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Effects of walk-off on cross-phase modulation induced modulation instability in an optical fibre with high-order dispersion*

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This paper investigates the effects of walk-off among optical pulses on cross-phase modulation induced modulation instability in the normal dispersion region of an optical fibre with high-order dispersion. The results indicate that, in the case of high-order dispersion, the walk-off effect takes on new characteristics and will influence considerably the shape, position and especially the number of the spectral regions of the gain spectra of modulation instability. Not only the group-velocity mismatch, but also the difference of the third-order dispersion of two optical waves will alter the gain spectra of modulation instability but in different ways. Depending on the values of the walk-off parameters, the number of the spectral regions may increase from two to at most four, and the spectral shape and position may change too.

Keywords: modulation instability, cross-phase modulation, walk-off effect, the difference of the third-order dispersion

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1. Introduction

It is well known that the interplay between the dispersion (in the temporal domain) or diffraction (in the spatial domain) or both (in the spatiotemporal domain) and the nonlinear effects of the optical material can result in modulation instability,[1−22] optical solitons,[23−26] and pulse compression,[26−30] etc. Among them, modulation instability has been investigated extensively whether in the temporal domain,[1−18] spatial domain,[19−21] or spatiotemporal domain,[22] for its significant theoretical and practical importance. This phenomenon manifests itself as the exponential growth of small optical perturbation with the propagation distance. As a result, the periodic temporal or spatial optical structures generate, which may evolve into temporal or spatial optical solitons. Thus, in this sense, modulation instability can be regarded as the precursor of optical solitons.

The temporal domain modulation instability, which includes self-phase modulation induced-modulation instability of one optical wave [1,4−10] and cross-phase modulation induced-modulation instability of two or more optical waves,[11−16] is generally thought to occur in optical fibres. Its significant applications lie in generation of high-repetition-rate soliton-like pulse trains,[1] all-optical switching,[4] generation of super-continuum spectra,[10] and so on. However, previous studies also indicate that the modulation instability is detrimental to long-haul optical fibre communication system.[5] For these reasons, modulation instability has attracted continuous interest since 1986 when it was experimentally demonstrated by Tai.

Self-phase modulation induced-modulation instability has been extensively studied in case of second-order[1,4−6] and high-order dispersions.[7−10] As for cross-phase modulation induced-modulation instability of two optical waves, however, previous studies were almost limited to the case of second-order dispersion.[11−15] To our knowledge, few reports involved the influences of the high-order dispersions[16] or, needless to say, the combined influences of the high-order dispersions and walk-off effects on cross-

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phase modulation induced-modulation instability. Although the high-order dispersion has been taken into account for the first time in Ref. [16], the walk-off effects have been neglected. However, as one knows, when the width of the optical pulse is short enough, or when the second-order group velocity dispersion approached its minimum value at the so-called zero-dispersion wavelength, the high-order dispersions will take effect.[7−10,16] Moreover, studies also show that the effect of walk-off among optical pulses can considerably modify the cross-phase modulation[28] and the gain spectra of cross-phase modulation induced modulation instability.[15] Therefore, generally speaking, both of the high-order dispersions and walk-off effect should be taken into consideration in study of cross-phase modulation induced-modulation instability.

The purpose of this paper is to study the effects of walk-off among two optical pulses on cross-phase modulation induced modulation instability when taking high-order dispersion of optical fibres into consideration. In this case, the walk-off effect may take on new characteristics and influence modulation instability considerably, as the following discussions will confirm.

2. Theoretical analysis

When two optical waves of different wavelengths with the same polarization co-propagate in the single-mode optical fibre, they may couple with each other through optical intensity induced change of refractive index by another optical wave due to cross-phase modulation. When taking into account the high-order dispersions, the slowly varying envelopes A_1 and A_2 of the two optical fields satisfy the following extended coupling nonlinear Schrödinger equations:[16]

$$\begin{aligned}&\frac{\partial A_j}{\partial z}+\frac{1}{V_{gj}}\frac{\partial A_j}{\partial t}+\frac{\mathrm{i}}{2}\beta_{2j}\frac{\partial^2 A_j}{\partial t^2}\\&-\frac{\beta_{3j}}{6}\frac{\partial^3 A_j}{\partial t^3}+\frac{\mathrm{i}}{24}\beta_{4j}\frac{\partial^4 A_j}{\partial t^4}\\&=\mathrm{i}\gamma_j(|A_j|^2+2|A_{3-j}|^2)A_j,\end{aligned}\tag{1}$$

where V_{gj} $(j=1,2)$ is the group velocity, β_{kj} $(k=2,3,4)$ is the k-order group velocity dispersion coefficient, γ_j is the nonlinear coefficient, z is the propagating distance, t is the time coordinate. To simplify the discussion, here we have omitted the loss of the optical fibre.

