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Linear matrix inequality approach to exponential synchronization of a class of chaotic neural networks with time-varying delays*

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In this paper, a synchronization scheme for a class of chaotic neural networks with time-varying delays is presented. This class of chaotic neural networks covers several well-known neural networks, such as Hopfield neural networks, cellular neural networks, and bidirectional associative memory networks. The obtained criteria are expressed in terms of linear matrix inequalities, thus they can be efficiently verified. A comparison between our results and the previous results shows that our results are less restrictive.

Keywords: chaotic neural networks, exponential synchronization, linear matrix inequalities
PACC: 0545

1. Introduction

In the past two decades, Hopfield neural networks, cellular neural networks and bidirectional associative memory networks have been studied extensively due to their applications in, for example, pattern recognition, associative memory, and combinatorial optimization.[1−20] Although neural networks can be implemented by very large-scale integrated electronic circuits, the finite switching speed of amplifiers and the inherent communication time of neurons inevitably induce time delays in the interaction between the neurons[21−23] and this may result in an oscillation phenomenon or network instability. Most of the existing work focused on the stability analysis and periodic oscillation of this kind of neural networks, and the stability and complex dynamical behaviour have been also investigated.[24] It is shown that these networks can exhibit some complicated dynamics and even chaotic behaviours if the network parameters and time delays are appropriately chosen.[25−27]

In Ref.[28], the authors have studied the synchronization problem of a class of chaotic neural networks with constant delays. The obtained result is based on Halanay inequality. However the criterion in Ref.[28] imposes a severe constraint on the weight matrix and the controller gain matrix. In Ref.[29], the exponential synchronization of a class of neural networks with time-varying delays is investigated by checking the Hamiltonian matrix with no eigenvalues on the imaginary axis. Algebraic criteria and matrix inequality conditions are difficult to check and find their feasible solution. In this paper, the linear matrix inequality (LMI) approach is applied to this problem and less conservative conditions are obtained. A comparison and numerical examples are used to illustrate the effectiveness of our method.

The following notations are used throughout this paper. For a vector $x \in R^n$, let $\|x\|$ denote the Euclidean vector norm, i.e. $\|x\| = (x^{\mathrm{T}}x)^{1/2}$ where x^{T} denotes the transpose of the vector x, and for $A \in R^{n\times n}$, let $\|A\|$ indicate the norm of A induced by the Euclidean vector norm, i.e. $\|A\| = (\lambda_{\max}(A^{\mathrm{T}}A))^{1/2}$, where $\lambda_{\max}(\cdot)(\lambda_{\min}(\cdot))$ represents the maximum (minimum) eigenvalue of matrix.

2. Problem formulation and some preliminaries

In this paper, we investigate the following delayed dynamical systems:

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$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -d_i\left(c_i(x_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} a_{ij}^{\tau} f_j(x_j(t-\tau_j(t))) + I_i\right),\ i = 1, 2, ..., n, \tag{1}$$

where $A = (a_{ij})_{n\times n}$ and $A^{\tau} = (a_{ij}^{\tau})_{n\times n}$ are the connection weight matrix and delayed connection weight matrix, respectively. $\tau_j(t)$ represents the time delays with $0 \le \tau_j(t) \le \tau$ and $\tau_j'(t) \le \mu < 1$ $(j = 1, 2, \ldots, n)$, where τ and μ are constants. $d_i > 0$ represents an amplification gain, and $c_i(x_i)$ is an appropriately behaved function such that the solution of the model given in Eq.(1) remains bounded. The activation function, f_j, describes the manner in which the neurons respond to each other, and I_i is a constant input to set the desired equilibrium point. The initial conditions of Eq.(1) are given by $x_i(s) = \psi_i(s)$, for $s \in [-\tau, 0]$, where $\psi_i \in C([-\tau, 0]),\ i = 1, ..., n$.

