Vol 16 No 7, July 2007
1009-1963/2007/16(07)/1934-07

Chinese Physics

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Hyperon–hyperon interaction in relativistic mean field model*

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(Received 22 September 2006; revised manuscript received 20 October 2006)

By using the new experimental data of ΛΛ potential, this paper has performed a full calculation for strange hadronic matter with different strangeness contents as well as its consequences on the global properties of neutron star matter in relativistic mean field model. It finds that the new weak hyperon–hyperon interaction makes the equations of state much stiffer than the result of the previous strong hyperon–hyperon interaction, and even stiffer than the result without consideration of hyperon–hyperon interaction. This new hyperon–hyperon interaction results in a maximum mass of $1.75M_\odot$ (where $M_\odot$ stands for the mass of the Sun), about 0.2–$0.5M_\odot$ larger than the previous prediction with the presence of hyperons. After examining carefully the onset densities of kaon condensation it finds that this new weak version of hyperon–hyperon interaction favours the occurrence of kaons in comparison with the strong one.

Keywords: hyperon–hyperon interaction, equations of state, neutron star, kaon condensation
PACC: 1375E, 2165, 9160F, 9760J

Nowadays hypernuclear physics is of great interest since many questions in heavy-ion physics and astrophysics are related to the effect of strangeness in matter. States with large strangeness per baryon have been supposed more stable than normal matter since years ago,[1] and the possible existence of such bound states[2−9] may have important consequence for the equation of state (EOS) of neutron stars at high densities[2−4] which remains unknown.

In relativistic mean field (RMF) theory,[5,10] if the hyperon–hyperon (YY) interaction is only slightly attractive through the usual σ and ω isoscalar field and the isovector ρ field, the potential $U_{\rm Y}^{(\rm Y')}$ (here $U_i^{(j)}$ denotes the potential depth of a baryon species i in matter of baryon species j), is rather weak (less than 30 MeV deep), and the strange hadronic matter is bound up to $f_{\rm s} \simeq 1.5$, where $f_{\rm s}$ stands for the number of strange quarks per baryon. When σ^* and ϕ were introduced[11] to reproduce experimentally the much deeper Λ potential well depth in Λ matter (i.e. $U_\Lambda^\Lambda \simeq 20$MeV), the matter gets more deeply bound at large $f_{\rm s}$, and even pure Ξ matter (where $f_{\rm s} = 2$) can reach a negative minimum binding (for example $E/A = -8.9$MeV[6]). Furthermore the introduction of these two hidden strange mesons results in a reduction of the predicted maximum neutron star mass,[2] i.e, from $M_{\rm max} = 1.8M_\odot$ down to 1.4–$1.6M_\odot$. Schaffner *et al*[2] also checked the effects of an enhanced YY interaction by increasing the coupling constants of Λ to the σ^* meson and in the mean time setting $g_{\sigma^*\Xi} = 2_{\sigma^*\Lambda}$, where $g_{\sigma^*\Xi}$ and $g_{\sigma^*\Lambda}$ are the coupling constants of σ^* to Ξ and Λ respectively.

While by the recent data analysis of ${}^{6}_{\Lambda\Lambda}$He double hypernucleus, Takahashi *et al*[12] obtained a much smaller ΛΛ interaction energy $\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.20^{+0.18}_{-0.11}$MeV, then the corresponding potential well depth has the value $U_\Lambda^\Lambda \simeq 5$MeV, indicating a much weaker attraction. By using this new weak version of Y–Y interaction several works[13,14] have been done to study EOSs of strange hadronic matter and neutron star matter. It is shown that a change of the hyperon coupling constants may have crucial effect on the sat-

*Project supported by the National Natural Science Foundation of China (Grant Nos 10575119 and 10235030), the Knowledge Innovation Project (Grant No KJCX2-SW-N02) of Chinese Academy of Sciences, the Major State Basic Research Development Program of China (Grant No G2000077400), the Major Prophase Research Project of Fundamental Research of the Ministry of Science and Technology of China (Grant No 2002CCB00200), and the Asia–Europe Link project (Grant No CN/ASIA-LINK/008(94791)) of the European Commission.
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uration properties and stabilities of strange hadronic matter, and also on the chemical composition of neutron stars.

