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The sea–air oscillator model of decadal variations in subtropical cells and equatorial Pacific SST*

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In this paper a time delay equation for sea–air oscillator model is studied. The aim is to create an approximate solving method of nonlinear equation for sea–air oscillator model. Employing the method of variational iteration, it obtains the approximate solution of corresponding equation. This method is an approximate analytic method, which can be often used for analysing other behaviour of the sea surface temperature anomaly of the atmosphere–ocean oscillator model.

Keywords: nonlinear, time delay, El Niño-Southern Oscillator, variational iteration, approximate solution.

PACC: 0545, 0200

1. Introduction

Interactions between the ocean and atmosphere contribute to climate fluctuations over a broad spectrum of time scales. Studies of those interactions have far focused on El Niño Southern Oscillation (ENSO) phenomena that have a period of 3 to 4 years and whose principal signature is in the tropical Pacific.[1,2] Equatorial Pacific sea surface temperature (SST) is of central importance to global climate. In addition to ENSO, eastern equatorial Pacific SST anomalies (SSTAs) also display significant decadal or interdecadal variations that modulate the amplitude and frequency of ENSO.[3] The cause of decadal SSTAs in the equatorial Pacific is not yet clear. Nonaka *et al*[4] consider that the nonlinearity of the tropical ocean–atmosphere can give rise to chaotic modulation of ENSO on decadal and longer time scales without involving the extratropics. Phenomenon of the ENSO is a very attractive object of study in the international academic circles.[5−8] Feng *et al*[5] considered an instable model of air–sea oscillator, Wang[7] studied an especial ENSO mechanism. Mo *et al*[9−16] Lin W T and Mo[17] and Lin Y H *et al*[18,19] considered also a class of problems in atmospheric physics, ocean weather, dynamic system and so on. This paper considers an ocean general circulation model of the Pacific.

Recently, many scholars have investigated the approximate theory.[20−22] Approximate methods have been developed and refined, including the method of averaging, boundary layer method, matched asymptotic expansion method and multiple scales method.[20] In this paper we consider a sea–air oscillator model using a simple and valid method for the variational iteration theory.[23−25]

2. Model of equatorial Pacific SSTA

There are experiments which demonstrate[4] that wind variability in the off-equatorial tropics is an effective mechanism for generating equatorial decadal SSTAs. A useful diagnostic is the heat-anomaly budget for the tropical upper ocean. It is defined by the volume integral of the model's temperature-anomaly

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equation in a box that extends across the basin from 11° S to 15° N and from –550 m to the ocean surface. The bottom is deep enough to encompass almost all of the mean subtropical cells. The resulting heat-anomaly equation can be written as[4]

$$\frac{\mathrm{d}H'}{\mathrm{d}t} = v' - Q', \tag{1}$$

where H' is the anomalous heat content in the box, t is a time variable, v' is the heat transport into the box through its sides and bottom, $Q' = \gamma \iint_A T_0' \,\mathrm{d}x\mathrm{d}y$ is the integral of surface heat flux out of the box over its horizontal A, and γ is a heat flux coefficient and T_0' is the solution's SSTA field. And the experiments demonstrate[4] that the Q' is very similar to decadal T in the eastern equatorial region 140° W∼90° W, 1° S∼1° N, and to a good approximation $Q' = bT$, $b = 2.83 \times 10^{14}\,\mathrm{W^\circ \cdot C^{-1}}$.[4] There is also a close relationship between Q' and H', with $H' = aQ'$, $a = 841\,\mathrm{m}$. These two relations imply that $T = \dfrac{1}{ab}H'$, this is a clear statement that equatorial SST anomalies are dynamically driven by vertical shifts in the depth of the thermocline. The preceding relationship can be rewritten for decadal time scales as

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{1}{a}T + \frac{1}{ab}v'. \tag{2}$$

From the demonstration of the experiments, and Eqs.(1), (2), we can consider that the heat transport v' signifies $T^2T(t-\delta)$, where δ is a time delay T. Thus we obtain the following time delay model of equatorial Pacific:

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t} = -\frac{1}{a}T(t) + \frac{\gamma}{ab}T^2(t)T(t-\delta). \tag{3}$$

3. Approximate solutions for model

Introduce a functional $F(T,\lambda)$:

$$F(T(t),\lambda) = T(t) - \int_0^t \lambda\left(\frac{\mathrm{d}T(\tau)}{\mathrm{d}\tau} + \frac{1}{a}T(\tau) - \frac{\gamma}{ab}\bar{T}^2(\tau)\bar{T}(\tau-\delta)\right)\mathrm{d}\tau, \tag{4}$$

where $\bar{T}(t)$, $\bar{T}(t-\delta)$ are restricted variables[23] of $T(t)$, $T(t-\delta)$ respectively, while λ is a Lagrange multiplier.