With regard to functions $c_i(x_i)$ and $f_i(t)$, the following assumptions are made:

(A1) Each function $c_i : R \to R$ is locally Lipschitz and there exists γ_i such that $c_i'(x) \ge \gamma_i > 0$ for all $x \in R$ at which $c_i(\cdot)$ is differentiable.

(A2) Each function $f_i : R \to R$ is bounded and satisfies the Lipschitz condition with a constant $G_i > 0$, i.e.

$$|f_i(x) - f_i(y)| \le G_i|x - y|,$$
$$i = 1, 2, ..., n, \forall x, y \in R,\ x \ne y.$$

Here the notations $\Gamma = \mathrm{diag}\{\gamma_1, ..., \gamma_n\}$ and $G = \mathrm{diag}\{G_1, ..., G_n\}$ have been used.

Remark 1 The class of delayed neural networks unifies several well-known neural networks such as Hopfield neural networks with or without delays and cellular neural networks with or without delays. If $d_i = 1,\ c_i(x_i) = c_i x_i$, and the activation function f_i is sigmoid, then Eq.(1) describes the dynamics of Hopfield neural networks. Similarly, if $d_i = 1,\ c_i(x_i) = c_i x_i$, and $f_i(x_i) = (|x_i + 1| - |x_i - 1|)/2$, then Eq.(1) describes the dynamics of cellular neural networks.

Chaos dynamics is extremely sensitive to initial conditions. Even infinitesimal changes in the initial condition will lead to an exponential divergence of orbits. In order to observe the synchronization behaviour in this class of chaotic neural networks, we have two chaotic neural networks where the drive system with state variable denoted by x_i drives the response system having identical dynamical equations denoted by state variable z_i. However, the initial condition imposed on the drive system is different from that on the response system. Therefore, two neural networks are described respectively by the following equations:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -d_i\left(c_i(x_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} a_{ij}^{\tau} f_j(x_j(t-\tau_j(t))) + I_i\right),\quad i = 1, 2, ..., n, \tag{2}$$

and

$$\frac{\mathrm{d}z_i(t)}{\mathrm{d}t} = -d_i\left(c_i(z_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(z_j(t)) - \sum_{j=1}^{n} a_{ij}^{\tau} f_j(z_j(t-\tau_j(t))) + I_i\right) + u_i,\quad i = 1, 2, ..., n, \tag{3}$$

with initial condition $z_i(s) = \phi_i(s)$, for $s \in [-\tau, 0]$, where $\phi_i \in C([-\tau, 0]),\ i = 1, ..., n$.

We need the following lemmas and notations, which will be used in the proof of main results.

Definition 1 The system (2) and the uncontrolled system (3) (i.e. $u \equiv 0$ in system (3)) are said to be exponentially synchronized if there exist constants $\eta \ge 1$ and $k > 0$ such that $\|x(t) - z(t)\| \le \eta \sup_{-\tau\le\theta\le 0} \|\psi(\theta) - \phi(\theta)\| \cdot \exp(-kt)$ for any $t \ge 0$, and $0 \le \tau(t) \le \tau$. Moreover, the constant k is defined as the exponential synchronization rate.

Lemma 1 For any vectors $a, b \in R^n$ and positive definite matrix $Y \in R^{n\times n}$, the following inequal-

ity holds:

$$2a^{\mathrm{T}}b \le a^{\mathrm{T}}Ya + b^{\mathrm{T}}Y^{-1}b.$$

Lemma 2 The following LMI

$$\begin{pmatrix} Q(x) & S(x) \\ S^{\mathrm{T}}(x) & R(x) \end{pmatrix} > 0,$$

where $Q(x) = Q^{\mathrm{T}}(x)$, $R(x) = R^{\mathrm{T}}(x)$ and $S(x)$ all depend affinely on x, is equivalent to

