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Characteristics of surface waves in anisotropic left-handed materials*

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We report the coexistence of TE and TM surface modes in certain same frequency domain at the interface between one isotropic regular medium and another biaxially anistotropic left-handed medium. The conditions for the existence of TE and TM polarized surface waves in biaxially anisotropic left-handed materials are identified, respectively. The Poynting vector and the energy density associated with surface modes are calculated. Depending on the system parameters, either TE or TM surface modes can have the time averaged Poynting vector directed to or opposite to the mode phase velocity. It is seen that the characteristics of surface waves in biaxially anisotropic left-handed media are significantly different from that in isotropic left-handed media.

Keywords: biaxially anisotropic left-handed material, surface wave, dispersion relation
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1. Introduction

Recently, left-handed (LH) materials[1] in which the direction of energy flow of an electromagnetic wave is opposite to the wavevector, have aroused great interest due to both the novelty of physical effects and tremendous prospects for applications.[2−4] LH electromagnetic materials with both negative permittivity ε and permeability μ in a certain frequency range exhibit negative refractive index which obeys the anomalous Snell law. The first experimental demonstration of LH materials over a band of frequencies in the microwave region was done on a two-dimensional array of split-ring resonators and wires.[2,3] Such a LH material slab can be used as a perfect lens[4] for focusing both propagating and evanescent waves due to the excitation of surface waves (or surface polaritons) at both interfaces of the slab. Surface waves are waves that propagate along the interface and decay in the transverse direction exponentially. Linear surface waves are known to exist under certain special conditions at an interface separating two different isotropic dielectric media. In particular, the existence of TM polarized surface waves requires that the dielectric constants of two dielectric materials separated by an interface have different signs, whilst for TE-polarized waves the magnetic permeability of the materials should have different signs.[5,6] Most of the recent theoretical works[5−9] were focused mainly on the calculation of the linear and nonlinear dispersion properties of modes localized at the interface of LH material and conventional right-handed (RH) material with the assumption that the LH materials are lossless and isotropic,[5−8] and on showing that the regions of existence of TE and TM mode do not overlap.[5,6] However, the LH materials that have been prepared successfully in experiments are actually anisotropic in nature,[1,10,11] and the anisotropy has attracted much attention in LH materials.[10−20] It is well known that the electrodynamic properties of anisotropic materials are quite different from that of isotropic materials. In the presence of anisotropy, negative refraction is not necessarily left-handed.[10,15,16] It is found that LH effect in the anisotropic medium does not require all the permittivity and permeability components being negative.[11,15−18] In this paper, we present a detailed investigation on the characteristics of surface waves in biaxially anisotropic LH media. We show under what conditions TE and TM surface modes shall coexist at the interface between one regular isotropic medium and another biaxially anistotropic LH medium. The Poynting vector and the energy density associated with surface modes are calculated. Depending on the

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system parameters, either TE or TM surface mode may have the time averaged Poynting vector directed to or opposite to the mode phase velocity. We show that the characteristics of surface waves in biaxially anisotropic LH media are significantly different from that in isotropic LH media.

2. Surface waves dispersion laws in anisotropic LH materials

We consider waves propagating along the z-axis situated on the interface ($x = 0$) between two semi-infinite media. The medium at $x < 0$ has a dielectric constant ε_1 and a permeability μ_1, both of which are frequency independent. The anisotropic LH medium occupies the $x > 0$ half-space. In the following we assume that both the permittivity and permeability are biaxially anisotropic, and consider an x-y-z coordinate system, in which the permittivity and permeability tensors are simultaneously diagonalizable, in the form[11] $\hat{\varepsilon} = \mathrm{diag}[\varepsilon_x, \varepsilon_y, \varepsilon_z]$ and $\hat{\mu} = \mathrm{diag}[\mu_x, \mu_y, \mu_z]$, where $\varepsilon_x, \varepsilon_y, \varepsilon_z$, and μ_x, μ_y, μ_z are the permittivity and permeability components in the principal coordinate system, respectively. From Maxwell's equations, we know that there are two types of linearly polarized waves, namely the TE and TM modes. For the case of the TE mode, the surface polarization fields are of the form

