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Teleportation of a two-atom entangled state via cavity decay*

Ye Sai-Yun(叶赛云)†

Department of Electronic Science and Applied Physics Fuzhou University Fuzhou 350002, *China*

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This paper proposes a scheme for teleporting a two-atom entangled state using leaky cavities. It uses resonant atom–cavity interaction to map the atomic state onto the cavity field. Then it utilizes the interference of polarized photons to establish the correlation between the distant sender and receiver. The advantage of the scheme is that the fidelity is independent of the cavity decay rate, atomic decay and detection efficiency.

Keywords: teleportation, entangled state, cavity decay
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Quantum teleportation, first introduced by Bennett *et al*,[1] is a process to transfer an unknown quantum state to a remote location without sending the original carriers of quantum state. Since it provides promising applications in quantum computation[2] and quantum communication,[3] much efforts have been devoted to implement the process. Theoretically, various protocols have been suggested in different contexts, such as trapped ions,[4] running wave[5] and cavity QED.[6] Experimentally, teleportation has also been demonstrated by several groups for discrete[7] and continuous variables.[8] Recently, two experiments for teleporting atomic states have been implemented in ion-trap systems by Riebe *et al*[9] and Barrett *et al*[10] respectively.

In the original teleporting protocol,[1] two key steps are the preparation of a maximally entangled state as a quantum channel and the realization of Bell-state measurement. However, the complete Bell-state measurement is still an experimental challenge within current technique. To overcome the obstacle, Zheng[11] has proposed an unique scheme for teleporting an atomic state in cavity QED, where the Bell-state measurement is not employed. Subsequently, various schemes[12] adopting the idea to simplify teleporting procedure have been proposed. On the other hand, since atoms are ideal for storing information as stationary qubits, and photons are ideal for carrying quantum information over long distances as flying qubits, some authors abandon entangled atomic states as quantum channels but utilize photon-interference to establish the correlation between the distant sender and receiver. For instance, Bose *et al*[13] have suggested a comparatively easy scheme to teleport an atomic state via cavity decay, where the flying qubits are photons and stationary qubits are atoms. Based on this method, Cho and Lee[14] have proposed an improved scheme using adiabatic passage, which is resistant to spontaneous decay and detection inefficiency. Recently, Zheng and Guo[15] have proposed a robust scheme via cavity decay, where the atom–cavity coupling strength is much smaller than the cavity decay rate and thus cavities of high quality factor are unnecessary. The schemes mentioned above mainly focus on the teleportation of an unknown atomic state.

In recent years, attention has been paid to entanglement teleportation in the realm of cavity QED. Cardoso *et al*[16] have proposed a Bell-state-measurement-free scheme for teleporting entangled field states with an approximately successful probability of 25%. Here, we propose a scheme for teleportation of a two-atom entangled state via cavity decay, where the fidelity is independent of the cavity decay rate, atomic decay and detection efficiency.

Inspired by the experimental setup in Ref.[17], which utilizes the photons leaking from four cavities to conditionally generate four-atom GHZ state, we employ it to teleport a two-atom entangled state. As shown in Fig.1, the setup is composed of four atoms, four optical cavities, four symmetric beam splitters and four single-photon detectors. Atoms 1, 2, 3 and 4 are trapped in cavities A, B, C and D, respectively.