$$R(x) > 0 \quad \text{and} \quad Q(x) - S(x)R^{-1}(x)S^{\mathrm{T}}(x) > 0.$$

3. Main results

Now we define the synchronization error signal $e_i(t) = x_i(t) - z_i(t)$, where $x_i(t)$ and $z_i(t)$ are the ith state variables of the drive and response neural networks, respectively. $e_i(t) \to 0$ as $t \to \infty$ means that the drive neural networks and the response neural networks are synchronized with each other. Therefore, the error dynamics between systems (2) and (3) can be expressed as

$$\begin{aligned}\frac{\mathrm{d}e_i(t)}{\mathrm{d}t} = -d_i\Big(c_i(e+z) - c_i(z) - \sum_{j=1}^{n} a_{ij}(f_j(e_j(t) \\ +z_j) - f_j(z_j)) - \sum_{j=1}^{n} a_{ij}^{\tau}(f_j(e_j(t-\tau_j(t)) \\ +z_j) - f_j(z_j))\Big) - u_i \end{aligned} \tag{4}$$

or the following matrix form:

$$\begin{aligned}\frac{\mathrm{d}e(t)}{\mathrm{d}t} = -D(C(e(t)) - Ag(e(t)) \\ -A^{\tau}g(e(t-\tau(t))) - u(t), \end{aligned} \tag{5}$$

where $e(t) = [e_1(t), e_2(t), ..., e_n(t)]^{\mathrm{T}} \in R^n$; $D = \mathrm{diag}\{d_1, d_2, ..., d_n\}$; $C(e) = [c_1(e_1 + z_1) - c_1(z_1), ..., c_n(e_n + z_n) - c_n(z_n)]^{\mathrm{T}}$; $g(e) = [f_1(e_1 + z_1) - f_1(z_1), ..., f_n(e_n + z_n) - f_1(z_n)]^{\mathrm{T}} \in R^n$ and $u(t) = [u_1(t), ..., u_n(t)]^{\mathrm{T}}$ denotes the input vector.

If the state variables of the drive system are used to drive the response system, then the control input vector with state feedback is designed as follows:

$$\begin{aligned}\begin{bmatrix} u_1(t) \\ \vdots \\ u_n(t) \end{bmatrix} &= \begin{bmatrix} \sum_{j=1}^{n} \omega_{1j}(x_j(t) - z_j(t)) \\ \vdots \\ \sum_{j=1}^{n} \omega_{nj}(x_j(t) - z_j(t)) \end{bmatrix} \\ &= \begin{bmatrix} \omega_{11} & \cdots & \omega_{1n} \\ \vdots & \vdots & \vdots \\ \omega_{n1} & \cdots & \omega_{nn} \end{bmatrix} \begin{bmatrix} x_1(t) - z_1(t) \\ \vdots \\ x_n(t) - z_n(t) \end{bmatrix} \\ &= \Omega \begin{bmatrix} e_1(t) \\ \vdots \\ e_n(t) \end{bmatrix}, \end{aligned} \tag{6}$$

where Ω is the controller gain matrix and will be appropriately chosen for exponentially synchronizing the drive system with the response system. With control law (6), the error dynamics can be expressed as the following compact form:

$$\begin{aligned}\frac{\mathrm{d}e(t)}{\mathrm{d}t} = -D(C(e(t)) - Ag(e(t)) \\ -A^{\tau}g(e(t-\tau(t))) - \Omega e(t). \end{aligned} \tag{7}$$

Theorem 1 Suppose that assumptions (A1) and (A2) are satisfied and $\mu \le 1$. If there exist positive diagonal matrices R, P, positive definite matrix Q and the controller gain matrix Ω in control law (6) is real symmetric and positive definite such that the following LMI holds:

(i)

$$\Sigma_1 := \begin{bmatrix} -2D\Gamma P - 2\Omega P + 2kP + R & PDA & PDA^{\tau} \\ A^{\mathrm{T}}DP & Q - RG^{-2} & 0 \\ (A^{\tau})^{\mathrm{T}}DP & 0 & -(1-\mu)\cdot \mathrm{e}^{-2k\tau}\cdot Q \end{bmatrix} < 0, \tag{8}$$

