beta t_k^{\rm c})-\frac{\kappa}{4\beta}\sin(\beta t_k^{\rm c})\Big]|g\rangle_{\rm A_k}|1\rangle_i-\frac{{\rm i}g}{\beta}\sin(\beta t_k^{\rm c})|e\rangle_{\rm A_k}|0\rangle_i\Big\}, \tag{26}$$

where

$$\beta=\Big(g^2-\frac{\kappa^2}{16}\Big)^{1/2}. \tag{27}$$

After interacting with each cavity mode for a time $t_k^{\rm c}=[\tan^{-1}(4\beta/\kappa)]/\beta$, the four atoms are engineered in the state

$$|\Psi_{\rm non}^2\rangle_{\rm A_1\sim A_4}=\frac{1}{N_{\rm a}}\Big[\Big(\frac{\kappa}{\alpha}+\frac{2g^2}{\alpha^2}\Big)|g\rangle_{\rm A_1}|g\rangle_{\rm A_2}|e\rangle_{\rm A_3}|e\rangle_{\rm A_4}+{\rm e}^{-\kappa\tau_2^{\rm c}}{\rm e}^{-\pi\kappa/(2\alpha)}|e\rangle_{\rm A_1}|e\rangle_{\rm A_2}|g\rangle_{\rm A_3}|g\rangle_{\rm A_4}\Big], \tag{28}$$

where $N_{\rm a}$ is a normalized factor equal to $N_{\rm c}$. The fidelity is

$$F_{\rm a}=|\langle\Psi_{\rm non}^{2}|\Psi^2\rangle_{\rm A_1\sim A_4}|^2=F_{\rm c}. \tag{29}$$

The probability of success is

$$P_{\rm a}={\rm e}^{-[\kappa\tan^{-1}(4\beta/\kappa)]/\beta}P_{\rm c}. \tag{30}$$

Figure 3(a) and 3(b) show the fidelity ($F_{\rm c}$) and the probability ($P_{\rm c}$) of the generated two-photon four-mode GHZ state with κ/g, respectively. Figure 3(c) shows the probability ($P_{\rm a}$) of the generated four-atom GHZ state with κ/g. The times $\tau_2^{\rm c}$ and $\tau_4^{\rm c}$ are chosen to be $\tau_2^{\rm c}=\tau_4^{\rm c}=5\pi/(2\alpha)$, which are as much as five times of the atom–cavity interaction time ($5\tau_1^{\rm c}$).[34] The scale of the independent variable is set to be $0\leq\kappa/g\leq0.1$. Let $\kappa=0.1g$, we have $F_{\rm c\,min}=F_{\rm a\,min}=0.8895$, $P_{\rm c\,min}=0.0835$, and $P_{\rm a\,min}=0.0715$. However, under the recent experimental condition, a photon damping lifetime of the millimetre-wave superconducting cavity is as long as 1 ms,[9,23] for the typical atom–cavity coupling constant $g=2\pi\times23.5$ kHz,[9,23] the decay rate should be $\kappa\simeq0.0068g$. Setting $\kappa=0.0068g$, we have $F_{\rm c}=F_{\rm a}=0.9994$, $P_{\rm c}=0.4358$, and $P_{\rm a}=0.4312$. Further, we calculate the atom–cavity interaction time to be $\tau_1^{\rm c}=\pi/(2\alpha)\simeq7.6$ μs, $\tau_3^{\rm c}=\pi/\alpha\simeq15.2$ μs, and $t_k^{\rm c}=[\tan^{-1}(4\beta/\kappa)]/\beta\simeq11$ μs. Thus one can send the atoms with a velocity $\sim10^2$ m/s through a few-cm-long cavities to achieve this purpose, and one can use a velocity selector and apply a Stark field adjustment in order to make the atom resonant with the cavity for the different lengths of time. Considering the imperfection of the atomic state detection,[35] we can only obtain the required states with probabilities of $P_{\rm c}'\simeq70\%$ $P_{\rm c}=0.3051$ and $P_{\rm a}'\simeq70\%$ $P_{\rm a}=0.3018$, respectively.

In the above, the atomic spontaneous emission is neglected. In respect that the Rydberg atoms used

commonly in cavity QED experiment has a lifetime as long as about 30 ms,[9] corresponding to a spontaneous emission rate $\Gamma \simeq 33.3\ \mathrm{s}^{-1}$, the condition $\Gamma(\simeq 2.2\times10^{-4}\ \mathrm{g}) \ll \kappa$ can be assumed to be satisfied and thus the spontaneous emission rate can be approximated to zero.

