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Critical behaviour in nuclear structure from spherical to axially symmetric deformed shape in IBM*

Zhang Jin-Fu(张进富)†, Lü Li-Jun(吕立君), and Bai Hong-Bo(白洪波)

Department of Physics, Chifeng College, Chifeng 024001, *China*

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To compare with the predictions of the transitional dynamical symmetry X(5) proposed by Iachello (2001 *Phys. Rev. Lett.* **87** 052502), the critical behaviours of U(5)–SU(3) are studied in the space of two control parameters in the interacting boson model (IBM). A simple-shaped phased diagram has been presented. It is found that X(5) predictions cannot be exactly reproduced by our calculations and that the best agreement is close to the calculations with boson numbers $N = 11$ and 12. By comparing with experimental data on X(5)-like nuclei, we find that X(5) predictions and IBM calculations can reproduce the energy ratios and E2 transition ones.

Keywords: nuclear structure, critical behaviour, X(5) nuclei, IBM
PACC: 2160F, 2160E

1. Introduction

In recent years, the theoretical and experimental information about the concept of the critical points in the shape phase transition has attracted great efforts(see, for example Refs.[1–4]). In fact, the concept in atomic nuclei was first discussed by making use of the coherent state formalism[5−7] of the interacting boson model (IBM).[8] The analytic descriptions of critical point symmetry, including E(5),[9] X(5),[10] Y(5),[11] Z(5)[12] and E(5/4),[13] have been proposed by Iachello and Bonatsos *et al* and applied to atomic nuclei. Meanwhile, various model (theoretical) studies,[14−23] such as IBM, have contributed greatly to the understanding of the behaviour of critical points in shape phase transition regions. This is a significant point since most nuclei are in transition regions of changing character as a function of nucleon number and, consequently, are usually treated in terms of complex numerical calculations[24−31] involving parametrized Hamiltonians.

The behaviour of critical points in the shape phase transition can be studied on nuclei in the framework of both the collective model and IBM. The critical point symmetries, such as X(5), are obtained by using a square well potential in Bohr Hamiltonian and attain predictions for both energies and E2 transition probabilities. It is our purpose to study the behaviour of critical points in the U(5)–SU(3) transition by using IBM, and investigate the connection between these two different approaches for describing phase transitional behaviour and critical point nuclei.

2. Critical point: a general analysis in terms of IBM

IBM is an algebraic model that has made a great success in medium-heavy nuclei. In this model, s and d bosons with lower-energy $L = 0$ state and higher-energy $L = 2$ state, respectively, interact with each other through a Hamiltonian that includes mainly one- and two-body interactions. The system is number conserving and rotationally invariant. The original version of the model, which carries no label to distinguish neutrons from protons, i.e. IBM-1, is used in this paper. The IBM-1 has an algebraic structure with U(6) as a dynamical group. Changing the free parameters of model Hamiltonian, one obtains three dynamical symmetries: U(5) (an anharmonic vibrator), SU(3) (an axisymmetric rotor), and O(6) (a γ – soft rotor).

In this work, we use the well-known two-para-

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†E-mail: zhjinfu@sohu.com

meter IBM-1 Hamiltonian[14]

$$\hat{H}_{(k,\chi)} = (1-k)\Big[-\frac{\hat{Q}_\chi \cdot \hat{Q}_\chi}{N}\Big] + k\hat{n}_{\rm d}, \qquad (1)$$

where $\hat{n}_{\rm d}$ and $\hat{Q}_\chi$ are the d boson number operator and quadrupole operator, respectively, with

$$\begin{aligned} &\hat{n}_{\rm d} = d^+ \cdot \tilde{d}, \\ &\hat{Q}_\chi = s^+\tilde{d} + d^+ s + \chi[d^+\tilde{d}]^{(2)}. \end{aligned} \qquad (2)$$

The dot denotes the scalar product. The coefficient of the one-body term changes in the range $k \in [0,1]$, and the structure parameter $\chi \in [-\sqrt{7}/2, 0]$. At certain points of the $k-\chi$ parameter plane, the above Hamiltonian contains three standard dynamical symmetries: U(5) for $k=1$ and χ arbitrary, SU(3) for $k=0$ and $\chi=-\sqrt{7}/2$, O(6) for $k=\chi=0$.