Our motivation here is to investigate the properties of strange hadronic matter with different strangeness contents by using this new weak version of YY interaction and also to illustrate its consequences on the global properties of neutron star matter, since at present this weak YY interaction may seem quite realistic as far as the available hyperon data as well as microscopic calculation[15,16] are concerned.

When we describe the properties of dense matter, it is also important to study the possibility of kaon condensation since it may affect the dynamical evolution of protoneutron stars[17] and the cooling rate of neutron stars via enhanced neutrino emissions.[18]

Since the proposition by Kaplan and Nelson,[19] the possibility of kaon condensation and its implications for astrophysical phenomena of neutron stars have been extensively discussed by many physicists.[17−31] Various theoretical models, however, gave different results addressing this issue. With the chiral Lagrangian approaches, Lee *et al*[18,20−23] predicted a relatively low critical density: $\sim$(2–4)b_0, where b_0 is the normal nuclear matter density. Brown *et al*[24] proposed a unified treatment of normal nuclear matter and kaon condensed matter by using BR scaling. The former is described by Walecka's mean field theory while the latter by mean field chiral perturbation theory. In this scheme the critical density was $\sim$(3–4)b_0. In addition, Ellis *et al*[25,26] explored this problem on the basis of Kaplan–Nelson Lagrangian for the kaon–baryon interactions and a Walecka-type relativistic field theoretical approach for the baryon–baryon interactions. They found that when the matter contained only nucleons and leptons, kaons condensed around $\sim$(3–4)b_0 as found earlier by using the chiral model, while the inclusion of hyperons in matter made the appearance of kaons very uncertain, even impossible. For a recent review one may refer to Ref.[27]. Westerberg *et al*[28] explored also the kaon condensation in the bound-state approach in the Skyrme model on a 3-sphere, and the resulting critical point was $\sim 3.7b_0$, which fell in the early predicted range. From the microscopic point of view, the nonrelativistic or relativistic reaction matrix theory, that is the so-called Brueckner–Hartree–Fock (BHF)[29,32,33] or Dirac–Brueckner method,[30] also gave very similar results with those phenomenological models in the absence of hyperons. Then in the framework of a chiral quark model Kim *et al*[31] studied the kaon condensation with the use of formalism that goes beyond the linear-density approximation commonly used in the literature, and the critical density was estimated to be $\sim$(3–4)b_0, where the linear-density approximation may still be valid. In this paper, we follow and extend the method of Ellis *et al*,[25,26] and mainly concentrate on the onset mechanisms of kaon condensation to explore the influence of YY interaction on the kaon threshold density.

We first review briefly the extended RMF model including hyperons adopted in our calculation, then apply it to the study of strange hadronic matter as well as the global properties of neutron stars. Finally the calculated onset densities of kaon condensation are discussed in neutron star matter.

The Lagrangian of the theory is

$$\begin{aligned}\mathcal{L}_B = & \sum_B \bar{\psi}_B(\mathrm{i}\gamma^\mu\partial_\mu - g_{\omega B}\gamma^\mu V_\mu - g_{\phi B}\gamma^\mu\phi_\mu \\ & -g_{\rho B}\gamma^\mu\tau^a\rho_\mu^a - M_B + g_{sB}\sigma + g_{\sigma^* B}\sigma^*)\psi_B \\ & -\frac{1}{4}G_{\mu\nu}G^{\nu\mu} + \frac{1}{2}m_\omega^2 V_\mu V^\mu + \frac{1}{4}c_3\left(V_\mu V^\mu\right)^2 \\ & +\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\ & -\frac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu} \\ & -\frac{1}{4}S_{\mu\nu}S^{\mu\nu} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu \\ & +\frac{1}{2}\left(\partial_\nu\sigma^*\partial^\nu\sigma^* - m_{\sigma^*}^2{\sigma^*}^2\right), \end{aligned} \tag{1}$$