$$\boldsymbol{E}^{(1)} = \hat{y} A \mathrm{e}^{k_1 x} \mathrm{e}^{\mathrm{i}qz} \mathrm{e}^{-\mathrm{i}\omega t}, \tag{1a}$$

$$\boldsymbol{H}^{(1)} = A\Big(-\frac{q}{\omega\mu_1}\hat{x} - \frac{\mathrm{i}k_1}{\omega\mu_1}\hat{z}\Big)\mathrm{e}^{k_1 x}\mathrm{e}^{\mathrm{i}qz}\mathrm{e}^{-\mathrm{i}\omega t}, \tag{1b}$$

in the region $x < 0$, and

$$\boldsymbol{E}^{(2)} = \hat{y} B \mathrm{e}^{-k_2 x} \mathrm{e}^{\mathrm{i}qz} \mathrm{e}^{-\mathrm{i}\omega t}, \tag{2a}$$

$$\boldsymbol{H}^{(2)} = B\Big(-\frac{q}{\omega\mu_x}\hat{x} + \frac{\mathrm{i}k_2}{\omega\mu_z}\hat{z}\Big)\mathrm{e}^{-k_2 x}\mathrm{e}^{\mathrm{i}qz}\mathrm{e}^{-\mathrm{i}\omega t} \tag{2b}$$

in the region $x > 0$. Here $\hat{x}, \hat{y}, \hat{z}$ are unit vectors in the x, y and z directions, respectively. For surface waves, the field amplitudes decay in an exponential fashion as one moves away from the interface into either medium, therefore both k_1 and k_2 have to be real, i.e., $k_{1,2} > 0$. The boundary conditions at $x = 0$ require the tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ to be continuous. This yields the surface wave dispersion relation

$$\frac{k_1}{\mu_1} + \frac{k_2}{\mu_z} = 0\,. \tag{3}$$

The wavevector components appearing in above equations are related by

$$k_1 = \Big(q^2 - \omega^2\varepsilon_1\mu_1\Big)^{1/2} \tag{4a}$$

and

$$k_2 = \Big(\frac{\mu_z}{\mu_x}q^2 - \omega^2\varepsilon_y\mu_z\Big)^{1/2}. \tag{4b}$$

Accounting for the Eqs.(4), the dispersion law Eq.(3) can be rewritten using the field decrements $k_{1,2}$ as

$$k_1^2 = \omega^2\varepsilon_1\mu_1\frac{1 - \varepsilon_y\mu_x/(\varepsilon_1\mu_1)}{\mu_z\mu_x/\mu_1^2 - 1} \tag{5a}$$

and

$$k_2^2 = \omega^2\varepsilon_1\mu_1\frac{1 - \varepsilon_y\mu_x/(\varepsilon_1\mu_1)}{\mu_z\mu_x/\mu_1^2 - 1}\cdot\frac{\mu_z^2}{\mu_1^2}. \tag{5b}$$

Similarly, the surface wave dispersion relation for the TM mode case is

$$\frac{k_1}{\varepsilon_1} + \frac{k_2}{\varepsilon_z} = 0\,. \tag{6}$$

The wavevector components appearing in the above equations are related by

$$k_1 = (q^2 - \omega^2\varepsilon_1\mu_1)^{1/2} \tag{7a}$$

and

$$k_2 = \Big(\frac{\varepsilon_z}{\varepsilon_x}q^2 - \omega^2\mu_y\varepsilon_z\Big)^{1/2}. \tag{7b}$$

Accounting for the Eqs.(7), the dispersion relation can be rewritten using the field decrements as

$$k_1^2 = \omega^2\varepsilon_1\mu_1\frac{1 - \varepsilon_x\mu_y/(\varepsilon_1\mu_1)}{\varepsilon_z\varepsilon_x/\varepsilon_1^2 - 1} \tag{8a}$$

and

$$k_2^2 = \omega^2\varepsilon_1\mu_1\frac{1 - \varepsilon_x\mu_y/(\varepsilon_1\mu_1)}{\varepsilon_z\varepsilon_x/\varepsilon_1^2 - 1}\cdot\frac{\varepsilon_z^2}{\varepsilon_1^2}. \tag{8b}$$