To analyse the critical behaviour of atomic nuclei, the intrinsic coherent state method is used. In such formalism, the wavefunction of the ground state of the system is

$$\begin{aligned} |N,\beta,\gamma\rangle = &\frac{1}{\sqrt{N!(1+\beta^2)^N}}\Big(s^+ + \beta\cos\gamma d_0^+ \\ &+ \frac{1}{\sqrt{2}}\beta\sin\gamma(d_2^+ + d_{-2}^+)\Big)^N|0\rangle, \end{aligned} \qquad (3)$$

where β and γ are the order parameters of the system. The shape phase transition will bring changes in the order parameters of the system. Much 'microscopic' information may be obtained by utilizing the intrinsic coherent state method. In details, the shape phase transition analysis of the Hamiltonian (1) yields the energy functional as follows:[14]

$$\begin{aligned} E(N,\beta,\gamma) &= \langle N,\beta,\gamma|\hat{H}_{(k,\chi)}|N,\beta,\gamma\rangle \\ &= -5(1-k) + \frac{1}{(1+\beta^2)^2} \\ &\quad \times \{[Nk-(1-k)\times(4N+\chi^2-8)]\beta^2 \\ &\quad + \Big[Nk-(1-k)\times\Big(\frac{2N+5}{7}\chi^2-4\Big)\Big]\beta^4 \\ &\quad + \Big[4\sqrt{\frac{2}{7}}(N-1)(1-k)\chi\Big]\beta^3\cos(3\gamma)\Big\}. \end{aligned} \qquad (4)$$

The critical point in k is found when E becomes flat at $\beta=0$ or $|\partial^2 E(k^{\rm c})/\partial\beta^2|_{\beta=0} = 0$. Thus, the transition from U(5) to SU(3) (for $\chi=-\sqrt{7}/2$) in the above expression can be obtained easily at the k value as follows:

$$k_N^{\rm c} = \frac{16N-25}{20N-25}. \qquad (5)$$

It can be said from formalism (5) that the critical point depends on N. This dependence is actually negligible for sufficiently large boson numbers. Meanwhile, we note that the critical point has a value of $k_\infty^{\rm c} = 0.8$ as N tends to infinity.

To explore the critical behaviour in the U(5)–SU(3) transition, in Figs.1–4, the energy functions (4) with coefficients according to $\chi=-\sqrt{7}/2$ and $\gamma=0$ are shown for four boson numbers, $N=9$, 10, 11 and 12. From all four graphs, we find that the energy surfaces exhibit a strong dependence on the control parameter k. For $k > k^{\rm c}$, the energy surfaces are very shallow while for $k=k^{\rm c}$, the surfaces display the typical flat-bottomed curve expected at the critical point. With decreasing k, the energy surfaces become sharpened and deep. These features indicate that the systems behave as a phase transition at $k=k_N^{\rm c}$.

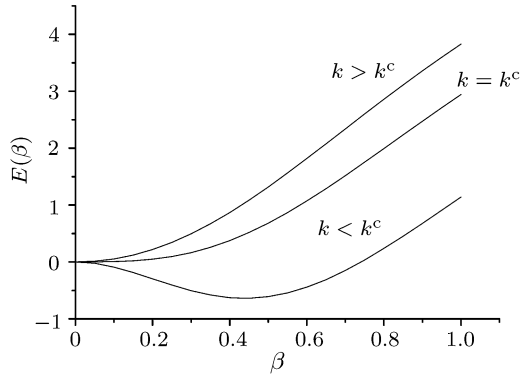


Fig.1. Energy surface in terms of the deformation parameter β for varying k values at $N=9$.