The steady-state solution of Eq.(1) is given by

$$\bar{A}_j=\sqrt{P_j}\exp(\mathrm{i}\phi_j).\tag{2}$$

Where P_j is the incident optical power and the nonlinear phase shift ϕ_j is of the following form:

$$\phi_j=\gamma_j P_j z+2\gamma_j P_{3-j}z.\tag{3}$$

Adopting the linear stability analysis, inserting weak perturbation a_j $(|a_j|^2\ll P_j)$ in Eq.(2), one obtains

$$A_j=(\sqrt{P_j}+a_j)\exp(\mathrm{i}\phi_j).\tag{4}$$

Substituting Eq.(4) into Eq.(1) and linearizing a_j, one can get the following coupled linearized nonlinear Schrödinger equations:

$$\begin{aligned}&\frac{\partial a_j}{\partial z}+\frac{1}{V_{gj}}\frac{\partial a_j}{\partial t}+\frac{\mathrm{i}}{2}\beta_{2j}\frac{\partial^2 a_j}{\partial t^2}\\&-\frac{\beta_{3j}}{6}\frac{\partial^3 a_j}{\partial t^3}+\frac{\mathrm{i}}{24}\beta_{4j}\frac{\partial^4 a_j}{\partial t^4}\\&=\mathrm{i}\gamma_j P_j(a_j+a_j^*)\\&\quad+2\mathrm{i}\gamma_j\sqrt{P_1P_2}(a_{3-j}+a_{3-j}^*).\end{aligned}\tag{5}$$

Here, we can assume that the general solution of Eq.(5) satisfies[15]

$$a_j(z,t)=U_j\cos(kz-\Omega t)+\mathrm{i}V_j\sin(kz-\Omega t),\tag{6}$$

where k and Ω are the wave number and angle frequency of the modulation wave respectively. Substituting Eq.(6) into Eq.(5) we get a set of four homogeneous equations for U_1, U_2, V_1, and V_2. This set of equations has a nontrivial solution only when the wave number k satisfies the following dispersion relation:

$$\begin{aligned}&\left[\left(k-\frac{\Omega}{V_{g1}}-\frac{1}{6}\beta_{31}\Omega^3\right)^2-f_1\right]\\&\times\left[\left(k-\frac{\Omega}{V_{g2}}-\frac{1}{6}\beta_{32}\Omega^3\right)^2-f_2\right]=C_{\mathrm{XPM}},\end{aligned}\tag{7}$$

where the parameters f_1, f_2 and C_{XPM} satisfy

$$\begin{aligned}f_j=&\left(\frac{1}{2}\beta_{2j}\Omega^2-\frac{1}{24}\beta_{4j}\Omega^4\right)\\&\times\left[\frac{1}{2}\beta_{2j}\Omega^2-\frac{1}{24}\beta_{4j}\Omega^4+2\gamma_j P_j\right],(j=1,\ 2),\end{aligned}\tag{8}$$

$$\begin{aligned}C_{\mathrm{XPM}}=&16\gamma_1\gamma_2P_1P_2\left(\frac{1}{2}\beta_{21}\Omega^2-\frac{1}{24}\beta_{41}\Omega^4\right)\\&\times\left(\frac{1}{2}\beta_{22}\Omega^2-\frac{1}{24}\beta_{42}\Omega^4\right)\end{aligned}\tag{9}$$

Obviously the expression of the dispersion relation (7) is similar to the case of second-order group velocity dispersion.[15] However, the definitions of parameters f_1, f_2 and C_{XPM} are quite different. Furthermore, the third-order dispersion terms have already appeared in the dispersion relation (7), which means that the

walk-off effect in case of high-order dispersion consists of group-velocity mismatch and the difference of the third-order dispersion. In other words, being different from the case of self-phase modulation induced modulation instability, where all the odd-order dispersion terms do not contribute to modulation instability,[7−10] the third-order dispersion here will influence the cross-phase modulation induced modulation instability. Eqs.(7)–(9) are in agreement with those of Ref.[15] when the third- and the fourth-order dispersions do not exist.

