Vol 16 No 7, July 2007
1009-1963/2007/16(07)/1827-05

Chinese Physics

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Mei symmetry and generalized Hojman conserved quantity for variable mass systems with unilateral holonomic constraints

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(Received 10 October 2006; revised manuscript received 4 November 2006)

This paper studies Mei symmetry which leads to a generalized Hojman conserved quantity for variable mass systems with unilateral holonomic constraints. The differential equations of motion for the systems are established, the definition and criterion of the Mei symmetry for the systems are given. The necessary and sufficient condition under which the Mei symmetry is a Lie symmetry for the systems is obtained and a generalized Hojman conserved quantity deduced from the Mei symmetry is got. An example is given to illustrate the application of the results.

Keywords: variable mass, unilateral holonomic constraint, Mei symmetry, generalized Hojman conserved quantity

PACC: 0320

1. Introduction

The study of symmetry for a dynamical system plays an important role in the fields of mathematics and physics. There are mainly three kinds of symmetries: the Noether symmetry,[1−5] the Lie symmetry[3,5−16] and the Mei symmetry[5,17−27] (or the form invariance). Noether symmetry is an invariance of a Hamilton action under the infinitesimal transformations of group. Lie symmetry is an invariance of differential equations under the infinitesimal transformations. The two symmetries can lead to Noether conserved quantity[1−5] and the Hojman conserved quantity[7] respectively. Mei gave the form invariance[17] (or the Mei symmetry), which means that the dynamical functions in the equation of motion still preserve the equation's primary form after infinitesimal transformations. Mei symmetry always leads to a new type conserved quantity. Luo *et al* obtained the special Hojman conserved quantity deduced from the form invariance of the nonholonomic system.[25] Li *et al* studied the Mei symmetry for variable mass systems with unilateral holonomic constraints, which leads to a Noether conserved quantity.[23] In this paper, we try to obtain a generalized Hojman conserved quantity deduced from the Mei symmetry for variable mass systems with unilateral holonomic constraints.

2. Definition and criterion of Mei symmetry

We consider a mechanical system which is composed of N particles, and its position is determined by n generalized coordinates $q_s(s = 1, \ldots, n)$. Let its motion be subjected to the g ideal unilateral holonomic constraints

$$f_\beta(t, \boldsymbol{q}) \geq 0 \ , \quad (\beta = 1, \ldots, g). \tag{1}$$

The mass of a representative point i of the system is

$$m_i = m_i(t, \boldsymbol{q}, \dot{\boldsymbol{q}}), \quad (i = 1, \ldots, N). \tag{2}$$

Then, the differential equations of the system can be expressed as follows:

$$\begin{aligned} &E_s(L) = Q_s + P_s + \lambda_\beta \frac{\partial f_\beta}{\partial q_s}, \\ &\lambda_\beta \geq 0, \quad f_\beta \geq 0, \quad \lambda_\beta f_\beta = 0, \\ &(s = 1, \ldots, n; \beta = 1, \ldots, g) \end{aligned} \tag{3}$$

where $L = L(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ is the Lagrangian, $Q_s = Q_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ are the nonpotential generalized forces, λ_β are the multipliers, $P_s = P_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ are the generalized counter thrusts, and we have

$$P_s = \ \dot{m}_i(\boldsymbol{u}_i + \dot{\boldsymbol{r}}_i)\frac{\partial \boldsymbol{r}_i}{\partial q_s} - \frac{1}{2}\dot{\boldsymbol{r}}_i \cdot \dot{\boldsymbol{r}}_i \frac{\partial m_i}{\partial q_s}$$

$$+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\dot{\boldsymbol{r}}_i\cdot\dot{\boldsymbol{r}}_i\frac{\partial m_i}{\partial\dot{q}_s}\right), \tag{4}$$

where $\boldsymbol{r}_i$ and $\dot{\boldsymbol{r}}_i$ are the vector radius and the velocity of the particle i respectively. $\boldsymbol{u}_i$ is the relative velocity of the small particle corresponding to the particle i, E_s are the Euler operators

$$E_s=\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial}{\partial\dot{\boldsymbol{q}}_s}-\frac{\partial}{\partial\boldsymbol{q}_s},\quad(s=1,\ldots,n). \tag{5}$$