where $G_{\mu\nu}(=\partial_\mu V_\nu - \partial_\nu V_\mu)$, $S_{\mu\nu}(=\partial_\mu\phi_\nu - \partial_\nu\phi_\mu)$ and $R_{\mu\nu}^a(=\partial_\mu\rho_\nu^a - \partial_\nu\rho_\mu^a + g_\rho\varepsilon_{abc}\rho_\mu^b\rho_\nu^c)$ are the ω field, the ϕ field and the ρ field strength tensor respectively. We take TM1[34] parameter set as nucleon parametrization in the following calculation, which is possibly capable to provide good descriptions of the properties of dense matter under the wide range of density[35] relevant to astrophysical interests. The coupling constants for the hyperons to the scalar σ and σ^* mesons are constrained by the hyperon potentials compatible with hypernuclear data[2,6]: $U_\Lambda^{(\mathrm{N})}(\rho_0) = -30\,\mathrm{MeV}, U_\Sigma^{(\mathrm{N})}(\rho_0) = +30\,\mathrm{MeV}, U_\Xi^{(\mathrm{N})}(\rho_0) = -18\,\mathrm{MeV}$, and $U_\Lambda^\Lambda(\rho_0) \simeq U_\Xi^\Xi(\rho_0) \simeq -20\,\mathrm{MeV}$(strong) or $-5\,\mathrm{MeV}$(weak). The coupling constants to the ω, ρ, ϕ vector mesons are fixed by using SU(6) symmetry.[36] Note that in the strong case this yielded $g_{\sigma^*\Lambda}/g_{\sigma\mathrm{N}} = 0.79$, $g_{\sigma^*\Xi}/g_{\sigma\mathrm{N}} = 1.25$ and in the weak case $g_{\sigma^*\Lambda}/g_{\sigma\mathrm{N}} = 0.37$, $g_{\sigma^*\Xi}/g_{\sigma\mathrm{N}} = 1.15$, which is obvious that for the latter one the value of $g_{\sigma^*\Lambda}/g_{\sigma\mathrm{N}}$ is far below the range of 0.7–1.2.

For infinite nuclear matter, introducing the mean field approximation, i.e. the meson fields are replaced by their mean values, the equations of motion can be simplified as

$$m_\sigma^2\sigma + g_2\sigma^2 + g_3\sigma^3 = \sum_B g_{\sigma B}\frac{2J_B+1}{2}n_B^{\rm s}, \quad (2)$$

$$m_\omega^2\omega + c_3\omega^3 = \sum_B g_{\omega B}\,(2J_B+1)\,k_B^3/(6\pi^2), \quad (3)$$

$$m_\rho^2\rho = \sum_B g_{\rho B}I_{3B}\,(2J_B+1)\,k_B^3/(6\pi^2), \quad (4)$$

$$m_{\sigma^*}^2\sigma^* = \sum_B g_{\sigma^* B}\frac{2J_B+1}{2}n_B^{\rm s}, \quad (5)$$

$$m_\phi^2\phi = \sum_B g_{\phi B}\,(2J_B+1)\,k_B^3/(6\pi^2), \quad (6)$$

where k_B is the Fermi momentum of the baryon species B. J_B and I_{3B} denote the spin and the isospin projection of baryon B. And baryon scalar density $n_B^{\rm s}$ is defined as

$$n_B^{\rm s} = \frac{1}{\pi^2}\int_0^{k_B}{\rm d}k\,k^2\frac{m_B^*}{\sqrt{m_B^{*2}+k_B^2}}, \quad (7)$$

where

$$m_B^* = m_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^* \quad (8)$$

is the effective mass of the baryon species B.