Under certain conditions, when k becomes complex for some angle frequencies Ω, modulation instability occurs and the corresponding power gain equals $2\mathrm{Im}(k)$, where Im stands for the imaginary part. If neglecting the walk-off effect, the analytical expression for the power gain spectra of modulation instability can be obtained as that shown in Ref.[16]:

$$g(\Omega) = 2\mathrm{Im}(k) = \sqrt{2}\,\{[(f_1+f_2)^2 + 4(C_{\mathrm{XPM}} - f_1 f_2)]^{1/2} - (f_1+f_2)\}^{1/2}. \quad (10)$$

Obviously, Eq.(10) is completely the same as the case of second-order dispersion. But the gain spectra are quite different from the latter case.[16]

In general, however, the walk-off effect should be taken into account and Eq.(7) should be numerically solved to discuss cross-phase modulation instability. To do so, one can set $\delta_1 = V_{g1}^{-1} - V_{g2}^{-1}$, $\delta_2 = \beta_{31} - \beta_{32}$, and $K = k - \Omega/V_{g2} - (1/6)\beta_{32}\Omega^3$, where δ_1 is the group velocity mismatch parameter and δ_2 the difference of the third-order dispersion. After substituting these assumptions into Eq.(7), one can obtain the following new dispersion relation:

$$\left[\left(K - \delta_1\,\Omega - \frac{1}{6}\delta_2\Omega^3\right)^2 - f_1\right][K^2 - f_2] = C_{\mathrm{XPM}}. \quad (11)$$

For the relation $\mathrm{Im}(k) = \mathrm{Im}(K)$ holds, the power gain of modulation instability, which equals to $2\mathrm{Im}(K)$, can be obtained by numerically solving Eq.(11). When taking into account the high-order dispersion, it is obvious to see from Eq.(11) that the walk-off effect among optical pulses includes not only the conventional group-velocity mismatch but also the difference of the third-order dispersion, both of which will alter the gain spectra of modulation instability. Moreover, one can conclude easily from Eq.(11) that the power gain has nothing to do with the concrete values of the group velocity and the third-order dispersion.

3. Calculations and discussions

In principle, it is well-known that the cross-phase modulation induced modulation instability can occur irrespective of the signs of the second-order group velocity dispersion coefficients of the two optical waves.[14,15] However, we will only discuss modulation instability in the normal dispersion region in what follows. For modulation instability in this case is solely due to cross-phase modulation.[15] In addition, Eq.(11) has four roots corresponding to the four gain spectra of modulation instability. But only the spectrum, which has the largest peak value as well as the most number of the separated spectral regions and can contain the other three ones, is adopted in the following discussion.

Without loss of generality, the wavelength λ_1 is chosen to be about 0.9μm, near this wavelength, the parameters of the conventional fibre are set as $\gamma_{11} = \gamma_{12} = 10\ \mathrm{W^{-1}\cdot\ km^{-1}}$, $\beta_{21} = \beta_{22} = 20\ \mathrm{ps^2\cdot\ km^{-1}}$ and $\beta_{41} = \beta_{42} = 0.1\ \mathrm{ps^4\cdot\ km^{-1}}$. The incident optical powers are set as $P_1 = 10$ W and $P_2 = 30$ W. The group velocity mismatch parameter and the difference of the third-order dispersion are set as $\delta_1 = 0 - 0.4\ \mathrm{ps.\,m^{-1}}$ and $\delta_2 = 0 - 4.8\times10^{-4}\ \mathrm{ps^3\cdot m^{-1}}$ respectively, which correspond to the maximum wavelength separations between the two optical waves of about 8.6 nm.

3.1. In the case of $\delta_1 \neq 0$ and $\delta_2 = 0$

In this case, the walk-off effect consists of only the conventional group velocity mismatch of two optical waves. The gain spectra of modulation instability for different parameters δ_1 have been shown in Fig.1. To observe more clearly, every complete spectrum has been divided into the first part (a) and the second ones (b) and (c).

