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Stochastic period-doubling bifurcation analysis of stochastic Bonhoeffer–van der Pol system*

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In this paper, the Chebyshev polynomial approximation is applied to the problem of stochastic period-doubling bifurcation of a stochastic Bonhoeffer–van der Pol (BVP for short) system with a bounded random parameter. In the analysis, the stochastic BVP system is transformed by the Chebyshev polynomial approximation into an equivalent deterministic system, whose response can be readily obtained by conventional numerical methods. In this way we have explored plenty of stochastic period-doubling bifurcation phenomena of the stochastic BVP system. The numerical simulations show that the behaviour of the stochastic period-doubling bifurcation in the stochastic BVP system is by and large similar to that in the deterministic mean-parameter BVP system, but there are still some featured differences between them. For example, in the stochastic dynamic system the period-doubling bifurcation point diffuses into a critical interval and the location of the critical interval shifts with the variation of intensity of the random parameter. The obtained results show that Chebyshev polynomial approximation is an effective approach to dynamical problems in some typical nonlinear systems with a bounded random parameter of an arch-like probability density function.

Keywords: Chebyshev polynomial approximation, stochastic Bonhoeffer–van der Pol system, stochastic period-doubling bifurcation, bounded random parameter

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1. Introduction

A system with random parameters is usually called a stochastic system. The adjective, 'stochastic', here implies 'dependent on some random parameter'. There are three basic mathematical methods available for the stochastic structure analysis. The first method is the Monte Carlo method,[1] which is simple and universal, but usually involved with quite a amount of computational effort. The second method is the stochastic perturbation method,[2] which is involved with the least computational effort, but usually restricted to the systems with random variables of small fluctuations only. The third method is the orthogonal polynomial approximation method, which was introduced in Refs.[3,4] and improved by Li.[5] This method does not require the assumption of small random perturbation, thus providing a larger applicability. The Chebyshev polynomial approximation and Gegenbauer polynomial approximation were first applied to the evolutionary random response problem of a stochastic structure,[6−9] and then further applied to the bifurcation and chaos problems of stochastic Duffing system and stochastic van der Pol system.[10−15]

In the present paper, we consider the BVP system which is simply a two-dimensional system and models an excited nerve membrane to describe the propagation of an electrical impulse along the membrane of a nerve cell.[16] With a harmonic excitation $f\cos(\omega t)$ the BVP system can be written as

$$\begin{cases} \dot{x} = x - \dfrac{1}{3}x^3 - y + f\cos(\omega t), \\ \dot{y} = c(x + a - by). \end{cases}$$

This BVP system, as an interesting dynamic system in biology and physics, exhibits plenty of deterministic nonlinear phenomena.[17−19] Now we are interested in studying a stochastic version of BVP system with a bounded random parameter. Recently stochastic bifurcation and chaos in some typical dynamic models were successfully analysed by using the Chebyshev polynomial approximation.[10,12] We will use the same strategy to explore the stochastic period-

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doubling bifurcation in the stochastic BVP system here.

2. Chebyshev polynomial approximation for stochastic BVP system

The random variable we considered is of an arch-like distribution, of which the probability distribution function (PDF) is shown in Fig.1 and can be expressed as follows:[6]

$$p(\xi) = \begin{cases} (2/\pi)\sqrt{1-\xi^2} & |\xi| \le 1, \\ 0 & |\xi| > 1. \end{cases} \tag{1}$$

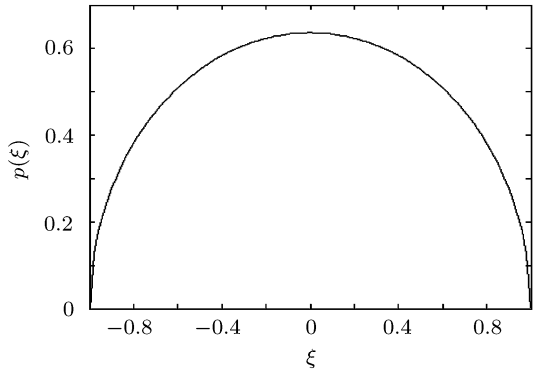


Fig.1. The arch-like PDF curve for random variable.

In the orthogonal polynomial approximation method, choosing the orthogonal polynomial basis depends on the PDF of the random variable in the equation. As the orthogonal polynomial basis for the arch-like PDF, the only choice is the second kind of Cheyshev polynomials which can be expressed as follows:

$$H_n(\xi) = \sum_{k=0}^{[n/2]} (-1)^k \frac{(n-k)!}{k!(n-2k)!}(2\xi)^{n-2k} \tag{2}$$

while the recurrent formula for the second kind of Chebyshev polynomials is

$$\xi H_n(\xi) = \frac{1}{2}[H_{n-1}(\xi) H_{n+1}(\xi)]. \tag{3}$$

The orthogonal relationships for the second kind of Chebyshev polynomials can be derived as

$$\int_{-1}^{1} \frac{2}{\pi}\sqrt{1-\xi^2} H_i(\xi) H_j(\xi) \mathrm{d}\xi = \begin{cases} 1, & i = j, \\ 0, & i \ne j. \end{cases} \tag{4}$$

Due to the orthogonality of Chebyshev polynomials, it is known that any function $f(\xi) \subset L^2$ can be expanded into a series of $H_i(\xi)$

$$f(\xi) = \sum_{i=0}^{\infty} c_i H_i(\xi), \tag{5}$$

where

$$c_i = \int_{-1}^{1} p(\xi) f(\xi) H_i(\xi) \mathrm{d}\xi. \tag{6}$$

It is obvious that in practice only a finite number of items can be taken in expression (5), and then the sum is merely an approximation with a minimum mean square residual.[20]