Equations (3) and (6) show that the relations $\mathrm{sign}(\mu_1) \neq \mathrm{sign}(\mu_z)$ and $\mathrm{sign}(\varepsilon_1) \neq \mathrm{sign}(\varepsilon_z)$ must be satisfied for the corresponding TE surface mode and TM surface mode to exist. For example, for TE mode, if we take μ_1 positive, then the condition μ_z being negative must be satisfied, which the other elements μ_x, μ_y do not need to be negative. And for TM mode, if ε_1 is positive, then the condition ε_z being negative must be satisfied, and other elements $\varepsilon_x, \varepsilon_y$ do not need to be negative. We note that, due to these inequalities, the coexistence of TE and TM polarized surface modes at the interface between the LH and RH materials is actually possible, which is different from the prediction of Ref.[6].

3. Domain of existence of TE and TM surface waves

We study linear surface waves localized at the interface separating a LH medium and a conventional or right-handed dielectric medium. We demonstrate that the interface can support both TE and TM polarized surface waves, and we study their properties.

The necessary condition of existence of surface mode is q^2 and $k_{1,2}^2$ being positive, therefore it is necessary to find domains where $k_{1,2}^2 > 0$. Equation (5) shows that for $k_{1,2}^2$ to be positive at $\mu_1 > 0$ and $\mu_z < 0$, the TE polarized surface waves at the interface of a conventional RH medium and an biaxially anisotropic LH medium shall exist provided that one of the four following conditions:

$$\mu_x < 0, \quad \varepsilon_y > 0, \quad \mu_x\mu_z > \mu_1^2, \tag{9a}$$

$$\mu_x < 0, \quad \varepsilon_y < 0, \quad \mu_x\mu_z > \mu_1^2, \quad \varepsilon_y\mu_x < \varepsilon_1\mu_1, \tag{9b}$$

$$\mu_x < 0, \quad \varepsilon_y < 0, \quad \mu_x\mu_z < \mu_1^2, \quad \varepsilon_y\mu_x > \varepsilon_1\mu_1 \tag{9c}$$

and

$$\mu_x > 0, \quad \varepsilon_y > 0, \quad \varepsilon_y\mu_x > \varepsilon_1\mu_1 \tag{9d}$$

is satisfied, and the values of $\varepsilon_x, \varepsilon_z, \mu_y$ do not affect the TE surface wave. Similarly, Eqs.(9) show that the TM polarized surface waves shall exist provided that one of the four following conditions:

$$\varepsilon_x < 0, \quad \mu_y > 0, \quad \varepsilon_x\varepsilon_z > \varepsilon_1^2, \tag{10a}$$

$$\varepsilon_x < 0, \quad \mu_y < 0, \quad \varepsilon_x\varepsilon_z > \varepsilon_1^2, \quad \varepsilon_x\mu_y < \varepsilon_1\mu_1 \tag{10b}$$

$$\varepsilon_x < 0, \quad \mu_y < 0, \quad \varepsilon_x\varepsilon_z < \varepsilon_1^2, \quad \varepsilon_x\mu_y > \varepsilon_1\mu_1 \tag{10c}$$

and

$$\varepsilon_x > 0, \quad \mu_y > 0, \quad \varepsilon_x\mu_y > \varepsilon_1\mu_1 \tag{10d}$$