Compute the variation δF of functional (4) for the variable t:

$$\delta F(t) = \delta T(t) + \delta\int_0^t \lambda(\tau)\left(\frac{\mathrm{d}T(\tau)}{\mathrm{d}\tau} + \frac{1}{a}T(\tau) - \frac{\gamma}{ab}\bar{T}^2(\tau)\bar{T}(\tau-\delta)\right)\mathrm{d}\tau.$$

Using the method of integration by parts and nothing that $\bar{T}(t)$, $\bar{T}(t-\delta)$ are restricted variables which denote parameters as operation of the variation, we have

$$\begin{aligned}\delta F(t) &= \delta T(t) + \delta\int_0^t \lambda(\tau)\left(\frac{\mathrm{d}T(\tau)}{\mathrm{d}\tau} + \frac{1}{a}T(\tau)\right)\mathrm{d}\tau \\ &= \delta T(t) - [(\lambda(\tau)\delta T(\tau))]_0^t \\ &\quad + \int_0^t \left(\frac{\mathrm{d}\lambda}{\mathrm{d}\tau} - \frac{1}{a}\lambda(\tau)\right)\delta T(\tau)\mathrm{d}\tau \\ &= \delta T(t) - (\lambda(\tau)\delta T(\tau))\,|_{\tau=t} \\ &\quad + \int_0^t \left(\frac{\mathrm{d}\lambda}{\mathrm{d}\tau} - \frac{1}{a}\lambda(\tau)\right)\delta T(\tau)\mathrm{d}\tau.\end{aligned}$$

Let

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\tau} - \frac{1}{a}\lambda = 0, \tag{5}$$

$$\lambda(t) = 1, \tag{6}$$

then we have $\delta F = 0$. From Eqs.(5) and (6), we have

$$\lambda = \exp\left(\frac{1}{a}(\tau - t)\right). \tag{7}$$

From Eqs.(4) and (7), we construct the following variational iteration:

$$\begin{aligned}T_{n+1}(t) &= T_n(t) - \int_0^t \exp\left(\frac{1}{a}(\tau-t)\right)\left(\frac{\mathrm{d}T_n(\tau)}{\mathrm{d}\tau} + \frac{1}{a}T_n(\tau) - \frac{\gamma}{ab}T_n^2(\tau)T_n(\tau-\delta)\right)\mathrm{d}\tau, \\ n &= 0, 1, 2, \ldots.\end{aligned} \tag{8}$$

Obviously, we can yield an uniformly convergent sequence in the finite interval $[0,\ t_0]$ [26] of function sequence $\{T_n(t)\}$ from Eq.(8), then $T(t) = \lim\limits_{n\to\infty} T_n(t)$ is a solution of Eq.(3).

Now we first select an initial approximate solution $T_0(t)$ which is a solution for the linear equation of Eq.(3):

$$\frac{\mathrm{d}T_0}{\mathrm{d}t} = -\frac{1}{a}T_0. \tag{9}$$

From Eq.(9), we have

$$T_0(t) = C\exp\left(-\frac{1}{a}t\right), \tag{10}$$

where C is an arbitrary constant.