(ii)

$$\Sigma_2 := \begin{bmatrix} -2D\Gamma P - 2\Omega P + 2kP + GQ^{-1}G + GQG & PDA^{\tau} & PDA \\ (A^{\tau})^{\mathrm{T}}DP & -(1-\mu)\cdot \mathrm{e}^{-2k\tau}\cdot Q & 0 \\ A^{\mathrm{T}}DP & 0 & -Q^{-1} \end{bmatrix} < 0, \tag{9}$$

then the exponential synchronization between systems (2) and (3) is obtained.

Proof Next, we will prove our results respectively.

Case 1:

Consider the Lyapunov functional

$$V(\mathrm{e}(t)) = \mathrm{e}^{2kt}\cdot \mathrm{e}^{\mathrm{T}}(t)Pe(t) + \int_{t-\tau(t)}^{t} \mathrm{e}^{2ks}\cdot g^{\mathrm{T}}(e(s))Qg(e(s))\mathrm{d}s. \quad (10)$$

The time derivatives of $V(z(t))$ along with the trajectories of system (7) satisfy

$$\begin{aligned}
&\frac{\mathrm{d}V(e(t))}{\mathrm{d}t}\\
&= 2k\mathrm{e}^{2kt}\cdot \mathrm{e}^{\mathrm{T}}(t)Pe(t) + 2\mathrm{e}^{2kt}\cdot \dot{\mathrm{e}}^{\mathrm{T}}(t)Pe(t)\\
&\quad +\mathrm{e}^{2kt}\cdot [g^{\mathrm{T}}(e(t))Qg(e(t)) - (1-\dot{\tau}(t))\\
&\quad \times \mathrm{e}^{-2k\tau(t)}\cdot g^{\mathrm{T}}(e(t-\tau(t)))Qg(e(t-\tau(t)))]\\
&\le \mathrm{e}^{2kt}\cdot [e^{\mathrm{T}}(t)(-2D\Gamma P - 2\Omega P + 2kP)e(t)\\
&\quad +2e^{\mathrm{T}}(t)PDAg(e(t))\\
&\quad +2e^{\mathrm{T}}(t)PDA^{\tau}g(e(t-\tau(t)))]\\
&\quad +\mathrm{e}^{2kt}\cdot [g^{\mathrm{T}}(e(t))Qg(e(t)) - (1-\tau)\cdot \mathrm{e}^{-2k\tau(t)}\\
&\quad \times g^{\mathrm{T}}(e(t-\tau(t)))Qg(e(t-\tau(t)))]\\
&\le \mathrm{e}^{2kt}\cdot [e^{\mathrm{T}}(t)(-2D\Gamma P - 2\Omega P + 2kP\\
&\quad +R)e(t) + 2e^{\mathrm{T}}(t)PDAg(e(t))\\
&\quad +2e^{\mathrm{T}}(t)PDA^{\tau}g(e(t-\tau(t)))] + \mathrm{e}^{2kt}\\
&\quad \times [g^{\mathrm{T}}(e(t))(Q - RG^{-2})g(e(t))\\
&\quad -(1-\mu)\cdot \mathrm{e}^{-2k\tau}\\
&\quad \times g^{\mathrm{T}}(e(t-\tau(t)))Qg(e(t-\tau(t)))]\\
&\le \mathrm{e}^{2kt}\cdot E_1^{\mathrm{T}}(t)\cdot \Sigma_1\cdot E_1(t), \qquad (11)
\end{aligned}$$

where

$$E_1^{\mathrm{T}} = [e^{\mathrm{T}}(t) \quad g^{\mathrm{T}}(e(t)) \quad g^{\mathrm{T}}(e(t-\tau(t)))],$$

and Σ_1 is given in expression (8).