Consider the BVP system with a random parameter, the differential equation for the response can be expressed as

$$\begin{cases} \dot{x} = x - \dfrac{1}{3}x^3 - y + f\cos(\omega t), \\ \dot{y} = c(x + a - by), \end{cases} \tag{7}$$

where $f\cos(\omega t)$ is a given harmonic excitation, a and c are deterministic constants, and b is a random parameter which can be expressed in the form

$$b = \bar{b} + \sigma\xi, \tag{8}$$

where $\bar{b}$ is the mean value of b, σ is a constant, and ξ is a bounded random variable defined on $[-1, 1]$ with a given arch-like PDF. The system itself is stochastic, so are the responses. Thus the responses of Eq.(7) should be a function of both time t and random variable ξ, namely

$$\begin{cases} x = x(t, \xi), \\ y = y(t, \xi). \end{cases} \tag{9}$$

According to the orthogonal polynomial approximation, the responses of Eq.(7) can be expressed as the following series:

$$\begin{cases} x(t, \xi) = \sum\limits_{i=0}^{N} x_i(t) H_i(\xi), \\ y(t, \xi) = \sum\limits_{i=0}^{N} y_i(t) H_i(\xi), \end{cases} \tag{10}$$

where the subscript i runs for the sequential number of Chebyshev polynomials, N represents the largest order of the polynomials we have taken. It is worth noting again that only if $N \to \infty$, then $\sum_{i=0}^{N} x_i(t) H_i(\xi)$ and $\sum_{i=0}^{N} y_i(t) H_i(\xi)$ are strictly equivalent to the responses of the stochastic BVP system, $x(t, \xi)$ and

$y(t,\xi)$, respectively. In this paper we take $N=4$, then

$$\begin{cases} x(t,\xi)=\sum\limits_{i=0}^{4} x_i(t)H_i(\xi), \\ y(t,\xi)=\sum\limits_{i=0}^{4} y_i(t)H_i(\xi), \end{cases} \tag{11}$$

which are approximate solutions with a minimal mean square residual error.

Substituting expressions (8) and (11) into Eq.(7), we have

$$\begin{aligned} \sum_{i=0}^{4}\dot{x}_i(t)H_i(\xi)=&\sum_{i=0}^{4}x_i(t)H_i(\xi)-\frac{1}{3}\Big[\sum_{i=0}^{4}x_i(t)H_i(\xi)\Big]^3 \\ &-\sum_{i=0}^{4}y_i(t)H_i(\xi)+f\cos(\omega t), \\ \sum_{i=0}^{4}\dot{y}_i(t)H_i(\xi)=&c\Big[\sum_{i=0}^{4}x_i(t)H_i(\xi)+a \\ &-\bar{b}\sum_{i=0}^{4}y_i(t)H_i(\xi) \\ &-\sigma\xi\sum_{i=0}^{4}y_i(t)H_i(\xi)\Big]. \end{aligned} \tag{12}$$

As for Chebyshev polynomials $H_i(\xi)$, we have

$$\begin{aligned} &H_0^3(\xi)=H_0(\xi), \\ &H_1^3(\xi)=H_3(\xi)+2H_1(\xi), \\ &\cdots \end{aligned} \tag{13}$$

Since any product of two Chebyshev polynomials can be reduced to a linear combination of individual Chebyshev polynomials, the final expansion of the cubic term $(\sum_{i=0}^{4}x_i(t)H_i(\xi))^3$ on the right-hand side of the first equation of system (12) can be denoted as

$$\Big[\sum_{i=0}^{4}x_i(t)H_i(\xi)\Big]^3=\sum_{i=0}^{12}x_i(t)H_i(\xi). \tag{14}$$

By the recurrent formulas (3) of Chebyshev polynomials, the last term of the second equation of system (12) can be reduced to

$$\begin{aligned} \sigma\xi\sum_{i=0}^{4}y_i(t)H_i(\xi)=&\frac{1}{2}\sigma\sum_{i=0}^{4}y_i(t)[H_{i-1}(\xi)+H_{i+1}(\xi)] \\ =&\frac{1}{2}\sigma\sum_{i=0}^{4}[y_{i-1}(t)+y_{i+1}(t)]H_i(\xi), \end{aligned} \tag{15}$$

where X_{-1} and X_5 are assumed to be zero, which also holds true for the succeeding equations. Substituting expressions (14), (15) into Eq.(12), we have

$$\begin{aligned} \sum_{i=0}^{4}\dot{x}_i(t)H_i(\xi)=&\sum_{i=0}^{4}x_i(t)H_i(\xi)-\frac{1}{3}\sum_{i=0}^{12}X_i(t)H_i(\xi) \\ &-\sum_{i=0}^{4}y_i(t)H_i(\xi)+f\cos(\omega t), \\ \sum_{i=0}^{4}\dot{y}_i(t)H_i(\xi)=&c\Big\{\sum_{i=0}^{4}x_i(t)H_i(\xi)+a \\ &-\bar{b}\sum_{i=0}^{4}y_i(t)H_i(\xi)-\frac{1}{2}\sigma\sum_{i=0}^{4}[y_{i-1}(t) \\ &+y_{i+1}(t)]H_i(\xi)\Big\}. \end{aligned} \tag{16}$$

Multiplying both sides of Eq.(16) in sequence by $H_i(\xi)$ $(i=1,2,3,4)$ and then taking expectation with respect to ξ, and using the orthogonal relationship of Chebyshev polynomials, we finally have

$$\begin{aligned} \dot{x}_0(t)&=-\frac{1}{3}X_0(t)-y_0(t)+x_0(t)+f\cos(\omega t), \\ \dot{y}_0(t)&=cx_0(t)+ac-c\bar{b}y_0(t)-\frac{1}{2}c\sigma y_1(t), \\ \dot{x}_1(t)&=-\frac{1}{3}X_1(t)-y_1(t)+x_1(t), \\ \dot{y}_1(t)&=cx_1(t)-c\bar{b}y_1(t)-\frac{1}{2}c\sigma[y_0(t)+y_2(t)], \\ \dot{x}_2(t)&=-\frac{1}{3}X_2(t)-y_2(t)+x_2(t), \\ \dot{y}_2(t)&=cx_2(t)-c\bar{b}y_2(t)-\frac{1}{2}c\sigma[y_1(t)+y_3(t)], \\ \dot{x}_3(t)&=-\frac{1}{3}X_3(t)-y_3(t)+x_3(t), \\ \dot{y}_3(t)&=cx_3(t)-c\bar{b}y_3(t)-\frac{1}{2}c\sigma[y_2(t)+y_4(t)], \\ \dot{x}_4(t)&=-\frac{1}{3}X_4(t)-y_4(t)+x_4(t), \\ \dot{y}_4(t)&=cx_4(t)-c\bar{b}y_4(t)-\frac{1}{2}c\sigma y_3(t). \end{aligned} \tag{17}$$