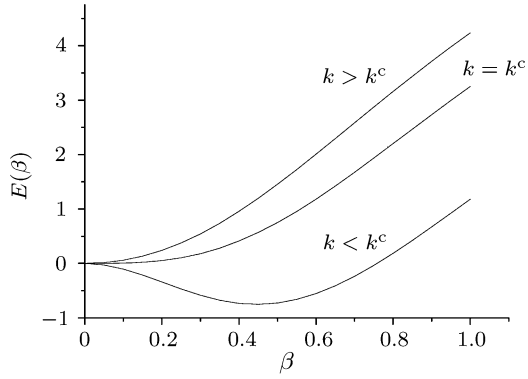


Fig.2. Energy surface in terms of the deformation parameter β for varying k values at $N=10$.

From Eq.(4), we know that $E(N,\beta,\gamma)$ depends on the order parameters β and γ, as well as on the control parameters k and χ. Taking the first-order derivative of Eq.(4) with respect to β, γ, separately, leads to the following equations in the limit of $N\to\infty$:

$$(5k-4)(1-\beta^2) + 2\Big[k - \frac{2}{7}\chi^2(1-k)\Big]\beta^2$$

$$+2\sqrt{\frac{2}{7}}(1-k)\chi(3-\beta^2)\beta\cos(3\gamma)=0, \tag{6a}$$

$$(1-k)\chi\beta^3\sin(3\gamma)=0. \tag{6b}$$

We have then the accessible β and γ for the U(5) symmetry (with $k=1$ in Eq.(1), $\beta=\pm\mathrm{i}$ and γ arbitrary, i.e. no real β exists for the U(5) symmetry. This indicates that the U(5) symmetry corresponds to a spherical shape with vibration. For the SU(3) symmetry (with $k=0$ and $\chi=-\sqrt{7}/2$ in Eq.(1), the allowed β and γ are $\beta=\sqrt{2}$ and $\gamma=0$ or $\pi/3$. This means that the nucleus in the SU(3) symmetry corresponds to a shape of ellipsoid with rotation (or axial rotation). Furthermore, for the U(5)–SU(3) transition (with $\chi=-\sqrt{7}/2$ and $k\in[1,0]$ in Eqs.(1) and (6a)), the equation for the shape equilibrium can be given as

$$\sqrt{2}(1-k)\beta^3+(3-2k)\beta^2 -3\sqrt{2}(1-k)\beta+5k-4=0. \tag{7}$$

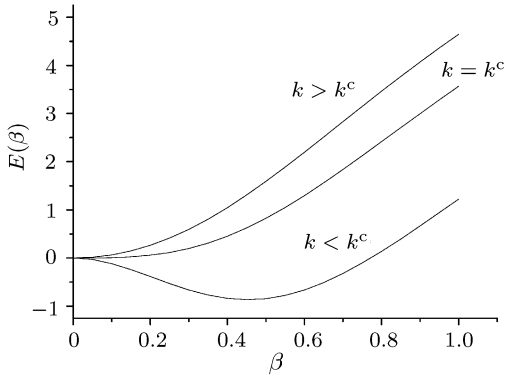


Fig.3. Energy surface in terms of the deformation parameter β for varying k values at $N=11$.

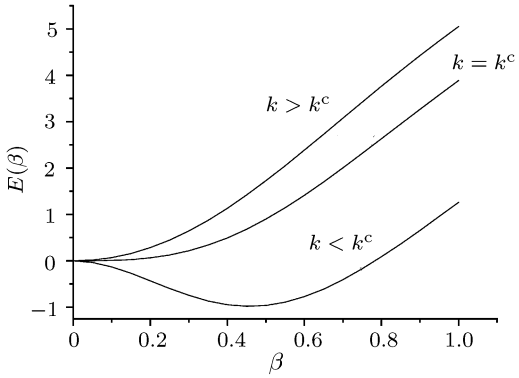


Fig.4. Energy surface in terms of the deformation parameter β for varying k values at $N=12$.

The variation of the order parameter β with respect to k expressed in Eq.(7) can be illustrated in Fig.5. It is evident that Fig.5 displays well the characteristic of the phase transition induced by the parameter k. We then obtain the phase diagram of the U(5)–SU(3) transition.