†E-mail: yesaiyun@fzu.edu.cn

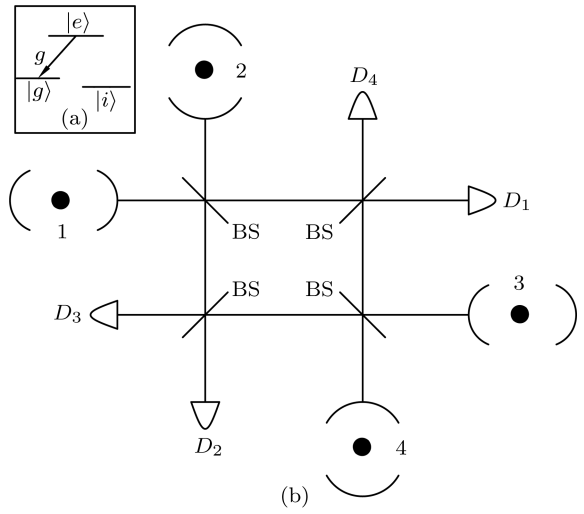


Fig.1. (a) The level configuration of each atom. The transition $|e\rangle \rightarrow |g\rangle$ is resonant with the cavity field with the coupling strength g. (b) The experimental setup. The four atoms are separately trapped in four leaky cavities. The four BS, four photondetectors D_j, the cavities l and 2 belong to Alice, while cavities 3 and 4 belong to Bob.

We here use Λ-type three-level atoms, where the atomic states are denoted by $\mid e\rangle$, $\mid g\rangle$ and $\mid i\rangle$. In each cavity, we assume that the transition frequency between $\mid e\rangle$ and $\mid g\rangle$ is resonant with a quantized cavity field. The transition frequency between $\mid i\rangle$ and $\mid e\rangle$ is highly detuned from the quantized cavity field and thus the level $\mid i\rangle$ is not affected by the atom–cavity interaction. Our present scheme is only based on resonant atom–cavity interaction, which is very different from the scheme in Ref.[17]. In the scheme of Ref.[17], it requires classical field driving-assisted resonant atom–cavity interaction, which aims at the generation of four-atom GHZ state.

We now take the cavity decay into consideration. Under the condition that no photon is detected by photon-detectors during the preparation stage, following a quantum trajectory description[13] the non-Hermitian Hamiltonian in the interaction picture can be expressed as

$$H_j = g\left(a_j s_j^+ + a_j^+ s_j^-\right) - \mathrm{i}\kappa a_j^+ a_j, \tag{1}$$

where a_j^+ and a_j are the creation and annihilation operators of the jth($j = 1,2,3,4$) cavity mode respectively, $s_j^+ = \mid e_j\rangle\langle g_j \mid$, $s_j^- = \mid g_j\rangle\langle e_j \mid$, g is the atom–cavity coupling strength, and κ is the cavity decay rate. Suppose that the atom is initially in the state $\mid e\rangle$ and the cavity is in a vacuum state. After an interaction time τ, the state of the system evolves into

$$\mid \Phi(\tau)\rangle = \alpha \mid e\rangle \mid 0\rangle + \beta \mid g\rangle \mid 1\rangle, \tag{2}$$

where

$$\alpha = \frac{\mathrm{e}^{-\frac{\kappa}{2}\tau}\left[\cos(\Omega_k\tau) + \dfrac{\kappa}{2\Omega_\kappa}\sin(\Omega_k\tau)\right]}{\sqrt{\left[\cos(\Omega_k\tau) + \dfrac{\kappa}{2\Omega_\kappa}\sin(\Omega_k\tau)\right]^2 + \dfrac{g^2}{\Omega_\kappa^2}\sin^2(\Omega_k\tau)}}, \tag{3}$$

$$\beta = -\frac{\mathrm{i}g\mathrm{e}^{-\frac{\kappa}{2}\tau}\sin(\Omega_k\tau)}{\Omega_\kappa\sqrt{\left[\cos(\Omega_k\tau) + \dfrac{\kappa}{2\Omega_\kappa}\sin(\Omega_k\tau)\right]^2 + \dfrac{g^2}{\Omega_\kappa^2}\sin^2(\Omega_k\tau)}}, \tag{4}$$

$$\Omega_k = \sqrt{g^2 - \kappa^2/4}. \tag{5}$$

If the interaction time τ is appropriately chosen to satisfy

$$\tan(\Omega_\kappa\tau) = -\frac{2\Omega_\kappa}{\kappa}, \tag{6}$$

then the atom–cavity system evolves into the state

$$\mid \Phi(\tau)\rangle = \beta \mid g\rangle \mid 1\rangle. \tag{7}$$

During the stage, the probability of no photon being detected is $P_{\mathrm{ND(single)}} = \mathrm{e}^{-\kappa\tau}\left[\dfrac{g^2}{\Omega_\kappa^2}\sin^2(\Omega_k\tau)\right]$.

