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Sudden death of entanglement of a quantum model*

Zhang Guo-Feng(张国锋)†

Department of Physics, School of Sciences, Beijing University of Aeronautics and Astronautics (BUAA), Beijing 100083, *China*

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In this paper the so-called sudden death effect of entanglement is investigated in a quantum model. The results show that one can expect the resurrection of the original entanglement to occur in a periodic way following each sudden death event. The length of the time interval for the zero entanglement depends not only on the degree of entanglement of the initial state but also on the initial state.

Keywords: entanglement, sudden death
PACC: 0365, 4250

Entanglement is one of the most striking features of quantum mechanics. It plays a fundamental role in almost all efficient protocols of quantum computation (QC) and quantum information processing (QIP).[1−5] In recent years, there has been an ongoing effort to characterize qualitatively and quantitatively the entanglement properties and apply them in quantum communication and information.[6−11] More recently, entanglement decoherence has been studied in connection with obstacles to realizing various QIP schemes. Particularly, entanglement can decay to zero abruptly in a finite time, this phenomenon is termed entanglement sudden death.[12,13] The purpose of this paper is to examine two interesting time-evolution quantum systems that have no route for mutual interaction, but whose mutual entanglement nevertheless evolves in an unusual way. Figure 1 shows the system configuration that we will consider.

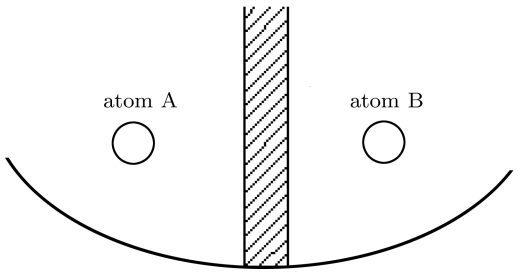


Fig.1. The system configuration diagram used in this paper.

The two atoms are in a perfect one-mode near-resonant and lossless cavity, but there is no interaction between the two atoms. We can get a mixed state of atom A and atom B by tracing over the cavity modes at time t. For a pair of qubits, we will adopt concurrence[14] $C(\rho)$ as the entanglement measure ($1 \geq C \geq 0$).

The system discussed here consists of two two-level atoms confined in a linear trap which has been surrounded by an optical cavity.[15] We refer to atom A and atom B. The Hamiltonian describing the system is[16] ($\hbar = 1$)

$$H = \frac{\omega_1}{2}\sigma_z^{\mathrm{A}} + \frac{\omega_2}{2}\sigma_z^{\mathrm{B}} + \omega_a a^\dagger a + \sum_{l=\mathrm{A}}^{\mathrm{B}} g(a^\dagger \sigma_-^l + a\sigma_+^l), \quad (1)$$

where a and $a^\dagger$ denote the annihilation and creation operators for the single mode cavity field respectively, and $\sigma_z^l = |e\rangle_{ll}\langle e| - |g\rangle_{ll}\langle g|$, $\sigma_+^l = |e\rangle_{ll}\langle g|$, $\sigma_-^l = |g\rangle_{ll}\langle e|(l = \mathrm{A}, \mathrm{B})$ are the atomic operators. ω_1 and ω_2 are the atomic transition frequencies of atom A and atom B, respectively. g is the coupling strength between the cavity and atom. Here we investigate two two-level atoms resonantly coupling with one mode optical cavity, i.e. $\omega_1 = \omega_3 = \omega_a = \omega$. Clearly there will be no interaction between atom A and atom B.

For greatest simplicity, we assume that the cavity is prepared initially in the vacuum state $|0\rangle$ and the two atoms are in a pure entangled state specified below. This allows a uniform measure of quantum entanglement concurrence. In principle, there are three different concurrences which provide information about

*Project supported by National Natural Science Foundation of China (Grant No 10604053).
†E-mail: gf1978zhang@buaa.edu.cn

the overall entanglements that may arise. With an obvious notation we can denote these as $C^{\rm AB}$, $C^{a\rm A}$, $C^{a\rm B}$. Here we confine our attention to $C^{\rm AB}$.