is satisfied, and the values of $\mu_x, \mu_z, \varepsilon_y$ do not affect the TM surface wave. Thus Eqs.(9) and (10) represent constraints on domains of existence of TE and TM surface waves, respectively. As we have mentioned above, the values of $\varepsilon_x, \varepsilon_z, \mu_y$ do not affect the TE surface waves, and the values of $\mu_x, \mu_z, \varepsilon_y$ do not affect the TM surface waves. For example, the dispersion relations Eq.(3) and Eq.(9a) for answer for that μ_x, μ_z are negative real numbers and $\mu_x\mu_z > \mu_1^2$, and ε_y is a positive real number, while all other diagonal components of permittivity and permeability can be positive or negative real numbers (such LH medium has already been realized using different resonant inclusions such as S- and Ω-shaped particles[21]—author's note) provided that one of Eqs.(10) is satisfied to support the TM surface waves. It is obvious that Eq.(10a) is compatible with some one of Eqs.(9) in certain frequency bands, and so on. For example, for two particular sets of media parameters, (a) $\varepsilon_1 = 1$, $\mu_1 = 1$, $\hat{\varepsilon} = \mathrm{diag}[-1.2, +1.8, -0.9]$, $\hat{\mu} = \mathrm{diag}[-1.5, +2.1, -1.3]$, Eqs.(10a) and (9a) are compatible, and (b) $\varepsilon_1 = 1$, $\mu_1 = 1$, $\hat{\varepsilon} = \mathrm{diag}[-1.5, -0.7, -1.3]$, $\hat{\mu} = \mathrm{diag}[-1.1, +1.8, -1.2]$, Eqs.(10a) and (9b) are compatible. This means that the corresponding regions of existence of TM and TE modes do overlap. The result is significantly different from that in isotropic left-handed media.[6]

4. The Poynting vector and energy density

In this section, we proceed with calculation of energy flow along the interface. The energy flow is described by the Poynting vector, which defines the time averaged energy flux. To this end we calculate the Poynting vectors $\boldsymbol{S}$ associated with surface modes. Let us deal with the TE modes only in this paper, the result for TM modes can be obtained from the duality. The time averaged Poynting vector of surface waves is directed along the x-axis and it amounts to

$$S_1 = \frac{|A|^2}{2}\frac{q}{\omega\mu_1}\mathrm{e}^{2k_1x} \tag{11a}$$

and

$$S_2 = \frac{|A|^2}{2}\frac{q}{\omega\mu_x}\mathrm{e}^{-2k_2x} \tag{11b}$$

in medium I and medium II, respectively. The corresponding total energy flow associated with the whole mode is determined by integrating over the x coordinate,

$$\begin{aligned}\langle S\rangle &= \int_0^\infty S_1\mathrm{d}x + \int_{-\infty}^0 S_2\mathrm{d}x \\ &= \frac{|A|^2}{4}\frac{q}{\omega}\Big(\frac{1}{\mu_1 k_1} + \frac{1}{\mu_x k_2}\Big).\end{aligned} \tag{12}$$

Let us recall the dispersion relation Eqs.(3) and (9); it can be inferred from Eqs.(11) and (12) that the guided energy flows can have opposite or the same directions in the adjacent media. Because the necessary condition for the excitation of TE polarized surface waves is $\mathrm{sign}(\mu_1) \neq \mathrm{sign}(\mu_z)$, so μ_x can be either positive or negative. For example, in the case of $\mu_1 > 0, \mu_x < 0$,

we have $S_1 > 0$, $S_2 < 0$, i.e. the energy flow in medium II is directed opposite to the phase velocity, and in the case of $\mu_1 > 0, \mu_x > 0$, we have $S_1 > 0$, $S_2 > 0$, i.e. the energy flow in medium II is directed to the phase velocity. The results are quite different from that in isotropic left-handed media, in which the energy flows are always in opposite directions on the sides of the interface. We remind that the direction of phase velocity of surface waves coincides with the direction of the z-axis. And the total energy flow is always positive at $\mu_1 > 0$, $\mu_x > 0$, which means that the surface waves are always forward. The total energy flow can be positive or negative, depending on the ratio between the two constituting energy flows at $\mu_1 > 0$, and $\mu_x < 0$, which the surface waves are forward or backward, correspondingly.

5. Conclusions

We have studied the characteristics of TE and TM polarized surface waves existing on the interface between conventional medium and biaxially anisotropic LH medium. The conditions for the existence of surface TE and TM polarized waves in anisotropic left-handed materials are identified. The results show that there exist certain bands in which the coexistence of TM and TE modes is possible, which is significantly different from that in isotropic left-handed media. The Poynting vector and the energy density associated with surface modes are calculated. Depending on the system parameters, either TE or TM surface modes can have the time averaged Poynting vector directed to or opposite to the mode phase velocity. We show that the characteristics of surface waves in biaxially anisotropic left-handed media are quite different from that in isotropic left-handed media.

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