The ω, ρ and ϕ fields can be obtained respectively by Eqs.(3), (4) and (6). The scalar fields σ and σ^* can be solved from the coupled Eqs.(2), (5), (7) and (8) by iteration. The total energy density of the uniform matter is then given by

$$\begin{aligned}\varepsilon = &\sum_B\frac{2J_B+1}{2\pi^2}\int_0^{k_B}\sqrt{k^2+m_B^{*2}}\;k^2{\rm d}k\\ &+\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{3}g_2\sigma^3+\frac{1}{4}g_3\sigma^4\\ &+\frac{1}{2}m_\omega^2\omega^2+\frac{3}{4}c_3\omega^4+\frac{1}{2}m_\rho^2\rho^2\\ &+\frac{1}{2}m_{\sigma^*}^2\sigma^{*2}+\frac{1}{2}m_\phi^2\phi^2. \end{aligned} \quad (9)$$

For symmetric strange hadronic matter there are only two conserved charges: the baryon number $n_{\rm b} = n_{\rm N} + n_\Lambda + n_\Sigma + n_\Xi$ and the strangeness number $n_{\rm s} = n_\Lambda + n_\Sigma + 2n_\Xi$, then one can define the parameter which specifies the strangeness contents in the system:

$$f_{\rm s} = \frac{n_{\rm s}}{n_{\rm b}} = \frac{n_\Lambda + n_\Sigma + 2n_\Xi}{n_{\rm b}}, \quad (10)$$

which is regarded more and more as another important degree of freedom, in similarity to the nuclear asymmetry $\beta = N/Z$. The hyperon fractions are determined through the Hugenholtz–van-Hove theorem so as to ensure chemical equilibrium written as $2\mu_\Lambda = 2\mu_\Sigma = \mu_{\rm N} + \mu_\Xi$ where

$$\mu_B = \sqrt{m_B^{*2}+k_B^2} + g_{\omega B}w_0 + g_{\phi B}\phi_0. \quad (11)$$

As usual, we would like to subtract the baryon masses in the total energy per baryon (9) of the strange matter and to study the binding energy per baryon expressed as

$$E_{\rm b} = \frac{1}{n_{\rm b}}(\varepsilon - Y_{\rm N}m_{\rm N} - Y_\Lambda m_\Lambda - Y_\Sigma m_\Sigma - Y_\Xi m_\Xi), \quad (12)$$

where $Y_B(B = {\rm N}, \Lambda, \Sigma, \Xi)$ denotes concentrations of particular baryons.

Figure 1 shows the minimum binding energy as a function of $f_{\rm s}$. The strong YY interaction leads to a much more bound state at large $f_{\rm s}$ than the other two cases. The calculated results with the weak YY interaction are very similar to (even less bound than) the one without strange mesons $(\sigma*, \phi)$, the hadronic matter is bound up to $f_{\rm s} \simeq 1.5$ consistent with former study[6] although with different parameter set. In order to see the role of strange mesons $(\sigma*, \phi)$ more clearly, we shall to discuss the possible implication for compact astrophysics neutral $\beta-$equilibrated neutron star matter.

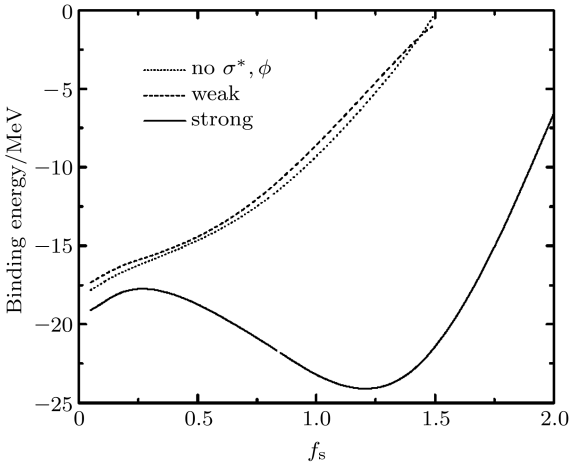


Fig.1. The minimized energy per baryon $E_{\rm b}$ as functions of strangeness contents $f_{\rm s}$ calculated for the strong and weak hyperon–hyperon interaction models in symmetric strange hadronic matter, in comparison with the one without strange mesons $(\sigma*, \phi)$. See text for details.