This is just the equivalent nonlinear deterministic system of Eq.(7), playing an important role in this paper. Since in the derivation we have taken some expectation with respect to ξ, the obtained equivalent deterministic system is a kind of weighted average system. So the response of the equivalent deterministic system must reflect some averaged characteristics of the original stochastic dynamic system.

If we sample the random parameter as $\xi=0$, namely $b=\bar{b}$, then the system is a sample of mean parameter system, of which the response may be ex-

pressed as

$$\begin{cases} x(t,0)=\sum_{i=0}^{4} x_i(t)H_i(0) = x_0(t)-x_2(t)+x_4(t), \\ y(t,0)=\sum_{i=0}^{4} y_i(t)H_i(0) = y_0(t)-y_2(t)+y_4(t). \end{cases} \tag{18}$$

And the ensemble mean response of the stochastic BVP system can be obtained as

$$\begin{cases} E[x(t,\xi)] = \sum_{i=0}^{4} x_i(t)E[H_i(\xi)] = x_0(t), \\ E[y(t,\xi)] = \sum_{i=0}^{4} y_i(t)E[H_i(\xi)] = y_0(t). \end{cases} \tag{19}$$

The stochastic BVP system (7) can be rewritten as

$$\begin{cases} \dot{x} = x - \frac{1}{3}x^3 - y + f\cos(\omega t), \\ \dot{y} = c[x + a - (\bar{b} + \sigma\xi)y]. \end{cases} \tag{20}$$

If $\sigma = 0$ or $\xi \equiv 0$, system (20) is reduced to the mean-parameter system (21) which is a deterministic system.

$$\begin{cases} \dot{x} = x - \frac{1}{3}x^3 - y + f\cos(\omega t), \\ \dot{y} = c(x + a - \bar{b}y). \end{cases} \tag{21}$$

Applying the Runge–Kutta method to systems (17) and (21), we obtain the responses $x_i(t)$ and $y_i(t)$ ($i = 0, 1, 2, 3, 4$) of the equivalent system and the deterministic responses $x(t)$ and $y(t)$ of the mean parameter system (21) respectively. Then the responses (18) and (19) can be worked out easily. We can also obtain the sample responses of systems (20) through the Monte Carlo method, which can then be taken as an expectation to form a weighted average response of the stochastic dynamic system.

3. Stochastic period-doubling bifurcation in stochastic BVP system

In this section, we explore various stochastic period-doubling bifurcation phenomena in a stochastic BVP system with a bounded random parameter (20) and show that the Chebyshev polynomial approximation is an effective way to study the dynamical behaviour of a typical stochastic nonlinear system. For this purpose, we offer the following four sets of response phase trajectories as the illustration: set SR for sample responses $x(t,0)$ and $y(t,0)$ of the response system (18); set DR for deterministic responses $x(t)$ and $y(t)$ of the deterministic system (21); set EMR for ensemble mean responses $E[x(t,\xi)]$ and $E[y(t,\xi)]$ of expression (19); and set WAR for the weighted average responses $x_{\rm w}(t)$ and $y_{\rm w}(t)$ of Eq.(20) obtained by Monte Carlo method. Then one may find the period-doubling bifurcation phenomena by observing the response phase trajectories in illustration for various situation of the given systems. SR is determined when $\sigma = 0$ or $\xi = 0$, so the characteristics of SR should be close to DR; meanwhile both EMR and WAR are some kinds of weighted average responses, so they should show similar characteristics. To some extent, we can verify the feasibility of Chebyshev polynomial approximation by comparing DR with SR and EMR with WAR separately.

There are abundant period-doubling bifurcation phenomena in the deterministic mean-parameter BVP system (21). For example, the period-doubling cascades from period $1T$ to period $2T$, and from period $2T$ to period $4T$ are shown in Fig.2(a); and the period-doubling cascades from period $2T$ to period $4T$, and from period $4T$ to period 8T along with the decreasing of the amplitude f are shown in Fig.2(b).

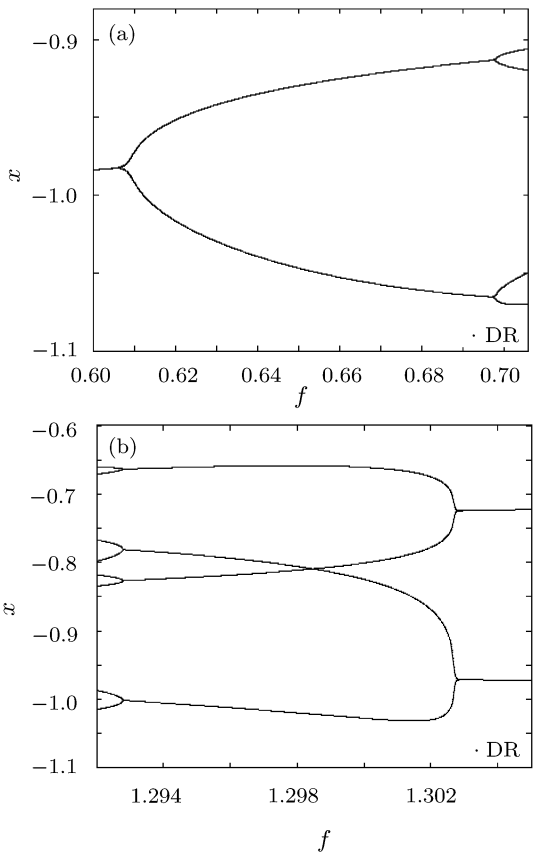


Fig.2. Bifurcation diagrams taking f as the bifurcation parameter in the ranges of (a) $0.6000 \le f \le 0.706$ and (b) $1.2920 \le f \le 1.3050$.