Now assume that the entangled state of atoms 1 and 2 which Alice wants to teleport is

$$\mid \phi\rangle_{1,2} = a \mid e\rangle_1 \mid g\rangle_2 + b \mid g\rangle_1 \mid e\rangle_2, \tag{8}$$

where a and b are unknown coefficients and satisfy $|a|^2 + |b|^2 = 1$. We further assume that initially each cavity is in a vacuum state, and thus the atom–cavity

system possessed by Alice is in the state

$$|\ \Psi(0)\rangle_A = (a\ |\ e\rangle_1\ |\ g\rangle_2 + b\ |\ g\rangle_1\ |\ e\rangle_2)\ |\ 0\rangle_1\ |\ 0\rangle_2. \tag{9}$$

The time evolution of the state is governed by the Hamiltonian in Eq.(1). After the interaction time τ given by Eq.(6), the atom–cavity system evolves into the state

$$|\ \Psi(\tau)\rangle_A = |\ g\rangle_1\ |\ g\rangle_2 (a\ |\ 1\rangle_1\ |\ 0\rangle_2 + b\ |\ 0\rangle_1\ |\ 1\rangle_2), \tag{10}$$

where we have discarded the common phase factor β. Here, we note that the atomic state has been mapped onto the cavity fields.

On the other hand, we assume that the atoms 3 and 4 are in the maximally entangled state

$$|\ \Psi\rangle_{3,4} = \frac{1}{\sqrt{2}} (|\ e\rangle_3\ |\ g\rangle_4 + |\ g\rangle_3\ |\ e\rangle_4). \tag{11}$$

Thus the initial atom–cavity system state possessed by Bob is

$$|\ \Psi(0)\rangle_B = \frac{1}{\sqrt{2}} (|\ e\rangle_3\ |\ g\rangle_4 + |\ g\rangle_3\ |\ e\rangle_4)\ |\ 0\rangle_3\ |\ 0\rangle_4. \tag{12}$$

We first use classical fields to make the single-qubit rotations on atoms 3 and 4 in their respective cavities,

$$|\ g\rangle_3 \rightarrow |\ i\rangle_3, |\ g\rangle_4 \rightarrow |\ i\rangle_4. \tag{13}$$

This leads to

$$|\ \Psi\rangle_B = \frac{1}{\sqrt{2}} (|\ e\rangle_3\ |\ i\rangle_4 + |\ i\rangle_3\ |\ e\rangle_4)\ |\ 0\rangle_3\ |\ 0\rangle_4. \tag{14}$$

Then atoms 3 and 4 resonantly interact with their respective quantized cavity fields. After the interaction time τ given by Eq.(6), the state of the atom–cavity system is

$$\begin{aligned} |\Psi(\tau)\rangle_B = \ & \frac{1}{\sqrt{2}}(|g\rangle_3|i\rangle_4|1\rangle_3\ |\ 0\rangle_4 \\ & + |\ i\rangle_3\ |\ g\rangle_4\ |\ 0\rangle_3|1\rangle_4). \end{aligned} \tag{15}$$

Now performing the following transformations:

$$|\ i\rangle_3 \rightarrow |\ e\rangle_3, |\ i\rangle_4 \rightarrow |\ e\rangle_4, \tag{16}$$

we find that the state of the atom–cavity system changes to

$$\begin{aligned} |\Psi'\rangle_B = \ & \frac{1}{\sqrt{2}}(|g\rangle_3|e\rangle_4|1\rangle_3|0\rangle_4 \\ & + |e\rangle_3|g\rangle_4|0\rangle_3|1\rangle_4). \end{aligned} \tag{17}$$

Now for Alice and Bob, the total system is in the state

$$|\ \Psi\rangle_{A,B} = |\ \Psi(\tau)\rangle_A \otimes |\ \Psi'\rangle_B. \tag{18}$$

Here, the successful probability during the preparation stage is the probability that no photon decays from the cavities, which is given by

$$\begin{aligned} P_{\mathrm{suc(prep)}} = \ & P_{\mathrm{ND}(A)} P_{\mathrm{ND}(B)} \\ = \ & P_{\mathrm{ND(single)}}(|a|^2 + |b|^2) \\ & \times \left(P_{\mathrm{ND(single)}} + P_{\mathrm{ND(single)}}\right)/2 \\ = \ & P^2_{\mathrm{ND(single)}}. \end{aligned} \tag{19}$$

Here, we do not take the successful probability of performing single qubit rotations into consideration during the Bob's preparation stage.