We assume that the two atoms are initially in a partially entangled atomic pure state which is a combination of Bell states usually denoted by $|\Psi^{\pm}\rangle$, we have

$$|\Psi_{\rm atom}\rangle = \cos[\alpha]|eg\rangle + \sin[\alpha]|ge\rangle, \tag{2}$$

where the first index denotes the state of atom A and the second denotes the state of atom B. Thus the initial state for the system (1) can be given by

$$\begin{aligned}|\Psi(0)\rangle &= (\cos[\alpha]|eg\rangle + \sin[\alpha]|ge\rangle) \otimes |0\rangle \\ &= \cos[\alpha]|eg0\rangle + \sin[\alpha]|ge0\rangle. \end{aligned}\tag{3}$$

The solution of the system in terms of the standard basis can be written as

$$|\Psi(t)\rangle = x_1|eg0\rangle + x_2|ge0\rangle + x_3|gg0\rangle, \tag{4}$$

the corresponding coefficients are

$$\begin{aligned} x_1 &= \frac{1}{2}\theta_+(\theta_- + \cos[x\pi]), \\ x_2 &= -\frac{1}{2}\theta_+(\theta_- - \cos[x\pi]), \\ x_3 &= -\frac{\rm i}{\sqrt{2}}\theta_+ \sin[x\pi], \end{aligned}\tag{5}$$

where $\theta_\pm = \cos[\alpha] \pm \sin[\alpha]$ and $x = \dfrac{\sqrt{2}gt}{\pi}$.

The information about the entanglement of two atoms is contained in the reduced density matrix $\rho^{\rm AB}$ for the two atoms which can be obtained from Eq.(4) by tracing out the photonic part of the total pure state. $\rho^{\rm AB}$ can be given in the basis $|ee\rangle$, $|eg\rangle$, $|ge\rangle$ and $|gg\rangle$ by

$$\rho^{\rm AB} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |x_1|^2 & x_1x_2^* & 0 \\ 0 & x_2x_1^* & |x_2|^2 & 0 \\ 0 & 0 & 0 & |x_3|^2 \end{pmatrix}, \tag{6}$$

where $*$ stands for the complex conjugate. It can be shown that the concurrence of the density matrix (6) is given by

$$C^{\rm AB}(t) = 2\max\{|x_1x_2^*|, 0\}. \tag{7}$$

Figure 2 shows that sudden death effect arises for a smaller α. Entanglement can fall abruptly to zero and will remain zero for a period of time before entanglement recovers for the smaller α for which the entanglement is weaker when $t = 0$. This is different from the results obtained by Yönac *et al.*[17]

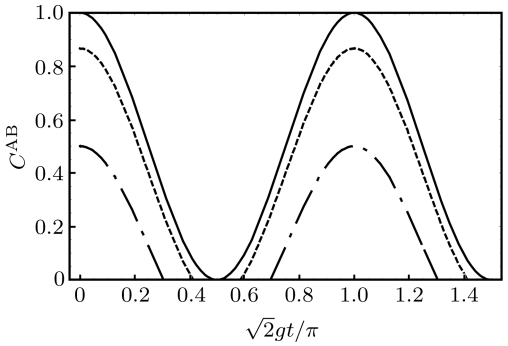


Fig.2. The concurrence for the two atoms with the initial atomic state $|\Psi_{\rm atom}\rangle = \cos[\alpha]|eg\rangle + \sin[\alpha]|ge\rangle$. (solid curve: $\alpha = \dfrac{\pi}{4}$; dashed curve: $\alpha = \dfrac{\pi}{6}$; dashed-dotted curve: $\alpha = \dfrac{\pi}{12}$).

It is assumed that the initial state for the total system is in a combination of the other two Bell states $|\Phi^{\pm}\rangle$:

$$|\Phi(0)\rangle = \cos[\alpha]|ee0\rangle + \sin[\alpha]|gg0\rangle. \tag{8}$$

In this case the state of the total system at time t can be expressed in the standard basis:

$$\begin{aligned}|\Phi(t)\rangle \;=\; & x_1|ee0\rangle + x_2|gg0\rangle \\ & + x_3|ge1\rangle + x_4|eg1\rangle + x_5|gg2\rangle, \end{aligned}\tag{9}$$

where the coefficients are given by:

$$\begin{aligned} x_1 &= \frac{1}{4}L_+^2 \cos[\alpha] \exp\left[-\frac{{\rm i}(2+\kappa)\pi x}{\sqrt{2}}\right], \\ x_2 &= \sin[\alpha] \exp\left[\frac{{\rm i}\kappa\pi x}{\sqrt{2}}\right], \\ x_3 &= x_4 = -\frac{\rm i}{2}\cos[\alpha] \sin\left[\frac{2\pi x}{\sqrt{2}}\right] \exp\left[-{\rm i}\frac{\kappa\pi x}{\sqrt{2}}\right], \\ x_5 &= \frac{1}{4}L_-^2 \cos[\alpha] \exp\left[-\frac{{\rm i}(2+\kappa)\pi x}{\sqrt{2}}\right] \end{aligned}\tag{10}$$