Now taking the amplitude f of the harmonic excitation as the bifurcation parameter, we explore the stochastic period-doubling bifurcation phenomena in the stochastic BVP system. The initial conditions (IC) for system (17) is expressed as

$$x(0) = [x_0(0), x_1(0), x_2(0), x_3(0), x_4(0)]^{\mathrm{T}},$$

$$y(0) = [y_0(0), y_1(0), y_2(0), y_3(0), y_4(0)]^{\mathrm{T}},$$

and the IC for systems (21) and (20) as

$$x(0) = x_0(0), \quad \text{and} \quad y(0) = y_0(0),$$

where

$$x_0(0) = -0.1, \quad y_0(0) = 0.1,$$

$$x_i(0) = y_i(0) = 0, \quad i = 1, 2, 3, 4.$$

Thus, the ICs are as (IC)

$$\begin{cases} x(0) = [-0.1,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}}, \\ y(0) = [\ 0.1,\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}}, \end{cases}$$

and

$$\begin{cases} x(0) = -0.1, \\ y(0) = \ \ 0.1. \end{cases}$$

In the numerical simulation the system parameters are chosen as follows:

$$a = 0.7, \quad \bar{b} = 0.8, \quad c = 0.1, \quad \omega = 1, \quad \text{and} \quad \sigma = 0.01.$$

At first we take f located in $0.6917 \le f \le 0.6995$. Numerical results are shown in Figs.3 and 4. As we take $f = 0.6917$, there is a periodic response with a period of $2T$, the phase trajectory and Poincaré section corresponding to IC are shown in Fig.3. While we take $f = 0.6995$, instead of the response with a period of $2T$ there is a periodic response with a period of $4T$, whose phase trajectory and Poincaré section corresponding to IC are shown in Fig.4. In each of these figures, one can see that it is difficult to distinguish the phase trajectories of different sets of responses, as they coincide with each other very well. The evident coherence between the phase trajectories in Figs.3(a) and 3(b), or in Figs.4(a) and 4(b), illustrates that the Chebyshev polynomial approximate method works very well.

From Figs.3 and 4 a qualitative conclusion can be drawn that a stochastic period-doubling bifurcation $2T \to 4T$ in the stochastic BVP system must happen at some point critically located within the parameter interval $0.6917 \le f \le 0.6995$. Though in principle the location of critical zone can be searched for approximately by numerical simulations, we do not have to do this for the moment.

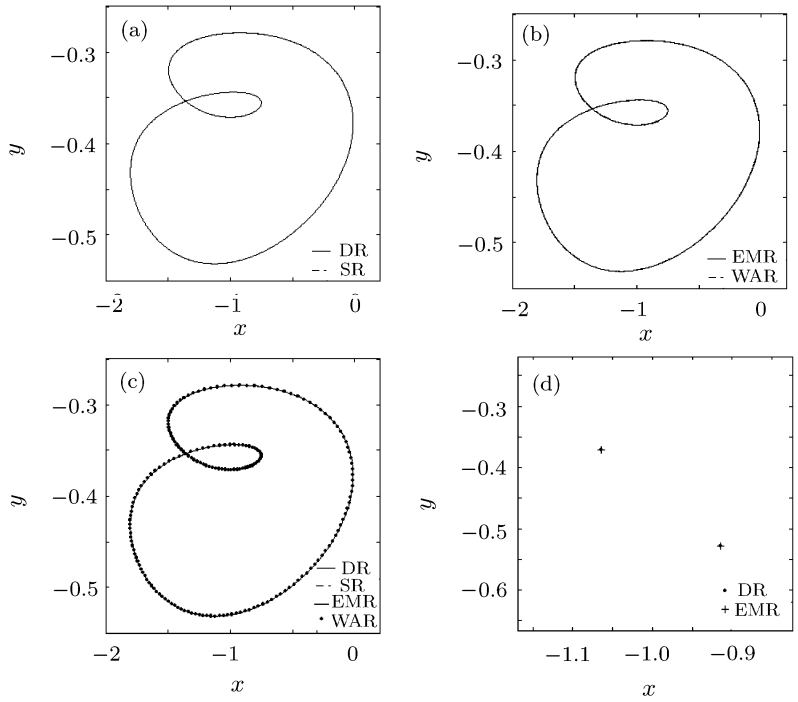


Fig.3. The phase portraits and Poincaré section for $f = 0.6917$.

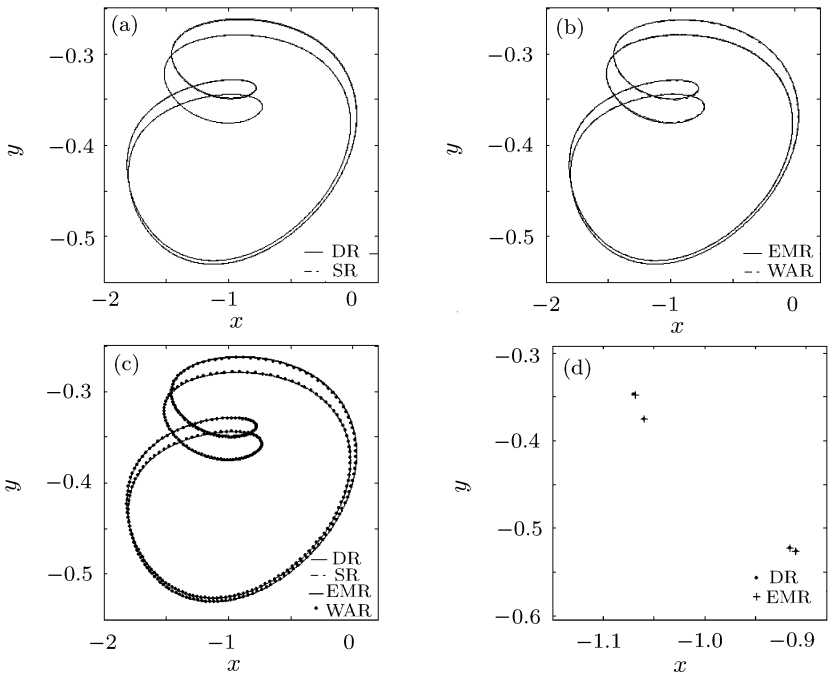


Fig.4. The phase portraits and Poincaré section for $f = 0.6995$.