If the system is under the constraints, i.e. constraints (1) take the sign of equality, then we have

$$f_\beta(t,\boldsymbol{q})=0\,,\quad(\beta=1,\ldots,g). \tag{6}$$

Considering a nonsingular system, i.e. $\det\left(\dfrac{\partial^2 L}{\partial\dot{q}_s\partial\dot{q}_k}\right)\neq 0$, before integrating the equations of motion, we can find λ_β as functions of $t,\boldsymbol{q},\dot{\boldsymbol{q}}$, and Eq.(3) can be written as

$$E_s(L)=Q_s+P_s+\Lambda_s,\quad(s=1,\ldots,n), \tag{7}$$

where $\Lambda_s=\Lambda_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})=\lambda_\beta\dfrac{\partial f_\beta}{\partial q_s}$ are the generalized holonomic reactive constraint forces. Expanding Eq.(7), we can seek all the generalized accelerations as

$$\ddot{q}_s=A_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}),\quad(s=1,\ldots,n). \tag{8}$$

If the system is free from the constraints, i.e. the sign of inequality in constraints (1) holds strictly, we have

$$f_\beta(t,\boldsymbol{q})>0\ ,\quad(\beta=1,\ldots,g) \tag{9}$$

then Eq.(3) becomes

$$E_s(L)=Q_s+P_s,\quad(s=1,\ldots,n). \tag{10}$$

From Eq.(10), we also find all the generalized accelerations as

$$\ddot{q}_s=B_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}),\quad(s=1,\ldots,n). \tag{11}$$

Choose the infinitesimal transformations of group with respect to time and coordinates as

$$\begin{aligned}&t^*=t+\varepsilon\tau(t,\boldsymbol{q},\dot{\boldsymbol{q}}),\\&q_s^*(t^*)=q_s(t)+\varepsilon\xi_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}),\\&(s=1,\ldots,n),\end{aligned} \tag{12}$$

where ε is an infinitesimal parameter, and τ, ξ_s are infinitesimal generators, under the infinitesimal transformations (12), L, Q_s, P_s, Λ_s and f_β become L^*, Q_s^*, P_s^*, Λ_s^* and f_β^*

$$\begin{aligned}L^*&=L\left(t^*,\boldsymbol{q}^*,\frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right)\\&=L(t,\boldsymbol{q},\dot{\boldsymbol{q}})+\varepsilon X^{(1)}(L)+O(\varepsilon^2),\end{aligned} \tag{13}$$

$$\begin{aligned}Q_s^*&=Q_s\left(t^*,\boldsymbol{q}^*,\frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right)\\&=Q_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})+\varepsilon X^{(1)}(Q_s)+O(\varepsilon^2),\end{aligned} \tag{14}$$

$$\begin{aligned}P_s^*&=P_s\left(t^*,\boldsymbol{q}^*,\frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*},\frac{\mathrm{d}^2\boldsymbol{q}^*}{\mathrm{d}t^{*2}}\right)\\&=P_s(t,\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})+\varepsilon X^{(2)}(P_s)+O(\varepsilon^2),\end{aligned} \tag{15}$$

$$\begin{aligned}\Lambda_s^*&=\Lambda_s\left(t^*,\boldsymbol{q}^*,\frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right)\\&=\Lambda_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})+\varepsilon X^{(1)}(\Lambda_s)+O(\varepsilon^2),\end{aligned} \tag{16}$$

$$\begin{aligned}f_\beta^*&=f_\beta(t^*,\boldsymbol{q}^*)\\&=f_\beta(t,\boldsymbol{q})+\varepsilon X^{(0)}(f_\beta)+O(\varepsilon^2),\end{aligned} \tag{17}$$

where

$$\begin{aligned}&X^{(0)}=\tau\frac{\partial}{\partial t}+\xi_s\frac{\partial}{\partial q_s},\\&X^{(1)}=X^{(0)}+(\dot{\xi}_s-\dot{q}_s\dot{\tau})\frac{\partial}{\partial\dot{q}_s},\\&X^{(2)}=X^{(1)}+[(\dot{\xi}_s-\dot{q}_s\dot{\tau})\dot{}-\ddot{q}_s\dot{\tau}]\frac{\partial}{\partial\ddot{q}_s}.\end{aligned} \tag{18}$$

If the system is under the constraints, we obtain definition 1.