with $\kappa = \dfrac{\omega}{g}$ and $L_\pm = \pm 1 + \exp[{\rm i}\sqrt{2}\pi x]$. In the basis of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$ and $|gg\rangle$ the reduced density matrix $\rho^{\rm AB}$ is now found to be

$$\rho^{\rm AB} = \begin{pmatrix} |x_1|^2 & 0 & 0 & x_1x_2^* \\ 0 & |x_4|^2 & x_4x_3^* & 0 \\ 0 & x_3x_4^* & |x_3|^2 & 0 \\ x_2x_1^* & 0 & 0 & |x_2|^2 + |x_5|^2 \end{pmatrix}, \tag{11}$$

and the concurrence for this matrix is given by

$$C^{\rm AB}(t) = \max\left\{\sin[2\alpha]\cos\left[\frac{\pi x}{\sqrt{2}}\right] - 2|x_3|^2, 0\right\}. \tag{12}$$

From Fig.3, we can see that sudden death effect also arises for this initial state (8). The smaller the initial degree of entanglement is, the longer the state will stay in the disentangled separable state. For the same initial entanglement value, comparing Fig.2 with Fig.3, we can see that the state (8) has longer length of the time interval for the zero entanglement than that of state (2).

In summary, we have investigated the entanglement existed in a quantum model consisting of two two-level atoms and a single mode cavity. There is no interaction between the two atoms. We found that sudden death effect arises when the initial state is a combination of typical Bell state for both $|\Psi^{\pm}\rangle$ and $|\Phi^{\pm}\rangle$. The length of the time interval for the zero entanglement depends not only on the degree of entanglement of the initial state but also on the initial state.

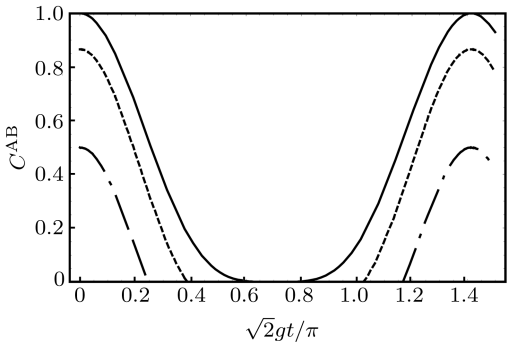


Fig.3. The concurrence for the two atoms with the initial atomic state $|\Psi_{\rm atom}\rangle = \cos[\alpha]|ee\rangle + \sin[\alpha]|gg\rangle$. (solid curve: $\alpha = \frac{\pi}{4}$; dashed curve: $\alpha = \frac{\pi}{6}$; dashed-dotted curve: $\alpha = \frac{\pi}{12}$).

References

[1] Bennett C H and Divincenzo D P 2000 *Nature* **407** 247

[2] Nielson M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[3] Hou X W, Hui Z, Ding R M, Chen X Y and Gao Y 2006 *Chin. Phys.* **15** 2510

[4] Li S S, Long G L, Bai F S, Feng S L and Zheng H Z 2001 *PNAS* **98** 11847

[5] Tan J and Fang M F 2006 *Chin. Phys.* **15** 2514

[6] Xu C T, Chen G, He M M and Liang J Q 2006 *Chin. Phys.* **15** 2828

[7] Abliz A, Gao H J, Xie X C, Wu Y S and Liu W M 2006 *Phys. Rev.* A **74** 052105

[8] Zhang G F, Yin W, Liang J Q and Yan Q W 2004 *Chin. Phys.* **13** 988

[9] Hao Y J and Liang J Q 2006 *Chin. Phys.* **15** 1161

[10] Xiong H N, Guo H, Jiang J, Chen J and Tang L Y 2006 *Acta Phys. Sin.* **55** 2720 (in Chinese)

[11] Jiang C L, Fang M F and Wu Z Z 2006 *Acta Phys. Sin.* **55** 4647 (in Chinese)

[12] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404

[13] Yu T and Eberly J H 2006 *Opt. Commun.* **264** 393

[14] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245

[15] Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565

[16] Plenio M, Huelga S F, Beige A and Knight P L 1999 *Phys. Rev.* A **59** 2468

[17] Yönac M, Yu T and Eberly J H 2006 *J. Phys.* B: *At. Mol. Opt. Phys.* **39** S621