Now with f reducing from 1.3033 to 1.3019 and the other parameters fixed, the numerical results for this case are shown in Figs.5 and 6. At $f = 1.3033$, the numerical results show that there is a periodic response with a period of $2T$, whose phase trajectory and Poincaré section corresponding to IC are shown in Fig.5. While at $f = 1.3019$, instead of the response with a period of $2T$ there is a periodic response with a period of $4T$ as shown in Fig.6. From Figs.5 and 6 a similar qualitative conclusion for a stochastic $2T \to 4T$ bifurcation can be drawn.

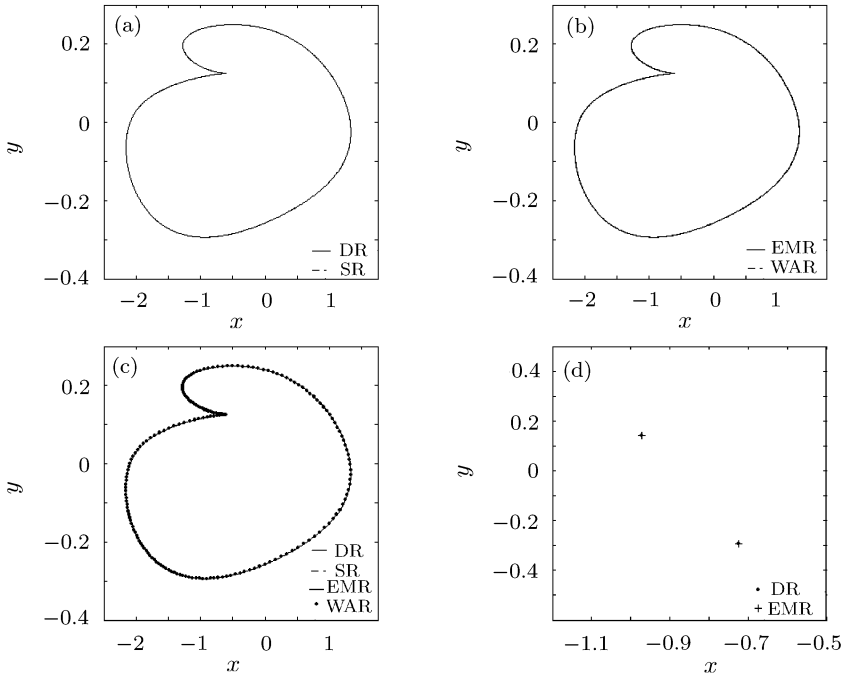


Fig.5. The phase portraits and Poincaré section for $f = 1.3033$.

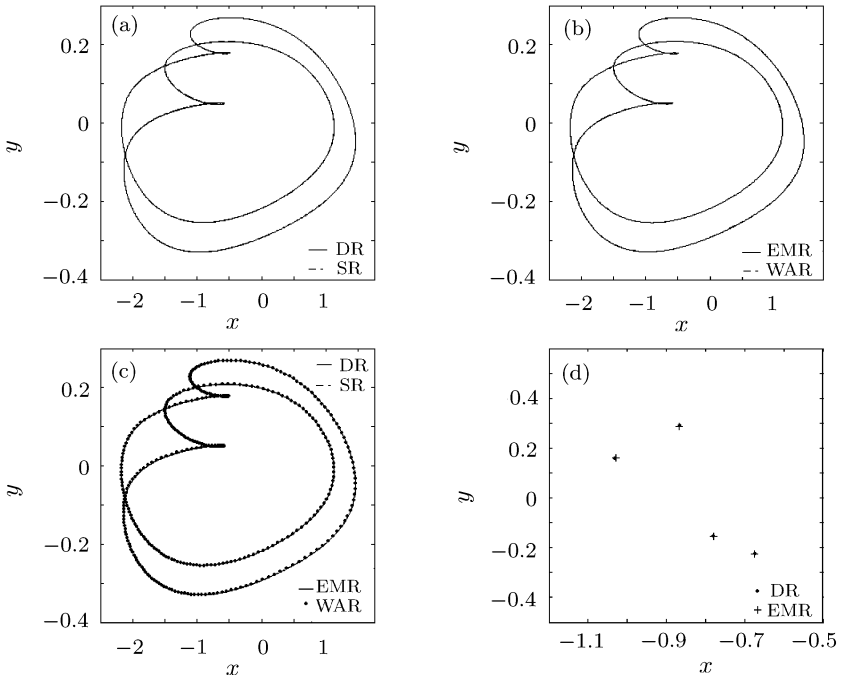


Fig.6. The phase portraits and Poincaré section for $f = 1.3019$.

We turn to the case where f is located in $1.2926 \leq f \leq 1.2935$. And the numerical results are shown in Figs.7 and 8. For $f = 1.2935$, the numerical results show that there is a periodic response with a period of $4T$, whose phase trajectory is a periodic response with a period of $4T$ as shown in Fig.7. while for $f = 1.2926$, instead of the response with a period of $4T$, there is a periodic response with a period of $8T$ as shown in Fig.8. From Figs.7 and 8 a qualitative conclusion for a $4T \to 8T$ period-doubling stochastic bifurcation can be drawn in a similar way.