From Fig.1, one can see that, when the walk-off effect is neglected ($\delta_1 = 0$ and $\delta_2 = 0$), every complete spectrum includes two spectral regions which can be seen in Figs.1(a) and 1(b) respectively. The first and the second spectral regions are near and far away from the centre frequencies of the main waves respectively. Moreover, the second spectral region is virtually made up of two sub-spectra which are linked together all the way. This simulation here is identical to the analytical result of Ref.[16]. When the group velocity mismatch is taken into account, however, the gain spectrum takes on new characteristics and the shape, the position and especially the number of the spectral peak are quite different from those of Ref.[16].

Concretely, when the mismatch parameter δ_1 is small, every complete gain spectrum comprises at most four spectral regions. With the increase of parameter δ_1, the third spectral region hardly varies, while the second and the fourth ones get far away from the third one, and the first one shifts to higher frequencies and its peak value increases before decreasing. Further increase of δ_1 will make the first and the second spectral regions overlap and disappear afterwards. In comparison, those spectral regions whose positions are far away from the centre frequencies have narrower spectral widths. In addition, the increase of δ_1 hardly alters the peak value of the fourth spectral region.

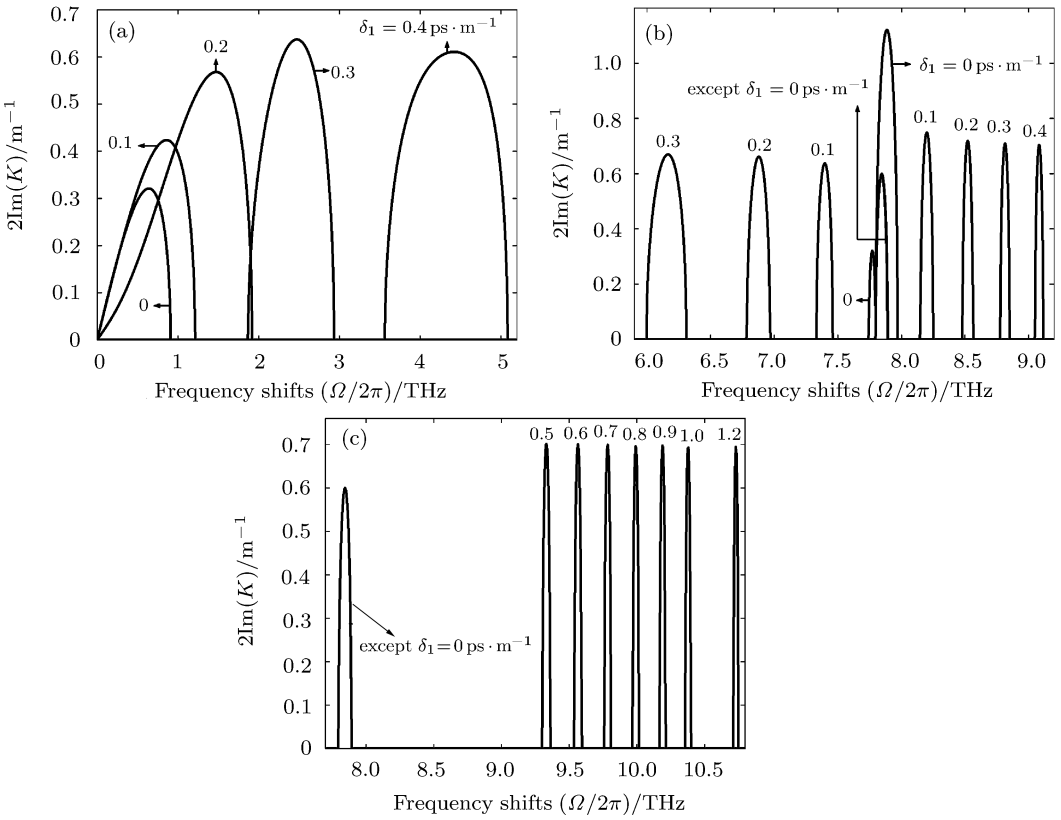


Fig.1. The first (a) and the second part ((b) and (c)) of the gain spectra of modulation instability for different parameters δ_1 when the parameter δ_2 is neglected.