Definition 1 If the form of constraints Eq.(6) and the differential equations (7) remains invariant under the infinitesimal transformations (12), i.e.

$$f_\beta^*=f_\beta(t^*,\boldsymbol{q}^*)=0,\quad(\beta=1,\ldots,g), \tag{19}$$

$$E_s(L^*)=Q_s^*+P_s^*+\Lambda_s^*,\quad(s=1,\ldots,n), \tag{20}$$

then the invariance is called the Mei symmetry of the equation of motion for variable mass systems with unilateral holonomic constraints, which is under the constraints.

If the system is free from constraints, we get definition 2.

Definition 2 If the form of the differential equations (10) remains invariant under the infinitesimal transformations (12), i.e.

$$E_s(L^*)=Q_s^*+P_s^* \tag{21}$$

then the invariance is called the Mei symmetry of the equation of motion for variable mass systems with unilateral holonomic constraints, which is free from the constraints.

Based on definition 1 and definition 2, we have definition 3.

Definition 3 If the form of Eqs.(6), (7), and (10) remains invariant under the infinitesimal transformations (12), then the invariance is called the Mei symmetry of the equation of motion for variable mass systems with unilateral holonomic constraints.

From definition 3, we have the following criterion.

Criterion For a variable mass system with unilateral holonomic constraints, which is expressed by Eqs.(1) and (3), if the infinitesimal generators τ, ξ_s satisfy the following relations:

$$\begin{aligned}&E_s\{X^{(1)}(L)\}=X^{(1)}(Q_s)+X^{(2)}(P_s)+X^{(1)}(\Lambda_s),\\&X^{(0)}(f_\beta)=0,\quad f_\beta=0,\\&E_s\{X^{(1)}(L)\}=X^{(1)}(Q_s)+X^{(2)}(P_s),\quad f_\beta>0,\\&(s=1,\dots,n;\beta=1,\dots,g)\end{aligned}\tag{22}$$

then it has Mei symmetry under the infinitesimal transformations (12).

Proof Substituting Eq.(17) into Eq.(19), using Eq.(16), substituting Eqs.(13)–(16) into Eq.(20), using Eq.(7), substituting Eqs.(13)–(15) into Eq.(21), using Eq.(10), then ignoring ε^2 and the higher infinitesimal terms, we get Eq.(22).

3. The necessary and sufficient condition under which Mei symmetry is a Lie symmetry

Based on the Lie theory of mechanical systems,[5] under the infinitesimal transformations (12), the determining equations of the Lie symmetry for variable mass systems with unilateral holonomic constraints are

$$\begin{aligned}&X^{(2)}\{E_s(L)\}-X^{(1)}(Q_s)-X^{(2)}(P_s)-X^{(1)}(\Lambda_s)=0,\quad X^{(0)}(f_\beta)=0,\quad f_\beta=0,\\&X^{(2)}\{E_s(L)\}-X^{(1)}(Q_s)-X^{(2)}(P_s)=0,\quad f_\beta>0,\quad (s=1,\dots,n;\beta=1,\dots,g),\end{aligned}\tag{23}$$

and their equivalent forms are

$$\begin{aligned}&\ddot{\xi}_s-\dot{q}_s\ddot{\tau}-2\dot{\tau}A_s=X^{(1)}(A_s),\\&\ddot{\xi}_s-\dot{q}_s\ddot{\tau}-2\dot{\tau}B_s=X^{(1)}(B_s),(s=1,\dots,n).\end{aligned}\tag{24}$$