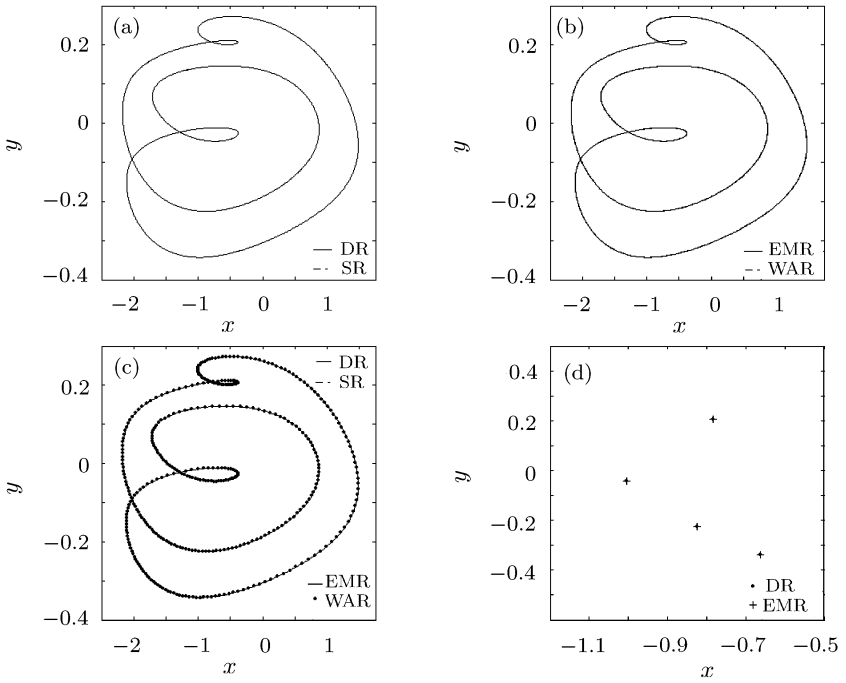


Fig.7. The phase portraits and Poincaré section for $f = 1.2935$.

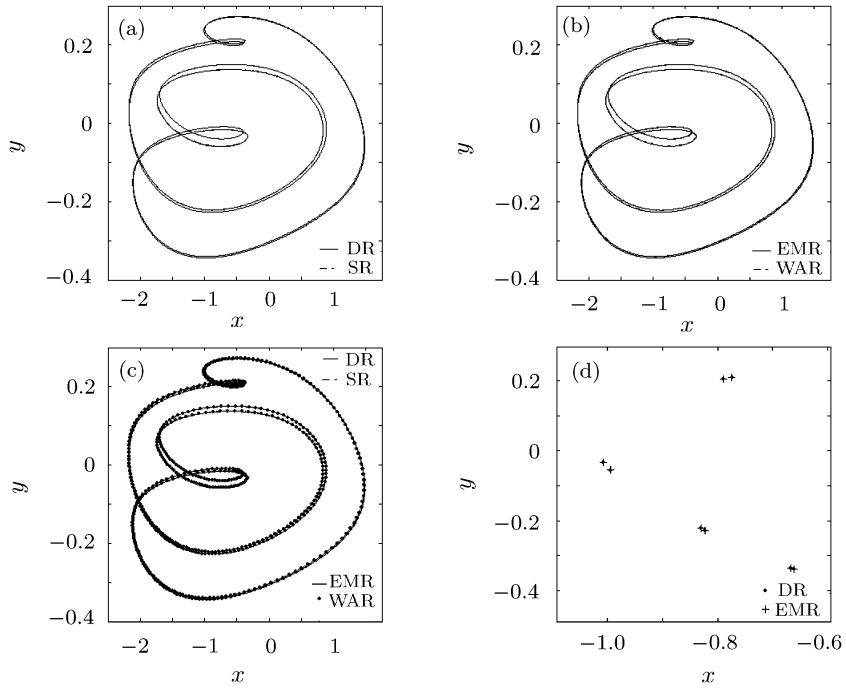


Fig.8. The phase portraits and Poincaré section for $f = 1.2926$.

4. Some other features of stochastic period-doubling bifurcation

In this paper the adjective, 'stochastic', means 'dependent on uncertainty of random parameters'. We have just studied the existence of stochastic period-doubling bifurcation of a stochastic BVP system through the Chebyshev polynomial approximation in Section 3. Yet there are some other features of stochastic period-doubling bifurcation depending on the random parameter to be studied.

4.1. Effect of the random parameter with fixed intensity on bifurcation diagram

For a stochastic nonlinear system with random parameters, its sample systems each are a deterministic nonlinear system. In general, the deterministic bifurcation should happen at a critical bifurcation point for a nonlinear system. However, it seems almost impossible for all the sample systems to bifurcate at the same critical point. Thus, a reasonable verdict is that for a stochastic nonlinear system there must be a critical interval, a line segment no matter how short it is, of the bifurcation parameter, at the beginning of which the stochastic bifurcation happens only in a few sample systems but at the end of which the stochastic bifurcation fully develops to all sample systems, i.e. the stochastic bifurcation must evolve from infant to mature. Since the EMR is some kind of average response of the stochastic BVP system, to some extent its evolution as shown in Fig.9 could partly reflect this feature. Comparing Fig.9 with the bifurcation diagram of a deterministic mean-parameter BVP system, Fig.2(a), one may obtain a better impression.

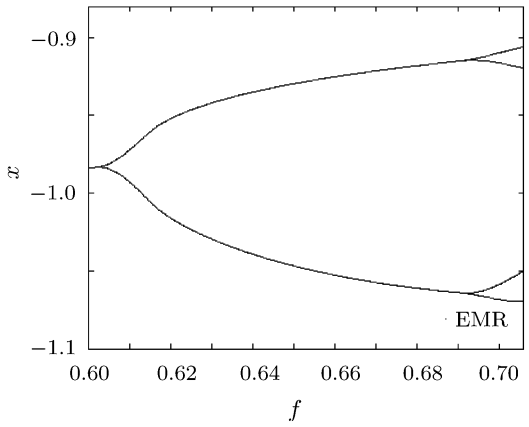


Fig.9. Bifurcation diagram taking f as the bifurcation parameter located in $0.60 < f < 0.7060$ with $\sigma = 0.01$.