To add leptons $L_{\rm l} = \sum_{L={\rm e},\mu}\bar{\psi}_L({\rm i}\gamma^\mu\partial_\mu - m_{\rm L})\psi_{\rm L}$ to the above Lagrangian of hadronic matter, the charge neutrality requires:

$$n_{\rm p} + n_{\Sigma^+} = n_{\rm e} + n_\mu + n_{\Sigma^-} + n_{\Xi^-}, \quad (13)$$

and equilibrium under the weak process (B_1 and B_2 denote baryons)

$$B_1 \to B_2 + L \qquad B_2 + L \to B_1$$

leads to the following relations among the involved chemical potentials:

$$\mu_{\rm p} = \mu_{\Sigma^+} = \mu_{\rm n} - \mu_{\rm e},\ \mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_{\rm n},$$
$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_{\rm n} + \mu_{\rm e},\ \mu_\mu = \mu_{\rm e}. \tag{14}$$

The EOSs of the system can be calculated as usual. In Fig.2 we give the calculated EOSs for the strong and weak YY interaction models, in comparison with the one without strange mesons ($\sigma*, \phi$) as well as the results without hyperons, namely (npeμ). Inevitably, the inclusion of hyperons makes the EOS much softer because of its serving as an additional degree of freedom. With the strong YY interaction one gets the expected softest EOS because of the attractive nature of YY interaction below $0.9\,{\rm fm}^{-3}$, but on higher density it may be stiffer than the one without ($\sigma*, \phi$). Also one can see clearly that the resulting EOS with the weak YY interaction is stronger than the corresponding EOS without strange mesons(which is indicated in the above figure of hadronic matter study).

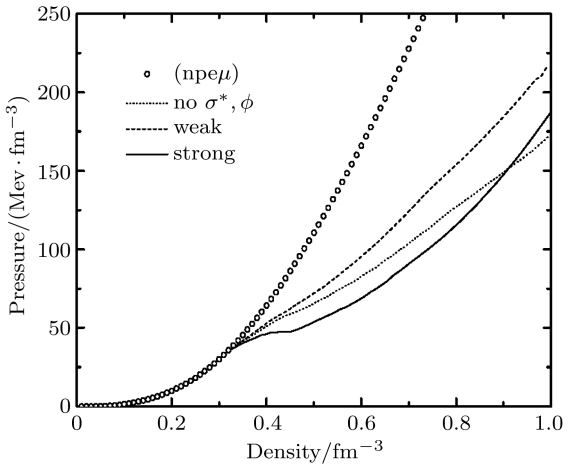


Fig.2. The equations of state calculated for the strong and weak hyperon–hyperon interaction models in neutron star matter, in comparison with the one without strange mesons ($\sigma*, \phi$), and also the results without hyperons. See text for details.

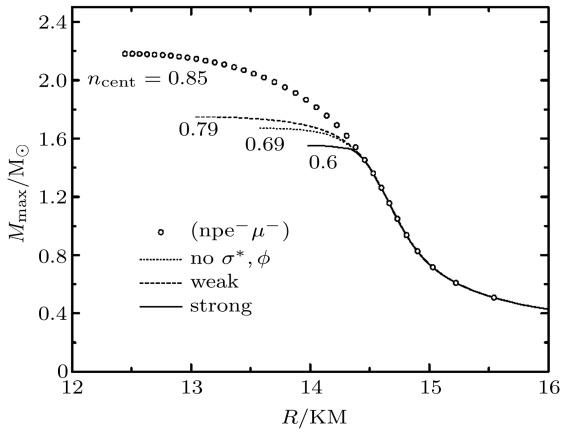


Fig.3. The mass–radius relation calculated for the strong and weak hyperon–hyperon interaction models, in comparison with the one without strange mesons ($\sigma*, \phi$), and also the results without hyperons. See text for details.