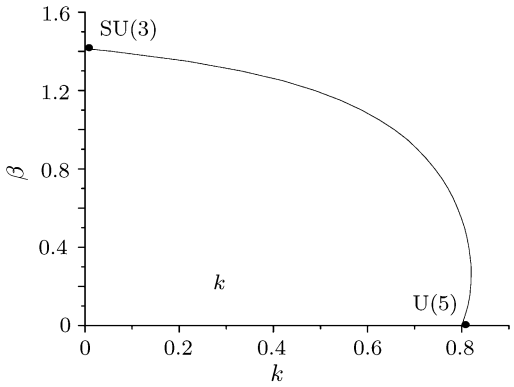


Fig.5. Shape–phase diagram for U(5)–SU(3) deformation, obtained from Eq.(7), against k.

3. Comparisons between X(5) and the IBM

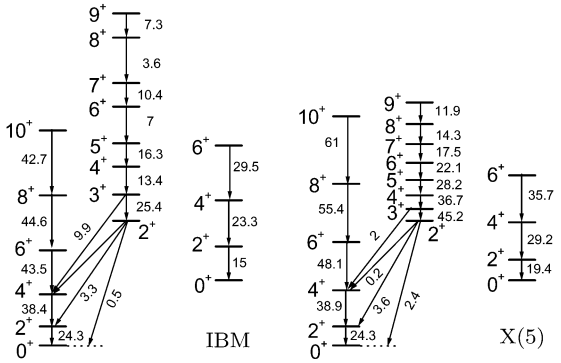


Fig.6. Level scheme of the X(5) symmetry (right) as compared with the IBM calculations (left) for $N=9$.

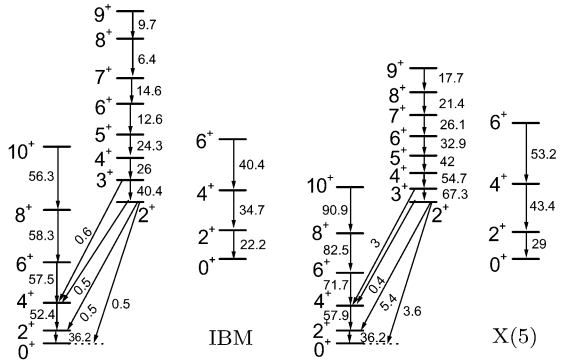


Fig.7. Level scheme of the X(5) symmetry (right) as compared with the IBM calculations (left) for $N=10$.

In Figs.6–9, we give the comparisons between the X(5)[10] predictions and the IBM calculations using the Hamiltonian (1) and the control parameters k_N^{c} (5) with the boson numbers $N=9$, 10, 11 and 12,

in which the spectra and B(E2) values are included. From Figs.6–9, we can see that IBM data can fit the X(5) predictions generally, especially in the states with $n_\gamma = 0$. The levels with $n_\gamma = 1$ are consistently higher in the IBM calculations than the X(5) predictions. Meanwhile, we find that the B(E2) values are consistently smaller in the IBM calculations than in X(5) predictions. It may be that the E2 operator takes on different forms in the IBM and in X(5) predictions. In the IBM the E2 operator contains two-body terms, while in the X(5) it contains one-body terms only. The X(5) predictions cannot be exactly reproduced by our calculations. However, we can see that the best agreement is close to the calculations with boson numbers $N = 11$ and 12 also.

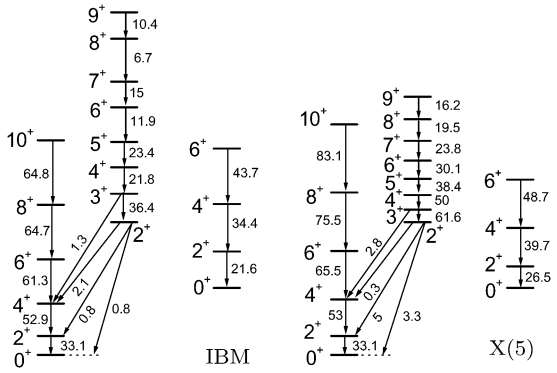


Fig.8. Level scheme of the X(5) symmetry (right) as compared with the IBM calculations (left) for $N = 11$.