3.2. In the case of $\delta_1 = 0$ and $\delta_2 \neq 0$

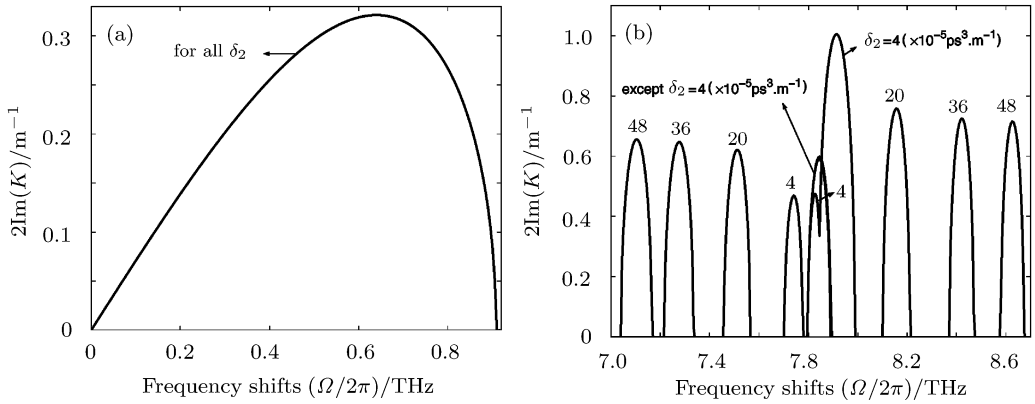


Fig.2. The first (a) and the second part (b) of the gain spectra of modulation instability for different parameters δ_2 when the parameter δ_1 is neglected.

In this case, the walk-off effect consists of only the difference of the third-order dispersions of two optical waves. The gain spectra of modulation instability for different parameters δ_2 have been shown in Fig.2. Similarly, every complete spectrum also comprises at most four spectral regions and is also divided into the first part (a) and the second one (b). With the increase of the parameter δ_2, it can be realized that the first spectral region is hardly influenced, the third one is only modified a little in terms of its peak value and width, and the second and the fourth ones also get far away from the third one as the case in Section 3.1.

3.3. In the case of $\delta_1 \neq 0$ and $\delta_2 \neq 0$

In this general case, both the group-velocity mismatch and the difference of the third-order dispersion will modify the gain spectra. The gain spectra of modulation instability for different parameters δ_1 and δ_2 have been shown in Fig.3. For $\delta_1 = 0.1,\ 0.2\,\mathrm{ps}\cdot\mathrm{m}^{-1}$ and $\delta_2 = 4\times10^{-5},\ 4\times10^{-4}\,\mathrm{ps}^3\cdot\ \mathrm{m}^{-1}$, every complete spectrum is divided into the first part (a) and the second part (b). While for $\delta_1 = 0.3,\ 0.4\,\mathrm{ps}\cdot\mathrm{m}^{-1}$ and $\delta_2 = 8\times10^{-5},\ 4\times10^{-4}\,\mathrm{ps}^3\cdot\ \mathrm{m}^{-1}$, every complete spectrum is divided into the first part (c) and the second part (d). Figure 3 indicates that the third spectral region is also hardly influenced by the walk-off parameters. The increase of δ_1 will also make the first spectral region shift to higher frequencies, and the peak value increases before decreasing. Being different from the case of $\delta_1 = 0$ and $\delta_2 \neq 0$, the increase of parameter δ_2 here will alter the peak value and width of the first spectral region to a small extent for the same δ_1. The second and the fourth spectral regions will also get far away from the third one with the increase of δ_1 or δ_2. Further increase of δ_1 or δ_2 will also make the first and the second spectral regions overlap and disappear.