Hence, if $\Sigma_1 < 0$, then $\frac{\mathrm{d}V(e(t))}{\mathrm{d}t} \le 0$, and $V(e(t)) \le V(e(0))$. Note that

$$\begin{aligned}
V(e(0)) &= e^{\mathrm{T}}(0)Pe(0) + \int_{-\tau(0)}^{0} \mathrm{e}^{2ks}\cdot g^{\mathrm{T}}(e(s))Qg(e(s))\mathrm{d}s\\
&\le \lambda_M(P)\|e(0)\|^2 + \lambda_M(Q)\int_{-\tau}^{0} e^{2ks}g^{\mathrm{T}}(e(s))g(e(s))\mathrm{d}s\\
&\le \lambda_M(P)\sup_{\theta\in[-\tau,0]}\|e(\theta)\|^2 + \lambda_M(Q)G_M^2\sup_{\theta\in[-\tau,0]}\|e(\theta)\|^2\int_{-\tau}^{0}\mathrm{e}^{2ks}\mathrm{d}s\\
&\le \left\{\lambda_M(P) + \lambda_M(Q)G_M^2\frac{1-\mathrm{e}^{-2k\tau}}{2k}\right\}\\
&\quad \times \sup_{\theta\in[-\tau,0]}\|e(\theta)\|^2, \qquad (12)
\end{aligned}$$

where $G_M = \max(G_1, G_2, ..., G_n)$, and $\lambda_M(P)$ denotes the maximum eigenvalue of matrix P.

On the other hand, we know

$$V(e(t)) \ge \lambda_M(P)\cdot \mathrm{e}^{2kt}\cdot \|\mathrm{e}(t)\|^2, \qquad (13)$$

where $\lambda_M(P)$ denotes the minimum eigenvalue of matrix P. From $e(t) = x(t) - z(t)$, we have

$$\|x(t) - z(t)\| \le \sqrt{\frac{\lambda_M(P) + \lambda_M(Q)G_M^2\frac{1-\mathrm{e}^{-2k\tau}}{2k}}{\lambda_m(P)}} \times \mathrm{e}^{-kt}\cdot \sup_{\theta\in[-\tau,0]}\|\psi(\theta) - \phi(\theta)\|. \quad (14)$$

From Definition 1, we know that Case 1 is proved.

Case 2:

We still consider the Lyapunov functional $V(e(t))$ used in Case 1. Calculating $\dot{V}(e(t))$ along with the solutions of system (7), we obtain

$$\begin{aligned}
\frac{\mathrm{d}V(e(t))}{\mathrm{d}t} &\le \mathrm{e}^{2kt}\cdot [e^{\mathrm{T}}(t)(-2D\Gamma P - 2\Omega P + 2kP)e(t)\\
&\quad +2e^{\mathrm{T}}(t)PDAg(e(t))\\
&\quad +2e^{\mathrm{T}}(t)PDA^{\tau}g(e(t-\tau(t)))]\\
&\quad +\mathrm{e}^{2kt}\cdot [g^{\mathrm{T}}(e(t))Qg(e(t))\\
&\quad -(1-\mu)\cdot \mathrm{e}^{-2k\tau}\\
&\quad \times g^{\mathrm{T}}(e(t-\tau(t)))Qg(e(t-\tau(t)))]. \qquad (15)
\end{aligned}$$

By using Lemma 1, we can obtain the following inequality:

$$2e^{\mathrm{T}}(t)PDAg(e(t)) \le e^{\mathrm{T}}(t)PDAQA^{\mathrm{T}}DPe(t) + g^{\mathrm{T}}(e(t))Q^{-1}g(e(t)). \quad (16)$$