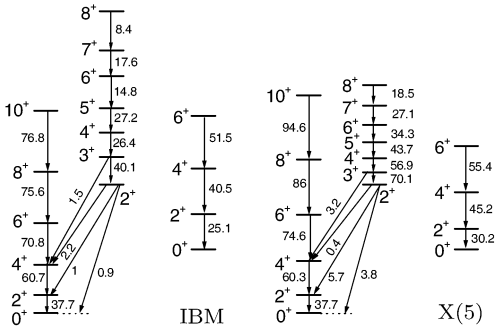


Fig.9. Level scheme of the X(5) symmetry (right) as compared with the IBM calculations (left) for $N = 12$.

In Figs.10 and 11, we give the comparisons of the ratios $R_L = E(L^+)/E(2_1^+)$ and $R = B(\mathrm{E2{:}L^+} \to (\mathrm{L\text{-}2})^+)/B(\mathrm{E2{:}2^+} \to 0^+)$ in the ground-state band (a usually adopted measure of nuclear collectivity), in X(5) predictions and the IBM calculations using the boson numbers $N = 9$, 10, 11 and 12. From Figs.10 and 11, we can see that IBM calculations fit the X(5) predictions generally. However, we find that the R_L and R values are consistently smaller in the IBM calculations than in the X(5) predictions, and that the best agreement is close to the calculations with boson numbers $N = 11$ and 12.

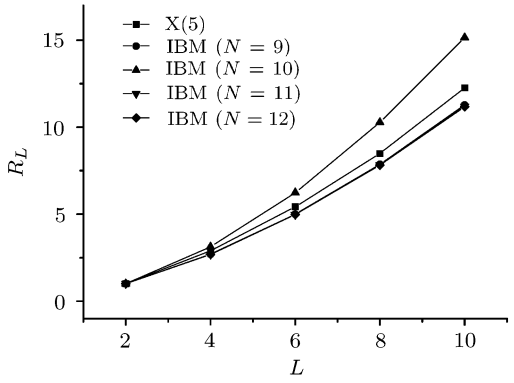


Fig.10. Comparison of the R_L values of the ground state bands in X(5) predictions with the IBM calculations with $N = 9$, 10, 11, and 12.

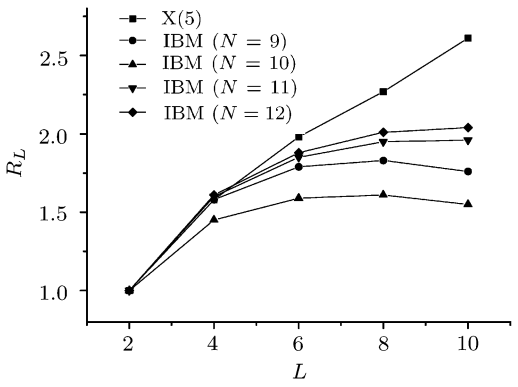


Fig.11. Comparison of the B(E2) values of the ground state bands in X(5) predictions with the IBM calculations with $N = 9$, 10, 11, and 12.

4. Comparisons between X(5), IBM and the experimental data

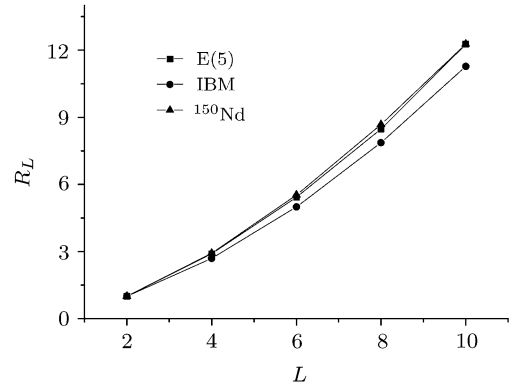


Fig.12. The experimental R_L values of the ground state bands of ^{150}Nd as compared to the E(5) and IBM calculations.