For a certain EOS the global properties of neutron star can be obtained from the hydrostatic equilibrium equations of Tolman, Oppenheimer and Volkoff. As shown in Fig.3, the predicted maximum masses increase in an order with the increasing of stiffness of the corresponding EOSs. The inclusion of hyperons lowers the value from $2.18{\rm M}_\odot$ at $n_{\rm b} = 0.85\,{\rm fm}^{-3}$ down to $1.66{\rm M}_\odot$ at $n_{\rm b} = 0.69\,{\rm fm}^{-3}$, then down to $1.56{\rm M}_\odot$ at $n_{\rm b} = 0.6\,{\rm fm}^{-3}$ if the strong YY interaction is considered, while the weak YY interaction lifts this value to $1.75{\rm M}_\odot$ at $n_{\rm b} = 0.79\,{\rm fm}^{-3}$. This predicted maximum mass is about 0.2–0.5 ${\rm M}_\odot$ which is larger than the usual prediction with hyperon interaction before.[2,36−39]

In the following we explore the influence of YY interaction on the kaon condensation in neutron star matter. The total systemic Lagrangian consists of two parts of contributions: $\mathcal{L}_{\rm H} = \mathcal{L}_{\rm B} + \mathcal{L}_{\rm K}$. We evaluate the kaon–nucleon sector in the energy density of the kaon-condensed phase of matter in terms of the effective chiral Lagrangian density which was suggested by Kaplan and Nelson[19] and extensively investigated afterwards.[20] In the Kaplan–Nelson model, the SU(3) × SU(3) chiral Lagrangian density is expressed as

$$\begin{aligned}\mathcal{L}_{\rm K} =\ & \frac{f^2}{4}{\rm Tr}\partial_\mu U\partial^\mu U^\dagger + {\rm Tr}\bar{B}({\rm i}\gamma^\mu D_\mu - m_B)B \\ & + F\,{\rm Tr}\bar{B}\gamma^\mu\gamma_5[\mathcal{A}_\mu, B] + D\,{\rm Tr}\bar{B}\gamma^\mu\gamma_5\{\mathcal{A}_\mu, B\} \\ & + c{\rm Tr}\mathcal{M}(U + U^\dagger) + a_1{\rm Tr}\bar{B}(\xi\mathcal{M}\xi + \xi^\dagger\mathcal{M}\xi^\dagger)B \\ & + a_2{\rm Tr}\bar{B}B(\xi\mathcal{M}\xi + \xi^\dagger\mathcal{M}\xi^\dagger) \\ & + a_3{\rm Tr}\bar{B}B\ {\rm Tr}(\xi\mathcal{M}\xi + \xi^\dagger\mathcal{M}\xi^\dagger). \end{aligned} \tag{15}$$

where we adopt the same notations and symbols as in Ref.[19]. The above Lagrangian density includes

an octet of pseudoscalar masons $\mathcal{M}$ and an octet of baryons B. The first four terms conserve the chiral symmetry. $f = 93\,\mathrm{MeV}$ is the pion decay constant. The constants D and F are given by $D = 0.81$ and $F = 0.44$. The coefficients $a_1 m_\mathrm{s}, a_2 m_\mathrm{s}$, $a_3 m_\mathrm{s}$, and c determine the strength of the chiral symmetry breaking in the last four terms. We adopt $a_1 m_\mathrm{s} = -67\,\mathrm{MeV}$ and $a_2 m_\mathrm{s} = 134\,\mathrm{MeV}$ obtained from the baryon mass splittings.[21] The constant c is related to the bare kaon mass by the Gell-Mann–Oakes–Renner relation $m_\mathrm{K}^2 = 2\,cm_\mathrm{s}/f^2$. The parameter $a_3 m_s$ had remained largely uncertain for many years due to our poor knowledge of the strangeness content of the proton and the kaon-nucleon sigma term Σ_{KN}.[19,21,22,40] Fortunately, the large ambiguity in this parameter has been settled recently by Dong, Lagaë and Liu[41] with small error based on the lattice calculations. In the present calculations we adopt the value of $a_3 m_\mathrm{s}$ in the range $-310\,\mathrm{MeV} < a_3 m_\mathrm{s} < -134\,\mathrm{MeV}$ as done in Refs.[18,23]. $a_3 m_\mathrm{s}$ measures the proton strangeness content and plays an essential role in determining the threshold density for kaon condensation[18] since it provides the dominant attraction in the kaon-nucleon interaction via the sigma term $\Sigma_{KN} = -(a_1 + 2a_2 + 4a_3)m_\mathrm{s}/2$. The above specified range of $a_3 m_3$ from $-310\,\mathrm{MeV}$ to $-134\,\mathrm{MeV}$ corresponds to a reasonable range $0.2 > \langle\bar{s}s\rangle_\mathrm{p}/\langle\bar{u}u + \bar{d}d + \bar{s}s\rangle_\mathrm{p} > 0$ of the proton strangeness content.[40]