Substituting inequality (16) into inequality (15), we have

$$\begin{aligned}
\frac{\mathrm{d}V(e(t))}{\mathrm{d}t} &\le \mathrm{e}^{2kt}\cdot [e^{\mathrm{T}}(t)(-2D\Gamma P - 2\Omega P + 2kP\\
&\quad +PDAQA^{\mathrm{T}}DP + GQ^{-1}G + GQG)e(t)\\
&\quad +2e^{\mathrm{T}}(t)PDA^{\tau}g(e(t-\tau(t)))]\\
&\quad -(1-\mu)\cdot \mathrm{e}^{-2k\tau(t)}\\
&\quad \times g^{\mathrm{T}}(e(t-\tau(t)))Qg(e(t-\tau(t)))\\
&\le \mathrm{e}^{2kt}\cdot E_2^{\mathrm{T}}(t)\cdot \Sigma_2'\cdot E_2(t), \qquad (17)
\end{aligned}$$

where

$$E_2^{\mathrm{T}} = [e^{\mathrm{T}}(t) \quad g^{\mathrm{T}}(e(t-\tau(t)))],$$

and

$$\Sigma_2' = \begin{bmatrix} -2D\Gamma P - 2\Omega P + 2kP + PDAQA^{\mathrm{T}}DP + GQ^{-1}G + GQG & PDA^{\tau} \\ (A^{\tau})^{\mathrm{T}}DP & -(1-\mu)\cdot \mathrm{e}^{-2k\tau}\cdot Q \end{bmatrix}.$$

By using Lemma 2 it is shown that Σ_2' is exactly Σ_2.

The following proof is similar to that for Case 1. We omit it here.

Remark 2 In Ref.[28], the authors have studied the synchronization problem of a class of chaotic neural networks with constant delays. The result obtained is based on Halanay inequality. However, the criterion in Ref.[28] imposes a severe constraint on the weight matrix and the controller gain matrix. In Ref.[29], the exponential synchronization of a class of neural networks with time-varying delays is investigated by checking the Hamiltonian matrix with no eigenvalues on imaginary axis. Algebraic criteria and matrix inequality conditions are difficult to check and find their feasible solution. Hamiltonian matrix condition is simple, but it is usually conservative for some situations. From a comparisons later, we can see that our approach can be verified efficiently with the Matlab LMI Control Toolbox[30] and is less conservative.

If we let $D = \Gamma = G = I$, where I is identity matrix, we have the following corollary.

Corollary 1 Suppose that the assumptions (A1) and (A2) are satisfied and $\tau(t)$ is the constant delay. If there exist positive diagonal matrices R, P, positive definite matrix Q and the controller gain matrix Ω in control law (6) is real symmetric and positive definite such that the following LMI holds:

(i)

$$\Sigma_1 := \begin{bmatrix} -2P - 2\Omega P + 2kP + R & PA & PA^{\tau} \\ A^{\mathrm{T}}P & Q - R & 0 \\ (A^{\tau})^{\mathrm{T}}P & 0 & -\mathrm{e}^{-2k\tau}\cdot Q \end{bmatrix} < 0, \tag{18}$$

(ii)

$$\Sigma_2 := \begin{bmatrix} -2P - 2\Omega P + 2kP + Q^{-1} + Q & PA^{\tau} & PA \\ (A^{\tau})^{\mathrm{T}}P & -\mathrm{e}^{-2k\tau}\cdot Q & 0 \\ A^{\mathrm{T}}P & 0 & -Q^{-1} \end{bmatrix} < 0, \tag{19}$$

then the exponential synchronization between systems (2) and (3) is obtained.

4. Comparison and numerical example

In this section we give two examples to make a comparison with the results in Ref.[28] to illustrate that our results are less restrictive.

Theorem 2.[28] For these drive-response chaotic neural networks (2) and (3) which satisfy assumptions (A1) and (A2), if the controller gain matrix Ω in control law (6) is real symmetric and positive definite, and satisfies $L(\|A\| + \|B\|)\max_i(d_i) < (\min\limits_{1\le i\le n}(d_i\gamma_i) + \lambda_{\min}(\Omega))$, where $L = \max\limits_{1\le i\le n}(L_i)$, then the exponential synchronization between systems (2) and (3) is obtained.