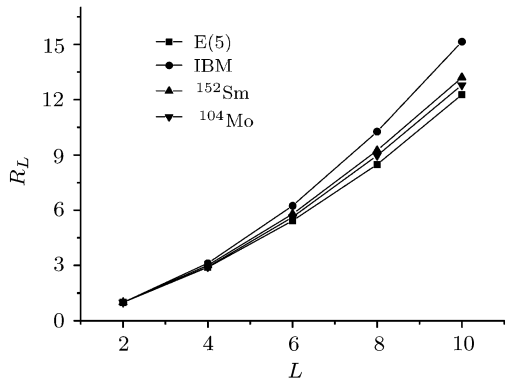


Fig.13. The experimental R_L values of the ground state bands of ^{152}Sm and ^{104}Mo, as compared to the E(5) and IBM calculations.

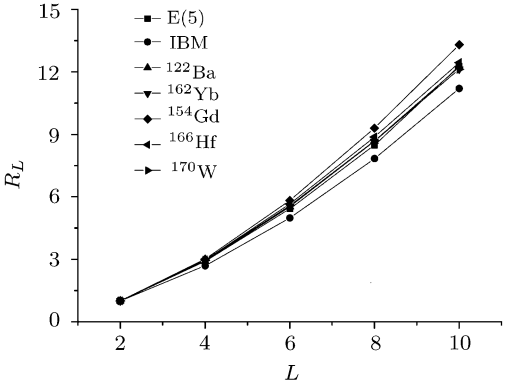


Fig.14. The experimental R_L values of the ground state bands of ^{122}Ba, ^{162}Yb, ^{154}Gd, ^{166}Hf and ^{170}W, as compared to the E(5) and IBM calculations.

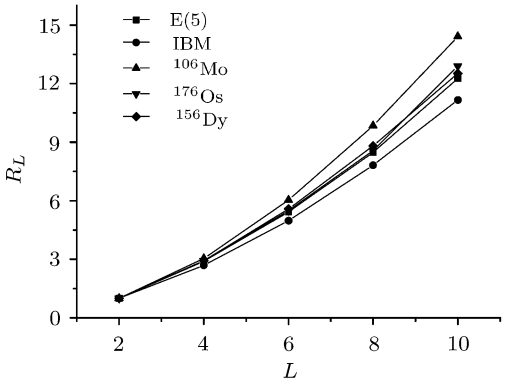


Fig.15. The experimental R_L values of the ground state bands of ^{106}Mo, ^{176}Os and ^{156}Dy, as compared to the E(5) and IBM calculations.

Nuclei in rare-earth elements are well-known examples of the U(5)–SU(3) transition. The nucleus ^{152}Sm was first proposed to be a typical one with symmetry[32]. Additional X(5)-like nuclei have then been proposed, which include ^{104,106}Mo,[33,34] ^{122}Ba,[35] ^{162}Yb,[36] ^{154}Gd,[37] ^{150}Nd,[38,39] ^{170}W,[40] ^{176}Os,[40] ^{166}Hf,[40] and ^{156}Dy.[41] In Figs.12–19, we give comparisons of the experimental R_L and R values for the X(5)-like nuclei $N = 9$, 10, 11 and 12 in the X(5) and IBM calculations. We can see that the R_L values are reproduced almost exactly in both IBM and X(5). However, the R values are not exactly reproduced by the X(5) and IBM calculations. Results in Figs.12–19 have been studied in detail in Ref.[32–41].

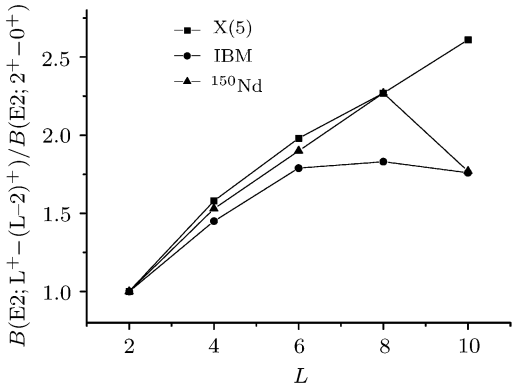


Fig.16. The experimental $B(E2;L^+-(L-2)^+)/B(E2;2^+-0^+)$ values of the ground state bands of ^{150}Nd as compared to the E(5) and IBM calculations.