Kaons are treated in the mean field approximation as usual[18]: $K^\pm = \dfrac{1}{\sqrt{2}} f\theta \mathrm{e}^{\pm \mathrm{i}\mu t}$, then as a symbol of the condensate amplitude, $\theta = 0$ accounts for the nonappearance of kaon condensation. All the contribution of kaons can be included in the effective baryon mass and chemical potential:

$$\begin{aligned} m_B^* = \; & m_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^* \\ & + [(a_1 + a_2)(1 + Y_B q_B) + (a_1 - a_2)(q_B - Y_B) \\ & + 4a_3] m_\mathrm{s} \sin^2 \frac{1}{2}\theta, \end{aligned} \tag{16}$$

and

$$\begin{aligned} \mu_B = \; & \sqrt{m_B^{*2} + k_B^2} + g_{\omega B}\omega_0 + g_{\rho B} t_{3B} b_0 + g_{\phi \mathrm{N}}\phi_0 \\ & - (Y_B + q_B)\mu \sin^2 \frac{1}{2}\theta, \end{aligned} \tag{17}$$

where Y_B denotes the supercharge and q_B the electric charge of the baryon species B, and $\mu = \mu_\mathrm{e} = \mu_\mu$ accounts for the chemical potential of kaons. Note here $g_{\sigma^* \mathrm{N}} = 0$ since the strange mesons do not couple to nucleons.

Particularly, taking notice of the possible isospin mixing of Λ and Σ^0, we find that the $\Lambda - \Sigma^0$ mass matrix needs to be diagonalized[42] in the first place of the grand potential calculation, as a result the corresponding coefficient in the square brackets of the last term of Eq.(16) would be $\dfrac{5}{3}(a_1 + a_2) + 4a_3$ for Λ and $\dfrac{1}{3}(a_1 + a_2) + 4a_3$ for Σ^0.

Employing $\theta = 0$, one can obtain the threshold density n_{crit} of condensation (That is Table 1) by solving the following equation:

$$\mu^2 + 2\mu\alpha - m_K^{*2} = 0, \tag{18}$$

where $\alpha = (2n_\mathrm{p} + n_\mathrm{n} - n_{\Sigma^-} + n_{\Sigma^+} + 2n_{\Xi^-} - n_{\Xi^0})/(4f_\pi^2)$, and the effective mass of kaons m_K^{*2} can be written elaborately as

$$\begin{aligned} m_\mathrm{K}^{*2} = \; & m_\mathrm{K}^2 + \Big[(2a_1 + 2a_2 + 4a_3) n_\mathrm{p}^\mathrm{s} + (2a_2 + 4a_3) n_\mathrm{n}^\mathrm{s} \\ & + \left(\frac{5}{3}(a_1 + a_2) + 4a_3 \right) n_\Lambda^\mathrm{s} \\ & + (2a_2 + 4a_3) n_{\Sigma^-}^\mathrm{s} + (2a_1 + 4a_3) n_{\Sigma^+}^\mathrm{s} \\ & + \left(\frac{1}{3}(a_1 + a_2) + 4a_3 \right) n_{\Sigma^0}^\mathrm{s} \\ & + (2a_1 + 2a_2 + 4a_3) n_{\Xi^-}^\mathrm{s} \\ & + (2a_2 + 4a_3) n_{\Xi^0}^s \Big] \frac{m_\mathrm{s}}{2f_\pi^2}. \end{aligned} \tag{19}$$