Example 1 Consider a chaotic cellular neural network without any delay as follows:[25,31]

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = -\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \times \left(\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix} - \begin{bmatrix}1.25&-3.2&-3.2\\-3.2&1.1&-4.4\\-3.2&4.4&1\end{bmatrix} \begin{bmatrix}f_1(x_1(t))\\ f_2(x_2(t))\\ f_3(x_3(t))\end{bmatrix} \right), \tag{20}$$

where $d_i = [1 \quad 1 \quad 1]^{\mathrm{T}}$, $c_i(x_i) = x_i$, and $f_i(x_i) = (|x_i + 1| - |x_i - 1|)/2$, $i = 1, 2, 3$. The delayed feedback matrix here is zero. The initial condition is $[x_1(0) \quad x_2(0) \quad x_3(0)] = [0.1 \quad 0.1 \quad 0.1]$. As shown in Fig.1, system (20) possesses a chaotic behaviour.

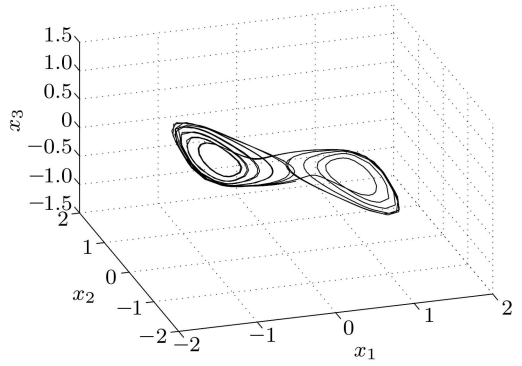


Fig.1. The chaotic behaviour of the cellular neural networks in Example 1.

We let the controller gain matrix $\Omega = \begin{bmatrix}135&0&3\\0&55&1\\3&1&5\end{bmatrix}$. By solving this LMI in Corollary 1 using the Matlab Toolbox,[30] the feasible solutions of LMI (18) are

$$P = \begin{bmatrix}0.0657&0.0011&-0.0085\\0.0011&0.1871&-0.0114\\-0.0085&-0.0114&0.3290\end{bmatrix},$$

$$Q = \begin{bmatrix}4.6841&0.7747&0.3279\\0.7747&4.5731&-0.4704\\0.3279&-0.4704&0.1637\end{bmatrix},$$

$$R = \begin{bmatrix}11.4075&0&0\\0&12.7477&0\\0&0&0.5893\end{bmatrix}, \qquad k = 4.577,$$

where k is the exponential convergence rate. We can find that the minimum eigenvalue of Ω is 4.9109. Using Theorem 2, these parameters yield $7.0099 = L(\|A\| + \|B\|)\max_i(d_i) < (\min\limits_{1\le i\le n}(d_i\gamma_i) + \lambda_{\min}(\Omega)) = 4.9109$, which fails to meet the condition. Figure 2 depicts the synchronization error for the initial condition [3 0.2 -0.2].

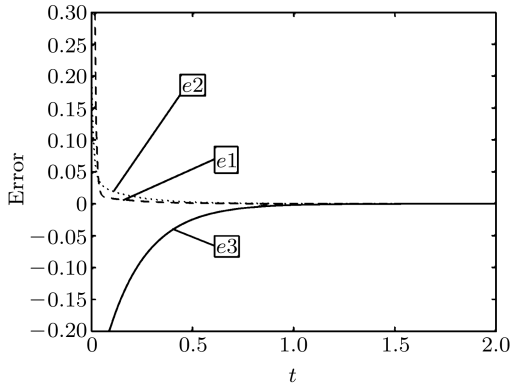


Fig.2. The synchronization error in Example 1.