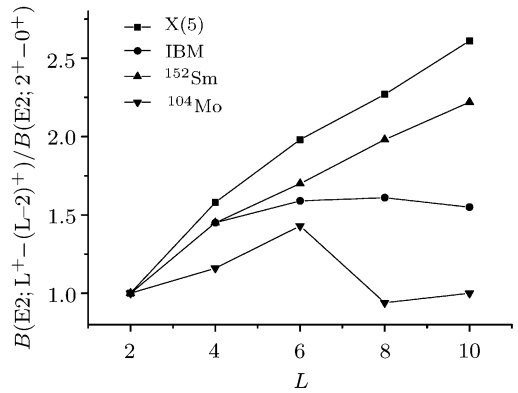


Fig.17. The experimental $B(E2;L^+-(L-2)^+)/B(E2;2^+-0^+)$ values of the ground state bands of ^{152}Sm and ^{104}Mo, as compared to the E(5) and IBM calculations.

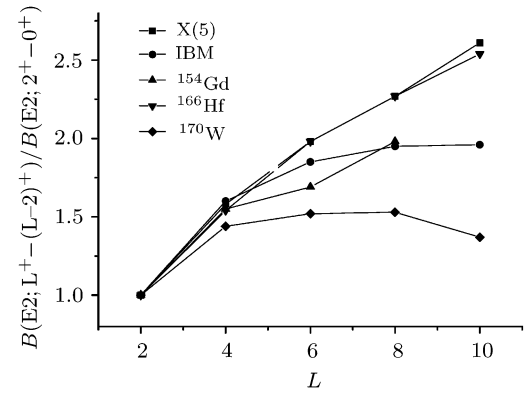


Fig.18. The experimental $B(E2;L^+-(L-2)^+)/B(E2;2^+-0^+)$ values of the ground state bands of ^{154}Gd, ^{166}Hf and ^{170}W, as compared to the E(5) and IBM calculations.

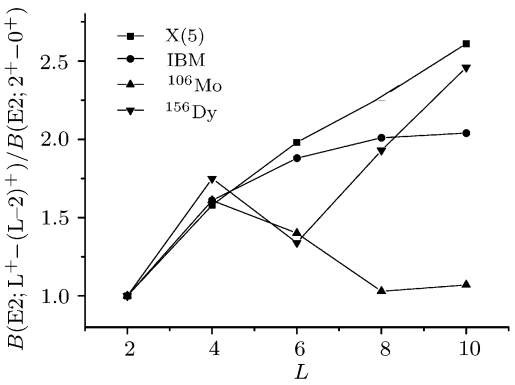


Fig.19. The experimental $B(\mathrm{E2};\mathrm{L}^+-(\mathrm{L}-2)^+)/B(\mathrm{E2};2^+-0^+)$ values of the ground state bands of ^{106}Mo and ^{156}Dy as compared to the E(5) and IBM calculations.

5. Summary

We have analysed the critical points of the shape phase transitional regions U(5)–SU(3) in the space of two control parameters k and χ, relying on the interacting boson model. A simple shape phase diagram, in which we plot the deformation parameter of the nuclear system versus the control parameters k, has been presented. We have also investigated the connection between the IBM calculations using the boson numbers consistent with real nuclei and the X(5) predictions which are obtained by a geometric approach. We find that X(5) predictions cannot be exactly reproduced by our calculations and that the best agreement is close to the calculations with boson number $N = 11$ and 12. By comparing with experimental data on X(5)-like nuclei, we find that X(5) predictions and IBM calculations can reproduce the energy ratios and E2 transition ratios.

Acknowledgement

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