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Re-evaluation of the ^{23}Mg(p,γ) ^{24}Al reaction rate*

Liu Hong-Lin(刘宏林), Liu Men-Quan(刘门全), Lai Xiang-Jun (赖祥军), and Luo Zhi-Quan(罗志全)†

Institute of Theoretical Physics, China West Normal University, Nanchong, Sichuan 637002, *China*

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Based on a new screening Coulomb model, this paper discusses the effect of electron screening on proton capture reaction of ^{23}Mg. The derived result shows that, in some considerable range of stellar temperatures, the effect of electron screening on resonant reaction is prominent; on the non-resonant reaction the effect is obvious only in the low stellar temperatures. The reaction rates of ^{23}Mg(p,γ)^{24}Al would increase 15%–25% due to the fact that the electron screening are considered in typical temperature range of massive mass white dwarfs, and the results undoubtedly affect the nucleosynthesis of some heavier nuclei in massive mass white dwarfs.

Keywords: electron screening, thermonuclear reaction, nucleosynthesis, stellar evolution
PACC: 2340, 9760, 9530C

1. Introduction

In the stellar evolution stage of explosive burning of hydrogen, for nuclear mass number $A > 20$, there is a large number of proton capture reactions and β^+-decays (rp-process).[1] The rp-process might be one of the most important mechanisms for energy production and nucleosynthesis in a variety of astrophysical sites with different temperature and density conditions. At stellar temperature below $T_9 = 0.1$ (T_9 is the temperature unit 10^9 K), the β^+-decay of ^{23}Mg and the subsequent reaction ^{23}Na(p,α)^{20}Ne convert ^{23}Mg back into ^{20}Ne, which is the so-called Ne–Na cycle. The timescale of the proton capture reaction of ^{23}Mg is shorter than that of the β^+-decay for sufficient high stellar temperature. In this case some of the ^{23}Mg would escape from the Ne–Na cycle by proton capture. The ^{23}Mg which leak from the Ne–Na cycle into the Mg–Al cycle result in a great deal of synthesis of heavier nuclei. Therefore, a meticulous calculation of the reaction rate of ^{23}Mg(p,γ)^{24}Al in the stellar environment is very obligatory for synthesis of the heavier nuclei.

The reaction rate of ^{23}Mg(p,γ)^{24}Al has been studied by many scholars due to its significance for nucleosynthesis in astrophysical sites. The reaction rate for ^{23}Mg(p,γ)^{24}Al was firstly estimated by Wallace and Woosley by considering a single resonance.[2] Afterwards, the rate was modified by Wiescher *et al*,[3] who considered three resonances and in addition a contribution from the direct capture process. Subsequently, the rate was re-estimated by Kubono *et al*[4] based on four resonances. The recent estimate for the rate was reported by Herndl *et al*[5] based on the new experimental information on ^{24}Al excitation energies. In Ref.[4] and Ref.[5], those authors studied the level structure of ^{24}Al near the proton threshold by using the ^{24}Mg(^{3}He,T)^{24}Al charge-exchange reaction, based the reaction rate estimate on their experimentally determined ^{24}Al excitation energies and spin-parity restrictions. Thus far, the effect of electron of screened Coulomb on thermonuclear reaction rate in high-density plasma has not been considered. Qualitatively, the electron screening is able to make thermonuclear reaction rate or cross section increases in high-density plasma circumstances. Therefore, it is extremely important to consider the effect of electron screening on thermonuclear reaction rate for meticulous analysis the reaction rate for ^{23}Mg(p,γ)^{24}Al.