Example 2 Consider the following delayed cellular neural networks:[26]

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = &-\begin{bmatrix}1&0\\0&1\end{bmatrix} \times \left(\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} - \begin{bmatrix}1+\dfrac{\pi}{4}&20\\0.1&1+\dfrac{\pi}{4}\end{bmatrix} \begin{bmatrix}f_1(x_1(t))\\ f_2(x_2(t))\end{bmatrix} \right.\\ &\left. - \begin{bmatrix}-\sqrt{2}\frac{\pi}{4}1.3&0.1\\0.1&-\sqrt{2}\frac{\pi}{4}1.3\end{bmatrix} \begin{bmatrix}f_1(x_1(t-1))\\ f_2(x_2(t-1))\end{bmatrix} \right),\end{aligned} \tag{21}$$

where $d_i = [1 \quad 1]^{\mathrm{T}}$, $c_i(x_i) = (|x_1 + 1| - |x_i - 1|)/2$, $i = 1, 2$. The system satisfies assumptions (A1) and (A2) with $G_1 = G_2 = 1$. It can be easily verified that $\gamma_1 = \gamma_2 = 1$. The chaotic behaviour of system (21) is shown in Fig.3.

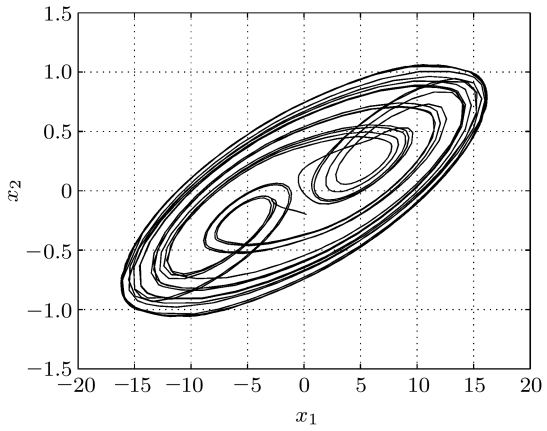


Fig.3. The chaotic behaviour of the cellular neural networks in Example 2.

We let the controller gain matrix $\Omega = \begin{bmatrix} 24 & -6 \\ -6 & 28 \end{bmatrix}$, and $k = 3.1$. By solving this LMI in Corollary 1 using the Matlab Toolbox,[30] the feasible solutions of LMI (18) are

$$P = \begin{bmatrix} 0.4962 & 0.3084 \\ 0.3084 & 1.9374 \end{bmatrix}, \qquad Q = \begin{bmatrix} 5.7276 & 0.1132 \\ 0.1132 & 25.1206 \end{bmatrix},$$

$$R = \begin{bmatrix} 6.9975 & 0 \\ 0 & 61.1392 \end{bmatrix}.$$

We can find that the minimum eigenvalue of Ω is 19.6754. Using Theorem 2 these parameters yield $21.7028 = L(\|A\| + \|B\|)\max_i(d_i) < (\min\limits_{1\le i\le n}(d_i\gamma_i) + \lambda_{\min}(\Omega)) = 20.6754$, which also fails to meet the condition. However, we can observe that the error is exponentially attractive according to Fig.4.

There examples show that the condition we proposed is less conservative and easily ensures the exponential synchronization of the class of neural networks. The exponential convergence rate can be assigned to our expected point by finding the proper parameter matrix.

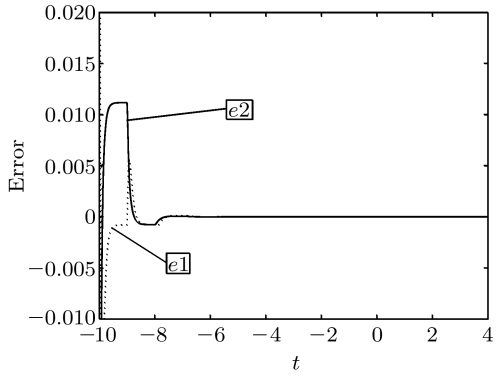


Fig.4. The synchronization error in Example 2.

5. Conclusion

In this paper, we have obtained new results for the exponential synchronization of a class of delayed chaotic neural networks with variable delays. A comparison between our results and the previous results has also been made, demonstrating that our criteria are less restrictive than those reported in the literature.

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