It is shown[19] that in the pure nucleonic case the kaon–nucleon sigma term Σ_KN provides a strongly attractive interaction between kaons and nucleons. This attraction reduces the K^- energy ω_K in nuclear medium and it becomes strong enough at a high enough baryon density above which the kaon condensed phase is energetically favourable, then K^- appears macroscopically as a Bose–Einstein condensate. The existence region of hyperons, however, may overlap with kaon condensation. Concerning the competition or coexistence problem of these phases, we point out that kaon threshold is pushed up to a higher baryon number density in the presence of the negatively charged hyperons in comparison with the neutron-star matter with no hyperons[25,26,43] which is shown clearly in Table 1. One may understand this in the following way. Since more and more negative charge of the system is carried by the negatively charged hyperons in place of the electrons to satisfy the charge neutrality with the increase of baryon number density, the charge chemical potential $\mu[= (3\pi^2 n_\mathrm{e})^{1/3}]$ gets smaller with the increase in the baryon number density (see Fig.4), which makes the onset condition for kaon condensation, $\omega_\mathrm{K} = \mu$, difficult to satisfy.

Table 1. Threshold densities for kaon condensation (fm^{-3}).

$a_3 m_s$/MeV	no Y	with Y (no YY)	with Y(with YY)	
			weak	strong
−310	0.42	0.45	0.44	0.47
−222	0.53	0.89	0.87	0.99
−134	0.67	—	—	—
[c]	0.85	0.69	0.79	0.60

[c] is the corresponding central density of a neutron star in the absence of kaons.

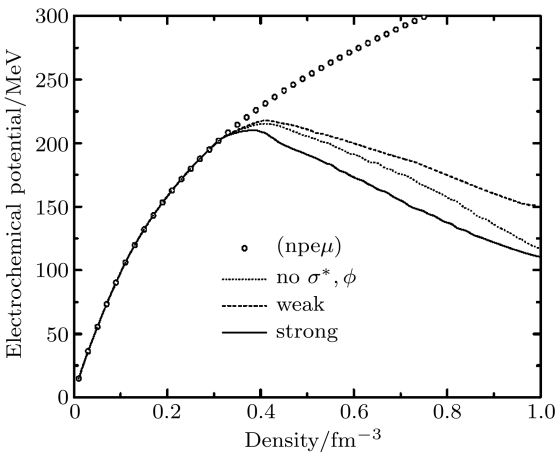


Fig.4. The electrochemical potentials calculated for the strong and weak hyperon–hyperon interaction models, in comparison with the one without strange mesons ($\sigma*, \phi$), and also the results without hyperons. See text for details.

Specifically, when there are no hyperons, with the increase of the magnitude of $a_3 m_s$, the critical point decreases: this results from the enhancement of kaon-nucleon attraction which makes the effective energy of kaons (ω_K) drops. In addition, the central density n_{cent} of a star, which contains only the corresponding baryons but no kaons, is more than the critical density n_{crit} with baryons as well as kaons in all cases. It implies that kaons will be present without doubt. This is also true with hyperons included when the magnitude of $a_3 m_s$ is the largest. When the magnitude of $a_3 m_s$ decreases, the onset densities are much postponed because of the presence of hyperons (larger than their corresponding n_{cent}s), so the kaon condensation would be impossible. The YY interaction may enhance the delay due to hyperons (see the strong case), or weaken it (see the weak case). Particularly, with the weak YY interaction the threshold density of kaons slightly moves up by 0.01–0.02 fm^{-3} because the delay appearance of hyperons.

In summary, we have shown that the presence of weak YY interaction makes the strange hadronic matter less bound than the strong case, stiffens the EOS of corresponding neutron star matter and results in a larger maximum mass prediction. In addition, it increases the chemical potential of electrons, therefore enhances the possibility of kaon condensation. Our procedure, however, is still far from good due to self-consistent problem, that is, the treatments of baryons and kaons are not on the same footing (the KY interaction is not incorporated properly), and also a fully understanding of the hyperon degrees of freedom is in bad need of the hyperon data. By improving these results our future work can be more decisive.

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