In the high-density plasma circumstances, the electron screening has been widely investigated on the basis of various screened Coulomb models.[6] Salpeter's ion-sphere model,[7] as well as Graboske–De Witt's model[8] and Mitler's model,[9] are widely used in weak screening. For example, those models are used to estimate uncertainties of the solar neutrino flux associated with the screening effect.[10] Also, the effect of electron screening on electron capture and prompt explosion energy for SNII had been discussed, and the result shows that it is extremely important

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†Corresponding Author. E-mail: zqluo@tom.com

in certain temperature-density conditions.[11−13] Recently a new screening model has been proposed by Liolios *et al* and is universally used.[14] In this paper the effect of electron screening on proton capture reaction rate of ^{23}Mg is discussed based on Liolios's screened Coulomb model, in which the four resonance and the contribution from the direct capture process will also be considered.

2. Nuclear reaction rates

The reaction rates of proton capture is the sum of contribution from the resonant reaction and non-resonant reaction (also called direct capture). The reaction rates of resonance is determined by narrow resonance in the phases of explosive stellar burning of hydrogen,[15] and its corresponding resonant reaction rate is given by

$$N_{\rm A}\langle\sigma v\rangle_r = 1.54\times10^{11}(\mu T_9)^{-2/3}\sum_i \omega\gamma_i \times\exp(-11.605E_{r_i}/T_9)\ {\rm cm^3\cdot mol^{-1}\cdot s^{-1}}, \quad (1)$$

where $N_{\rm A}$ is Avogadro's constant. μ is the reduced mass of the two collision partners in a.m.u., E_{r_i} is resonance energy and $\omega\gamma_i$ is the strength of the resonance in MeV. The values of the E_{r_i} and corresponding $\omega\gamma_i$ are listed in Table 1 from Refs.[3,5], and the average values of them are adopted in order to reduce the uncertainty of the reaction rates in this paper. The non-resonant cross section varies smoothly with proton bombarding energy E and is usually converted into the astrophysical $S(E)$-factor, defined by[16]

$$S(E) = \sigma(E)E\exp(2\pi\eta), \quad (2)$$

where $\eta(E)$ is the Sommerfeld parameter, $\eta(E) = Z_1Z_2e^2\sqrt{\mu/2\hbar^2}E^{-1/2}$. The non-resonant reaction rates are obtained by

$$N_{\rm A}\langle\sigma v\rangle_{nr} = 4.339\times10^8\tau^2\frac{1}{\mu Z}e^{-\tau}S_{\rm eff}\ {\rm cm^3\cdot mol^{-1}\cdot s^{-1}}, \quad (3)$$

where

$$S_{\rm eff} = S(0)\Big[1+\frac{5}{12\tau}+\frac{S'(0)}{S(0)}\Big(E_0+\frac{35}{36}kT\Big) + \frac{1}{2}\frac{S''(0)}{S(0)}\Big(E_0^2+\frac{89}{36}E_0kT\Big)\Big]\ {\rm MeV\cdot b},$$

$$E_0 = 0.122(Z_1Z_2\mu T_9^2)^{1/3}, \quad \tau = 4.428\Big(\frac{Z_1Z_2\mu}{T_9}\Big)^{1/3}. \quad (4)$$

Usually the measure of the value $S(0)$ was extremely difficult, and the direct capture reaction not determines the reaction rates at the high stellar temperatures. According to Ref.[5], for bombarding energies below 1 MeV, the $S(E)$-factor can be expressed as

$$S(E) = 22.5 - 1.1\times10^{-2}E + 6.9\times10^{-6}E^2\ {\rm keV\cdot b}. \quad (5)$$

In terms of the Hauser–Feshbach statistical model (SM),[17] the thermonuclear reaction rate of ^{23}Mg(p,γ)^{24}Al can be described by

$$N_{\rm A}\langle\sigma v\rangle_{SM} = \exp(a_0 + a_1T_9^{-1} + a_2T_9^{-1/3} + a_3T_9^{1/3} + a_4T_9 + a_5T_9^{5/3} + a_6\ln T_9)\ {\rm cm^3\cdot mol^{-1}\cdot s^{-1}}. \quad (6)$$

The parameters a_i, $i = 0, 1, 3, \ldots, 6$ are listed in Table 2.[17]

Table 1. Resonance parameters for the reaction ^{23}Mg(p,γ)^{24}Al.