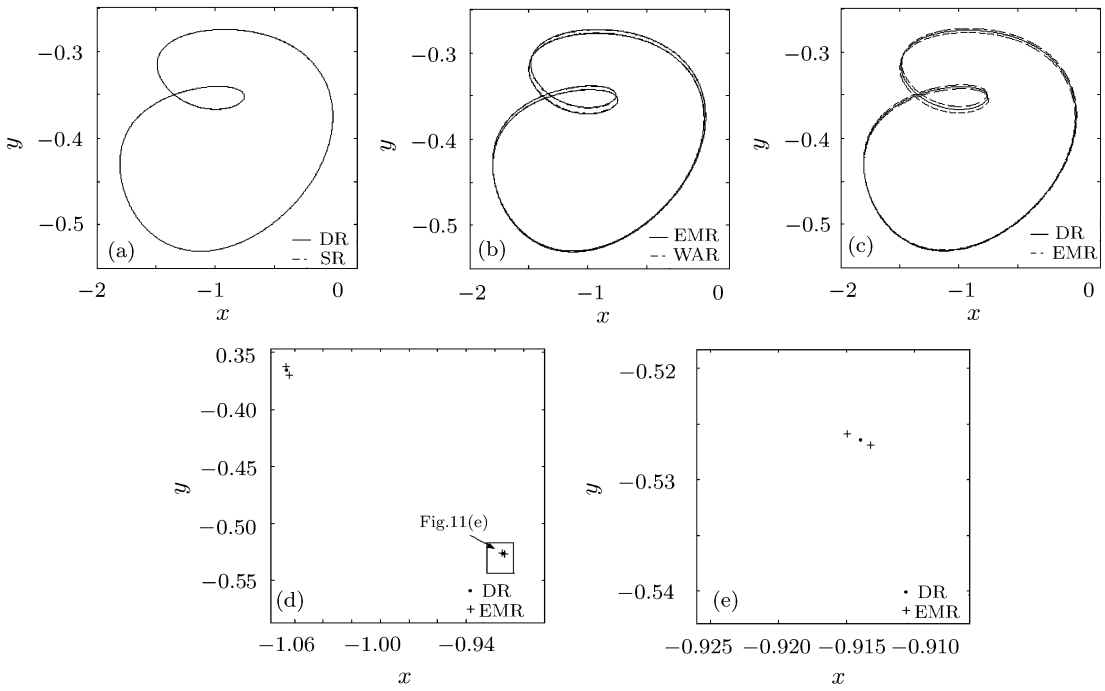


Fig.10. The phase portraits and Poincaré section with $f = 0.6950$.

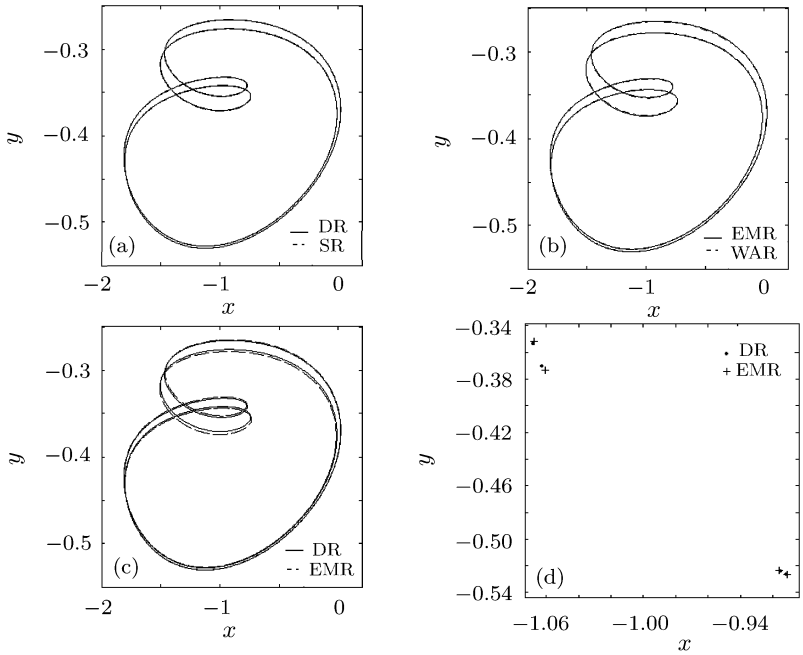


Fig.11. The phase portraits and Poincaré section with $f = 0.6985$.

By investigating the phase portrait diagrams, we can acquire more detailed information on this feature. We have learned in Section 3 that when $f \in [0.6917, 0.6995]$, there exists a stochastic period-doubling bifurcation $2T \to 4T$ in the stochastic BVP system. For either $f = 0.6917$ or $f = 0.6995$, the four sets of responses DR, SR, EMR and WAR coincide with each other very well (see Figs.3 and 4). Now letting $f = 0.6950$, corresponding to (IC), we find that DR and SR are of steady-state period responses, whose phase trajectories as shown in Fig.10(a) are similar to those with a period of $2T$ as shown in Fig.3; but EMR and WAR are of steady-state period responses with a period of $4T$, whose phase trajecto-

ries as shown in Fig.10(b) are somewhat apart. And a larger difference between DR and EMR as shown in Figs.10(c) and 10(d) may infer that the period-doubling bifurcation has occurred in a part of sample BVP systems, but it has not yet occurred in the deterministic mean-parameter BVP system (21). Then, by taking $f = 0.6985$, corresponding to (IC), the four sets of responses are all period responses with a period of $4T$ as shown in Figs.11(a) and 11(b). But there is still some difference between DR and EMR as shown in Figs.11(c) and 11(d), which infers that the period-doubling bifurcations occur in both stochastic and deterministic BVP systems, but they have not fully developed yet in the stochastic BVP system. However, with the increase of the bifurcation parameter f, the difference between DR and EMR must become smaller and smaller. Finally, when $f = 0.6995$, the 4 sets of phase trajectories will coincide with each other well as shown in Figs.3(c) and 3(d).