Substituting initial approximate solution (10) into Eq.(8), we can obtain the first order approximate solution of Eq.(3):

$$T_1(t) = T_0(t) - \int_0^t \exp\left(\frac{1}{a}(\tau - t)\right)\left(\frac{\mathrm{d}T_0(\tau)}{\mathrm{d}\tau} + \frac{1}{a}T_0(\tau) - \frac{\gamma}{ab}T_0^2(\tau)T_0(\tau-\delta)\right)\mathrm{d}\tau.$$

From Eq.(10), we have

$$T_1(t) = C\exp\left(-\frac{1}{a}t\right) - \int_0^t \exp\left(\frac{1}{a}(\tau - t)\right)\left(\frac{\mathrm{d}}{\mathrm{d}\tau}\left[C\exp\left(-\frac{1}{a}\tau\right)\right] + \frac{C}{a}\exp\left(-\frac{1}{a}\tau\right) - \frac{\gamma}{ab}\left[C\exp\left(-\frac{1}{a}t\right)\right]^2\left[C\exp\left(-\frac{1}{a}(\tau-\delta)\right)\right]\right)\mathrm{d}\tau.$$

Thus first order approximate solution of model (3) is

$$T_1(t) = C\exp\left(-\frac{1}{a}t\right) + \frac{\gamma C^3}{b}\left[\exp\left(-\frac{1}{a}(t-\delta)\right) - \exp\left(-\frac{1}{a}(3t-\delta)\right)\right]. \tag{11}$$

Using the same method we can obtain higher order approximate solution of Eq.(3).

4. Comparison of precisions

In order to show the precision of above result Eq.(11), now we compare the special case as $0 < \gamma \ll 1$. We can obtain the asymptotic solution $\bar{T}$ by using the perturbation method as follows.

Let

$$\bar{T} = \sum_{i=1}^{\infty} \bar{T}_i(t)\gamma^i. \tag{12}$$

Substituting Eq.(12) into Eq.(3), developing nonlinear term in γ and equating coefficients of the same powers of γ, for $i = 0, 1, \ldots$, we obtain

$$\frac{\mathrm{d}\bar{T}_0(t)}{\mathrm{d}t} = -\frac{1}{a}\bar{T}_0(t), \tag{13}$$

$$\frac{\mathrm{d}\bar{T}_1(t)}{\mathrm{d}t} = -\frac{1}{a}\bar{T}_1(t) + \frac{1}{ab}\bar{T}_0^2(t)\bar{T}_0(t-\delta). \tag{14}$$

From Eqs.(13) and (14), we have

$$\bar{T}_0(t) = C\exp\left(-\frac{1}{a}t\right), \tag{15}$$

$$\bar{T}_1(t) = \frac{1}{ab}\exp\left(-\frac{1}{a}t\right) \times \int_0^t \exp\left(\frac{1}{a}\tau\right)\bar{T}_0^2(\tau)\bar{T}_0(\tau-\delta)\mathrm{d}\tau, \tag{16}$$

where C is an arbitrary constant. Substituting Eq.(15) into Eq.(16) we have

$$\bar{T}_1(t) = \frac{C^3}{b}\left[\exp\left(-\frac{1}{a}(t-\delta)\right) - \exp\left(-\frac{1}{a}(3t-\delta)\right)\right]. \tag{17}$$

From Eqs.(12), (15) and (17), we obtain the first order asymptotic solution

$$\bar{T}(t) = C\exp\left(-\frac{1}{a}t\right) + \frac{\gamma C^3}{b}\left[\exp\left(-\frac{1}{a}(t-\delta)\right) - \exp\left(-\frac{1}{a}(3t-\delta)\right)\right] + O(\gamma^2), \quad 0 < \gamma \ll 1. \tag{18}$$

Comparing Eq.(11) with Eq.(18), we find that they are same as $0 < \gamma \ll 1$. So we know that the first order approximate solution (11) of model (3) obtained by using the variational iteration (8) and the

first order asymptotic solution (18) by the perturbation method are same as ε is small enough. Therefore using the method of variational iteration, we know that the approximate solution possesses finer approximate degree.

5. Discussions

Atmospheric physics studies a very complicated natural phenomenon. Hence we need to modify basic models for the sea–air oscillator. And we solve them by using the approximate method. The method of variational iteration is a simple and valid method.

The method of variational iteration is an approximate analytic method, which differs from general numerical method. The expansions of solution using the method of variational iteration can be continuously analytic operation. Thus, from Eq.(11), we can study further the qualitative and quantitative behaviours of the temperature anomaly in the equatorial eastern Pacific, the thermocline anomaly and the zonal wind stress anomaly. In this paper we do not discuss further.

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