We make the calculation and get

$$\begin{aligned}&E_s\{X^{(1)}(L)\}-X^{(2)}\{E_s(L)\}\\=&\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\tau}{\partial\dot{q}_s}\frac{\partial L}{\partial t}\right)-\frac{\partial\tau}{\partial q_s}\frac{\partial L}{\partial t}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_k}{\partial\dot{q}_s}\frac{\partial L}{\partial q_k}\right)-\frac{\partial\xi_k}{\partial q_s}\frac{\partial L}{\partial q_k}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\xi}_k}{\partial\dot{q}_s}\frac{\partial L}{\partial\dot{q}_k}\right)-\frac{\partial\dot{\xi}_k}{\partial q_s}\frac{\partial L}{\partial\dot{q}_k}\\&-\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\tau}}{\partial\dot{q}_s}\dot{q}_k\frac{\partial L}{\partial\dot{q}_k}\right)+\frac{\partial\dot{\tau}}{\partial q_s}\dot{q}_k\frac{\partial L}{\partial\dot{q}_k}-\ddot{\tau}\frac{\partial L}{\partial\dot{q}_s}.\end{aligned}\tag{25}$$

Therefore, we have

Proposition 1. For a variable mass system with unilateral holonomic constraints, the necessary and sufficient condition under which Mei symmetry is a Lie symmetry is that the right hand side of formula (25) vanishes, i.e.

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\tau}{\partial\dot{q}_s}\frac{\partial L}{\partial t}\right)-\frac{\partial\tau}{\partial q_s}\frac{\partial L}{\partial t}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\xi_k}{\partial\dot{q}_s}\frac{\partial L}{\partial q_k}\right)-\frac{\partial\xi_k}{\partial q_s}\frac{\partial L}{\partial q_k}+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\xi}_k}{\partial\dot{q}_s}\frac{\partial L}{\partial\dot{q}_k}\right)\\&-\frac{\partial\dot{\xi}_k}{\partial q_s}\frac{\partial L}{\partial\dot{q}_k}-\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\tau}}{\partial\dot{q}_s}\dot{q}_k\frac{\partial L}{\partial\dot{q}_k}\right)+\frac{\partial\dot{\tau}}{\partial q_s}\dot{q}_k\frac{\partial L}{\partial\dot{q}_k}-\ddot{\tau}\frac{\partial L}{\partial\dot{q}_s}=0.\end{aligned}\tag{26}$$

Proof Substituting Eq.(26) into Eq.(25), we obtain

$$E_s\{X^{(1)}(L)\} - X^{(2)}\{E_s(L)\} = 0 \tag{27}$$

and we have

$$\begin{aligned} &E_s\{X^{(1)}(L)\} - X^{(1)}(Q_s) - X^{(2)}(P_s) - X^{(1)}(\Lambda_s) \\ &= X^{(2)}\{E_s(L)\} - X^{(1)}(Q_s) - X^{(2)}(P_s) - X^{(1)}(\Lambda_s), \quad f_\beta = 0; \end{aligned} \tag{28}$$

$$E_s\{X^{(1)}(L)\} - X^{(1)}(Q_s) - X^{(2)}(P_s) = X^{(2)}\{E_s(L)\} - X^{(1)}(Q_s) - X^{(2)}(P_s), \quad f_\beta > 0, \tag{29}$$

according to Eqs.(22), (28) and (29), we know that Mei symmetry is a Lie symmetry.

4. The generalized Hojman conserved quantity deduced from the Mei symmetry

Using the Mei symmetry, we can find the generalized Hojman conserved quantity.

Proposition 2 For a variable mass system with unilateral holonomic constraints, under the infinitesimal transformations (12), if the infinitesimal generators τ, ξ_s satisfy Eqs.(22), (26), and there is a function $\mu = \mu(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ such that

$$\begin{aligned} &\frac{\partial A_s}{\partial \dot{q}_s} + \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\mu = 0, \quad f_\beta = 0, \\ &\frac{\partial B_s}{\partial \dot{q}_s} + \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\mu = 0, \quad f_\beta > 0, \end{aligned} \tag{30}$$

where

$$\begin{aligned} &\frac{\bar{\mathrm{d}}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \dot{q}_s\frac{\partial}{\partial q_s} + A_s\frac{\partial}{\partial \dot{q}_s}, \quad f_\beta = 0, \\ &\frac{\bar{\mathrm{d}}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \dot{q}_s\frac{\partial}{\partial q_s} + B_s\frac{\partial}{\partial \dot{q}_s}, \quad f_\beta > 0, \end{aligned} \tag{31}$$