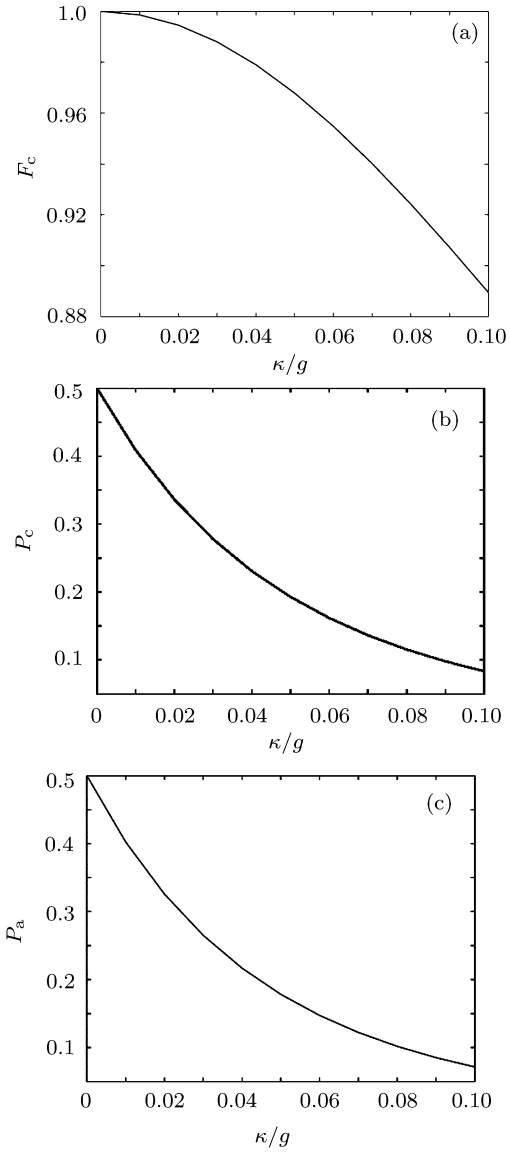


Fig.3. Schematic diagrams of the fidelity and the probability of the generated four-particle GHZ states. (a) The fidelity (F_c) of the generated two-photon four-mode GHZ state as a function of κ/g. (b) The probability (P_c) of the generated two-photon four-mode GHZ state as a function of κ/g. (c) The probability (P_a) of the generated four-atom GHZ state as a function of κ/g.

4. Conclusion

In conclusion, we have proposed a scheme for preparing four-particle Greenberger–Horne–Zeilinger states of the type of Eq.(7) and Eq.(11) in cavity QED. The scheme is an alternative to the previous one proposed by Gerry.[24] For preparing a two-photon four-mode GHZ state, we use the resonant interaction; the atom can make resonant single-photon transition between two neighbouring states. Thus the required states can also be engineered in a faster way. In principle, we can obtain the required state with the fidelity of unity and the success probability of $\frac{1}{2}$. In the scheme of Ref.[24], the atom is made to undergo two-photon transition between the upper and the lower states, with the intermediate state adiabatically eliminated, under the condition that the transition frequencies for two pairs of the neighbouring states are highly detuned from the frequencies for the corresponding cavity modes, and the required state can only be prepared in an approximate way, provided that the atom–cavity interaction time is small enough. If this assumption is not satisfied, the fidelity of the generated state would be decreased. Since the required states can only be engineered on condition that $\lambda_p t \ll 1$ (see Ref.[24]), the probability of success ($2|\lambda_p t|^2$) for the generated state is very small.

In contrast to the schemes in Refs.[12–15], the present scheme may reduce the number of atoms, as the cavity field in the present scheme does not have to be initially prepared in a coherent state or a superposition of zero- and Fock- states. Thus it is unnecessary to employ more atoms to detect or to prepare the field states, which might diminish the difficulty of experimental implementation of the scheme.

Moreover, taking into account the cavity decay, we obtain that the fidelity for the generated state is very close to unity if cavities with high quality factor are used (with storage lifetime 1 ms for a photon). Although the efficiency for the atomic state detection is not perfect (about 70% at present), one only has to repeat the preparation process four times before it can reach its successful conclusion (with the probability of success about 30%). The careful selection of velocity for the atoms in order to achieve the precise interaction times is no longer a great problem in the present cavity QED experiment (see Ref.[23]). A challenge remains: the first atom should be made to be resonant simultaneously with the two modes with different frequencies in a bimodal cavity. Based on the current cavity QED technique or anticipated improvements, the present scheme might be achievable.

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