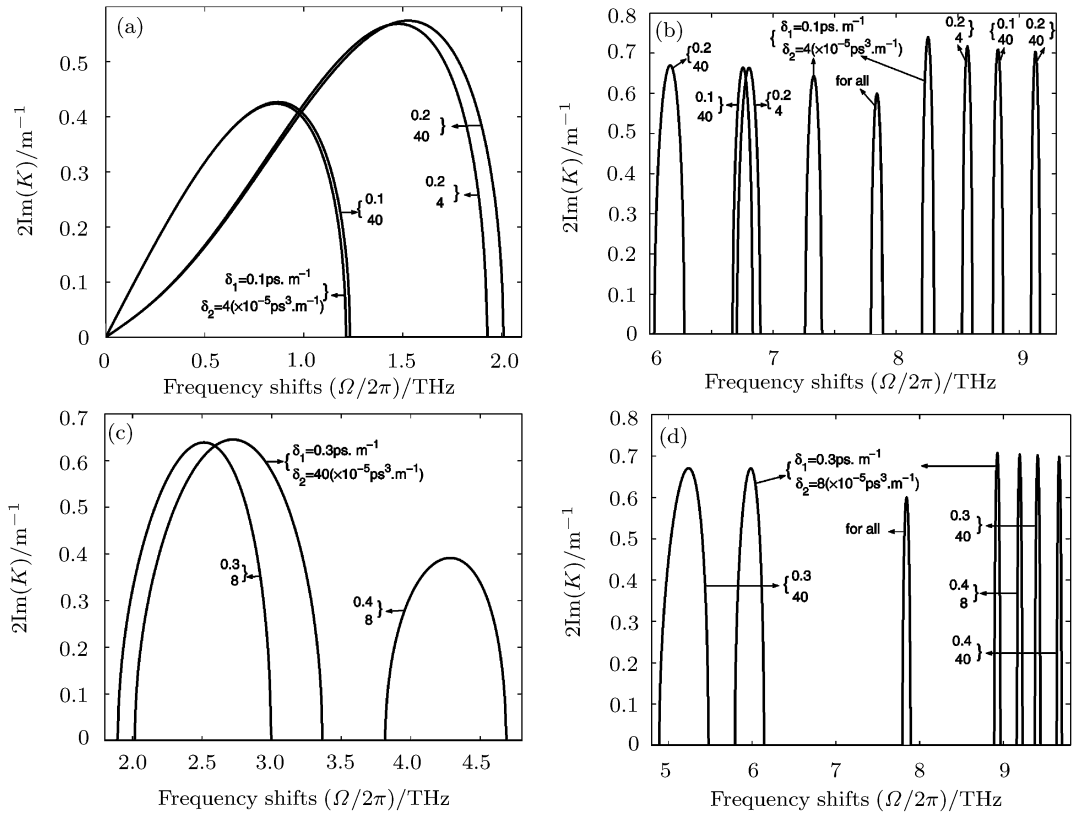


Fig.3. The gain spectra of modulation instability for different parameters δ_2 and δ_1. For $\delta_1 = 0.1,\ 0.2\ \mathrm{ps}\cdot\mathrm{m}^{-1}$ and $\delta_2 = 4\times10^{-5},\ 4\times10^{-4}\,\mathrm{ps}^3\cdot\ \mathrm{m}^{-1}$, every complete spectrum is divided into the first part (a) and the second part (b). While for $\delta_1 = 0.3,\ 0.4\,\mathrm{ps}\cdot\mathrm{m}^{-1}$ and $\delta_2 = 8\times10^{-5},\ 4\times10^{-4}\,\mathrm{ps}^3\cdot\ \mathrm{m}^{-1}$, every complete spectrum is divided into the first part (c) and the second part (d).

When the high-order dispersion is neglected, the relations $\beta_3 = \beta_4 = 0$ and $\delta_2 = 0$ holds, and then the simulation results here according to Eqs.(8), (9) and (11) are completely in agreement with those of Ref.[15].

4. Conclusions

In conclusion, the effects of walk-off among two optical pulses on cross-phase modulation induced modulation instability in the normal dispersion region of an optical fibre with high-order dispersion are investigated and the gain spectra of modulation instability for different walk-off parameters are obtained. The results indicate that, in case of high-order dispersion, the walk-off effect takes on new characteristics and both the group-velocity mismatch and the difference of the third-order dispersion will alter the gain spectra of modulation instability considerably in terms of the shape, position and especially the number of the spectral regions, but in different ways. When the walk-off effect and high-order dispersion are neglected, our simulation results are completely in agreement with those of Refs.[16] and [15]. Depending on the walk-off parameters δ_1 and δ_2, the number of the spectral regions may increase from two to at most four, and the spectral shape and position may change too.

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