then the Mei symmetry will lead to the generalized Hojman conserved quantity

$$\begin{aligned} I = &\frac{\partial \tau}{\partial t} + \frac{\partial \xi_s}{\partial q_s} + \frac{\partial}{\partial \dot{q}_s}\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_s - \dot{q}_s\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau\right) \\ &+\tau\frac{\partial}{\partial t}\ln\mu + \xi_s\frac{\partial}{\partial q_s}\ln\mu \\ &+\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_s - \dot{q}_s\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau\right)\frac{\partial}{\partial \dot{q}_s}\ln\mu - \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau \\ = &\ \text{const.} \end{aligned} \tag{32}$$

Proof If the infinitesimal generators τ, ξ_s satisfy Eq.(26), using proposition 1, we know that the generators τ, ξ_s are Lie symmetrical, then we can subsequently get the conserved quantity (32) by using the proof method[14] of the generalized Hojman conserved quantity.

5. Illustrative example

The mass of a particle with variable mass is $m(t) = m_0 t$ (where m_0 is constant), the Lagrangian is

$$L = \frac{1}{2}m(t)(\dot{q}_1^2 + \dot{q}_2^2), \tag{33}$$

the nonpotential generalized forces are

$$Q_1 = m_0(\dot{q}_1 + t), \quad Q_2 = m_0\dot{q}_2, \tag{34}$$

the relative velocity of the small particle corresponding to particle i is

$$\boldsymbol{u} = -\dot{\boldsymbol{r}}_i, \tag{35}$$

the unilateral constraint is

$$f = q_2 - q_1 \geq 0, \tag{36}$$

we try to study the Mei symmetry and the generalized Hojman conserved quantity.

From Eq.(4), we have

$$P_1 = P_2 = 0. \tag{37}$$

The differential equations of motion of the system are

$$\begin{aligned} &\dot{m}\dot{q}_1 + m\ddot{q}_1 = m_0\dot{q}_1 + m_0 t - \lambda, \\ &\dot{m}\dot{q}_2 + m\ddot{q}_2 = m_0\dot{q}_2 + \lambda, \\ &\lambda \geq 0,\ f \geq 0,\ \lambda f = 0. \end{aligned} \tag{38}$$

If the system is under the constraints, from Eqs.(36) and (38), we have

$$\lambda = \frac{m_0 t}{2}, \quad \ddot{q}_1 = \frac{1}{2}, \quad \ddot{q}_2 = \frac{1}{2}, \tag{39}$$

the generalized holonomic reactive constraint forces are

$$\Lambda_1 = -\frac{m_0 t}{2}, \qquad \Lambda_2 = \frac{m_0 t}{2}. \tag{40}$$

If the system is free from the constraints, from Eq.(38), we obtain

$$\ddot{q}_1 = 1, \quad \ddot{q}_2 = 0, \tag{41}$$

Taking infinitesimal transformation generators

$$\tau = 1, \quad \xi_1 = \xi_2 = 1, \tag{42}$$

it is easy to prove that Eq.(42) satisfy Eqs.(22) and (26), by using proposition 1, we find that the quantities in Eq.(42) are both Mei symmetrical and Lie symmetrical infinitesimal transformation generators.

Expression (30) gives

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t} \ln \mu = 0, \tag{43}$$

then we have

$$\mu = (\dot{q}_1 + \dot{q}_2)t - q_1 - q_2 - \frac{1}{2}t^2. \tag{44}$$

From Eqs.(42) and (43), by using proposition 2, from the conserved quantity expression (32) we obtain

$$I = \frac{\dot{q}_1 + \dot{q}_2 - t - 2}{(\dot{q}_1 + \dot{q}_2)t - q_1 - q_2 - \frac{1}{2}t^2} = \text{const.} \tag{45}$$

Equations (45) is a generalized Hojman conserved quantity of the system deduced from the Mei symmetry.

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