Next we turn to the detection stage. Now the atomic transition $|\ g\rangle \longleftrightarrow |\ e\rangle$ is tuned far off-resonant with the cavity field so that the atom–cavity interaction is frozen during the detection stage. Then if there is a click in the detector D_j $(j = 1, 2, 3, 4)$, the coherent evolution is interrupted by a quantum jump. The corresponding operators of the action of quantum jumps can be described as[17]

$$\begin{aligned} D_1 &= \frac{1}{2}(a_1 + a_2 - a_3 + a_4), \\ D_2 &= \frac{1}{2}(-a_1 + a_2 + a_3 + a_4), \\ D_3 &= \frac{1}{2}(a_1 - a_2 + a_3 + a_4), \\ D_4 &= \frac{1}{2}(-a_1 - a_2 - a_3 + a_4). \end{aligned} \tag{20}$$

Assume that D_1 clicks at the time t_1. The total atom–cavity system collapses into the state

$$\begin{aligned} |\ \Psi(t_1)\rangle_{A,B} = \ & \frac{1}{\sqrt{2}} e^{-2\kappa t_1}\ |\ g\rangle_1\ |\ g\rangle_2 \{|\ g\rangle_3\ |\ e\rangle_4 [a(|\ 0\rangle_1\ |\ 0\rangle_2\ |\ 1\rangle_3\ |\ 0\rangle_4 - |\ 1\rangle_1\ |\ 0\rangle_2\ |\ 0\rangle_3\ |\ 0\rangle_4) \\ & + b(|\ 0\rangle_1\ |\ 0\rangle_2\ |\ 1\rangle_3\ |\ 0\rangle_4 - |\ 0\rangle_1\ |\ 1\rangle_2\ |\ 0\rangle_3\ |\ 0\rangle_4)] + |\ e\rangle_3\ |\ g\rangle_4 [a(|\ 0\rangle_1\ |\ 0\rangle_2\ |\ 0\rangle_3\ |\ 1\rangle_4 \\ & + |\ 1\rangle_1\ |\ 0\rangle_2\ |\ 0\rangle_3\ |\ 0\rangle_4) + b(|\ 0\rangle_1\ |\ 0\rangle_2\ |\ 0\rangle_3\ |\ 1\rangle_4 + |\ 0\rangle_1\ |\ 1\rangle_2\ |\ 0\rangle_3\ |\ 0\rangle_4)]\}. \end{aligned} \tag{21}$$

Then we wait for another time $\triangle t$. Assume that a second photon is detected by D_3 at the time $t_1 + \triangle t$. The

whole system collapses into the state

$$\mid \Psi\left(t_1+\triangle t\right)\rangle_{A,B}=\frac{1}{\sqrt{2}}\mathrm{e}^{-2\kappa t_1-\kappa\triangle t}\mid g\rangle_1\mid g\rangle_2(a\mid e\rangle_3\mid g\rangle_4+b\mid g\rangle_3\mid e\rangle_4)\mid 0\rangle_1\mid 0\rangle_2\mid 0\rangle_3\mid 0\rangle_4. \tag{22}$$