4.2. Effect of the intensity of the random variable on bifurcation diagrams

Through further exploring the stochastic period-doubling bifurcation in the stochastic BVP system, we find that the stochastic bifurcation can be also affected by the intensity σ of the random variable.

In Section 3, we have investigated the stochastic period-doubling bifurcation in the stochastic BVP system with a fixed intensity $\sigma = 0.01$. In this section, we will take some different values of intensity σ, while keep the other parameters fixed. When f is located within $0.56 \leq f \leq 0.62$, the numerical results for EMR show that there appears a stochastic period-doubling bifurcation $1T \to 2T$ with the increase of the bifurcation parameter f. For the values of intensity σ be 0.01, 0.02, 0.03, and 0.04, the bifurcation diagrams are shown in Figs.12(a)–12(d) respectively.

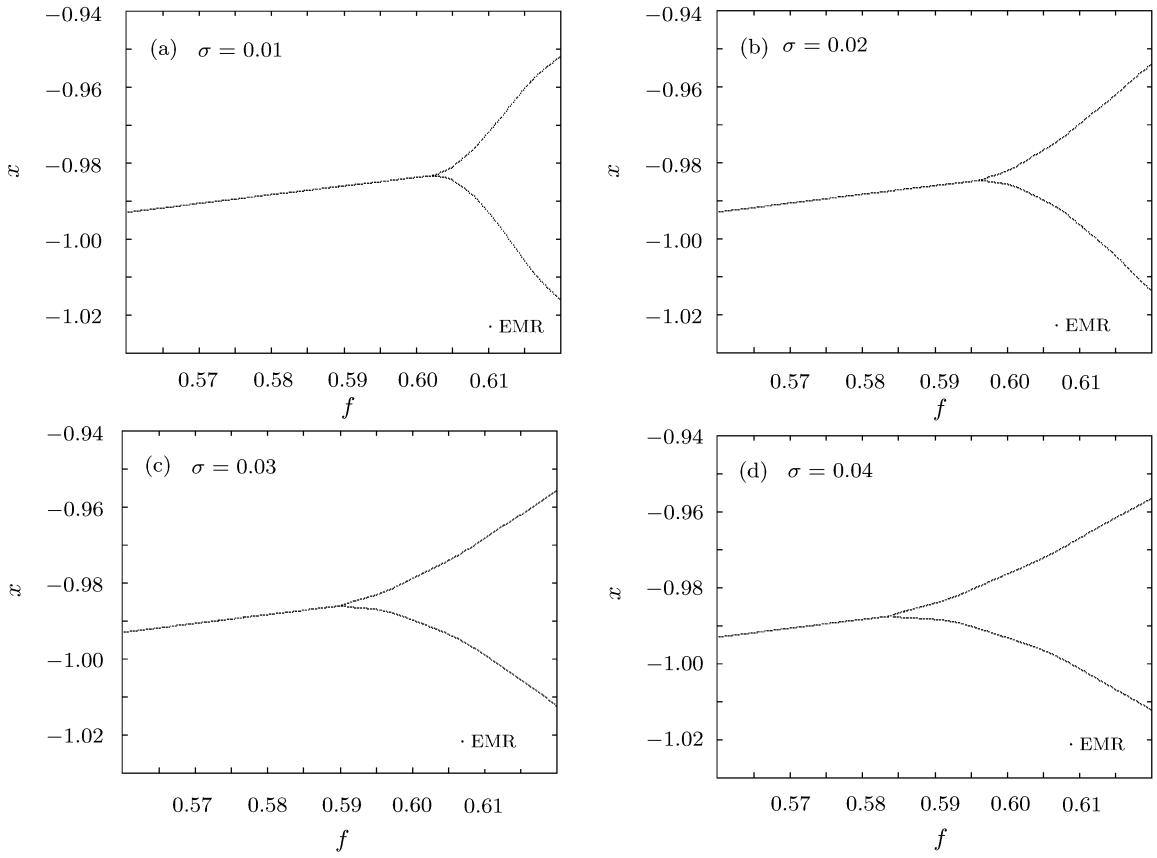


Fig.12. The bifurcation diagrams with $0.56 \leq f \leq 0.62$, for values of σ (a) 0.01, (b) = 0.02, (c) 0.03, and (d) = 0.04 respectively.

From Fig.12 one can see that the bigger the intensity σ is, the smaller the value of f at which the stochastic period-doubling bifurcation occurs. In other words, the critical interval of stochastic period-doubling bifurcation will shift ahead with the increase of intensity σ.

5. Conclusions

The typical period-doubling phenomena of a stochastic BVP system are studied by the Chebyshev polynomial approximation. This method can reduce the stochastic nonlinear system to an equivalent deterministic nonlinear system, and then any effective numerical method of dealing with deterministic systems can be applied to studying the dynamical behaviours of the stochastic nonlinear systems. In this paper, primary numerical results show that the equivalent deterministic nonlinear system preserves the similar nonlinear phenomena of period-doubling bifurcation in the deterministic mean-parameter BVP system. Besides, further numerical results show some other features of stochastic period-doubling bifurcation in the stochastic BVP system, especially the effects of random parameter on the bifurcation diagrams of the system. The bifurcation actually occurs in a small critical interval of the bifurcation parameter, and the critical interval will shift ahead with the increase of the intensity of the random variable. We think that the proposed Chebyshev polynomial approximation could be a promising approach to the exploration of other abundant nonlinear phenomena in the stochastic BVP system, and it could be applied to any nonlinear stochastic system with algebraic product nonlinearities.

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