Here, we note that $\frac{1}{\sqrt{2}}\mathrm{e}^{-2\kappa t_1-\kappa\triangle t}$ is a common factor. Thus the fidelity of state $\mid \Psi\left(t_1+\triangle t\right)\rangle_{A,B}$ is actually independent of the cavity decay rate κ, the detection time t_1 and $t_1+\triangle t$. Now the state of atoms 3 and 4 possessed by Bob is

$$\mid \Psi\left(t_1+\triangle t\right)\rangle_{B-\mathrm{atom}}=a\mid e\rangle_3\mid g\rangle_4+b\mid g\rangle_3\mid e\rangle_4, \tag{23}$$

which is the exact state that Alice wants to teleport. Reversely, if D_3 clicks first and D_1 clicks at a later time, the atoms possessed by Bob also collapse into the state of Eq.(23). Moreover, if D_1 and D_3 click at the same time, the atoms possessed by Bob still collapse into the state of Eq.(23). Thus, only if D_1 and D_3 click, we can conclude that the present teleporting protocol succeeds. Similarly, we can find that: (1) if D_2 and D_4 click, the atoms possessed by Bob collapse into the state $a\mid e\rangle_3\mid g\rangle_4+b\mid g\rangle_3\mid e\rangle_4$; (2) if D_1 and D_2 click, or D_3 and D_4 click, the atoms possessed by Bob collapse into the state $a\mid g\rangle_3\mid e\rangle_4+b\mid e\rangle_3\mid g\rangle_4$; (3) if D_1 and D_4 click, or D_2 and D_3 click, the state of the atoms possessed by Bob collapses into 0. If one wants to transform the resultant state of Bob's atom in the case of (2) into the case of (1), he (she) can apply two classical fields to perform the following transitions:

$$\begin{aligned}&\mid g\rangle_3\longrightarrow\mid e\rangle_3,\\&\mid e\rangle_3\longrightarrow\mid g\rangle_3;\end{aligned} \tag{24}$$

and

$$\begin{aligned}&\mid g\rangle_4\longrightarrow\mid e\rangle_4,\\&\mid e\rangle_4\longrightarrow\mid g\rangle_4,\end{aligned} \tag{25}$$

in cavities 3 and 4 respectively. Suppose that we wish to wait for a time $t_{\mathrm{w},}$ The probability of success during the detection stage is

$$P_{\mathrm{dect}}=\frac{1}{2}\left(1-\mathrm{e}^{-2\kappa t_w}\right)^2, \tag{26}$$

Thus the total probability of success is

$$\begin{aligned}P_{\mathrm{suc}}&=P_{\mathrm{suc(prep)}}\times P_{\mathrm{dect}}\\&=\frac{1}{2}\mathrm{e}^{-2\kappa\tau}\left[\frac{g^2}{\Omega_\kappa^2}\sin^2\left(\Omega_k\tau\right)\right]^2\left(1-\mathrm{e}^{-2\kappa t_w}\right)^2,\end{aligned} \tag{27}$$

If t_{w} is sufficiently long and thus satisfies $\mathrm{e}^{-2\kappa t_{\mathrm{w}}}\ll 1$, the total probability of success is about $\frac{1}{2}\mathrm{e}^{-2\kappa\tau}\left[\frac{g^2}{\Omega_\kappa^2}\sin^2\left(\Omega_k\tau\right)\right]^2$.

Finally, we give a brief discussion of the fidelity and success probability of the present scheme. If two photons are detected in the expected detectors as in the cases of (1) and (2), we can conclude that the present teleporting protocol succeeds, and the fidelity is independent of the cavity decay rate and the detection time. But if the detector is imperfect, there is a probability that only one photon or no photon is detected, while actually two photons have leaked from the cavities. Moreover, considering the effects of the atomic spontaneous emission, we cannot detect the photons decaying from atomic spontaneous emissions by the photon-detectors, as they run with random directions. Hence, both the atomic spontaneous emission and imperfection of the photon-detectors reduce the probability of success. But it should be noted that they have no effects on the fidelity of the scheme.

In summary, a scheme for teleporting a two-atom entangled state via cavity decay has been proposed. It utilizes quantum interference of the photons leaking from four symmetric cavities to establish the correlation between the distant sender and receiver, which destroys the which-path information. We have shown that the fidelity of the present scheme is independent of the cavity decay rate, atomic decay and detection efficiency.

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