E_{r_i}/meV	$\omega\gamma_i$/meV[5]	$\omega\gamma_i$/meV[3]
0.478	25	27
0.663	58	130
0.939	52	11
1.029	12	16

Table 2. The parameters for thermonuclear reaction rates of ^{23}Mg(p,γ)^{24}Al.

a_0	a_1	a_2	a_3	a_4	a_5	a_6
5.9288×10	-2.2420	5.4295×10	-1.1449×10^2	7.2303	-4.1795×10^{-1}	5.0744×10

3. The effect of electron screening

The resonant energies declined relatively due to the effect of electron screening, namely $E'_{r_i} < E_{r_i}$, E'_{r_i} are the corrected resonance energies. Thus Eq.(1) can be rewritten as

$$N_{\rm A}\langle\sigma v\rangle'_r = 1.54\times10^{11}(\mu T_9)^{-2/3}\sum_i \omega\gamma_i \times\exp(-11.605E'_{r_i}/T_9)\ {\rm cm^3\cdot mol^{-1}\cdot s^{-1}}. \quad (7)$$

Generally speaking, the values of E'_{r_i} should be measured by experiment, but it is too hard to provide sufficient data. In general and approximate analyses, take $E'_{r_i} = E_{r_i} - U_0$.[18] In the recent Liolios screened Coulomb model,[13]

$$U_0 = \frac{15}{8}\frac{Z_1Z_2e^2}{a} \quad \text{and} \quad a = \Big(\frac{15}{8Z_1^2\pi}\Big)^{1/3}a_0\,, \quad (8)$$

where a_0 is the Bohr radius. We introduce a screening enhancement factor (SEF) for considering the screening effect. The SEF for the resonant reaction can be expressed as

$$f_r = \exp(11.605U_0/T_9). \quad (9)$$

Obviously, the resonant SEF is determined by stellar temperature as well as screening potentials altogether. Considering the screening effect, the contribution from non-resonant would be modified as

$$N_{\rm A}\langle\sigma v\rangle_{nr'} = 4.339\times10^8\tau^2 f_{nr} \times \frac{1}{\mu Z}e^{-\tau}S_{\rm eff}\ {\rm cm}^3\cdot{\rm mol}^{-1}\cdot{\rm s}^{-1}, \quad (10)$$

where the non-resonant SEF f_{nr} has been given in Ref.[10], namely

$$f_{nr}(E) = \exp(\pi\eta(E)U_eE^{-1}), \quad (11)$$

where U_e is the screening energy, and

$$U_e \approx \frac{15}{8}\frac{Z_1Z_2e^2}{a}. \quad (12)$$

The relative bombarding energies are equal to corresponding Gamow peak value, and the f_{nr} can be rewritten as

$$f_{nr} = \exp((0.122)^{-3/2}2/\sqrt{2}(Z_1Z_2)^{1/2}\pi e^2U_eT_9^{-1}). \quad (13)$$

By the above discussion, the total reaction rates of ^{23}Mg(p,γ)^{24}Al, due to the screening effect, should be corrected as

$$N_{\rm A}\langle\sigma v\rangle = f_r\cdot N_{\rm A}\langle\sigma v\rangle_r + f_{nr}\cdot N_{\rm A}\langle\sigma v\rangle_{nr}. \quad (14)$$

4. Numerical analysis and discussion

In Fig.1 SEF as a function of the stellar temperature for the reaction ^{23}Mg(p,γ)^{24}Al is presented. It is apparent that the SEF declines exponentially with the increasing temperature. The reaction rate increases at least 5% at the low stellar temperatures $T_9 < 1$. One can see from Fig.1 that, at the stellar temperatures $T_9 = 0.1 - 1.0$ range, the effect of electron screening on resonant contribution is prominent, and on non-resonant contribution is obvious only in the low stellar temperatures. Therefore, it is extremely important to consider the influences of electron screening in certain temperature-density conditions, which is consistent with previous results given by Refs.[10–13] and references therein.

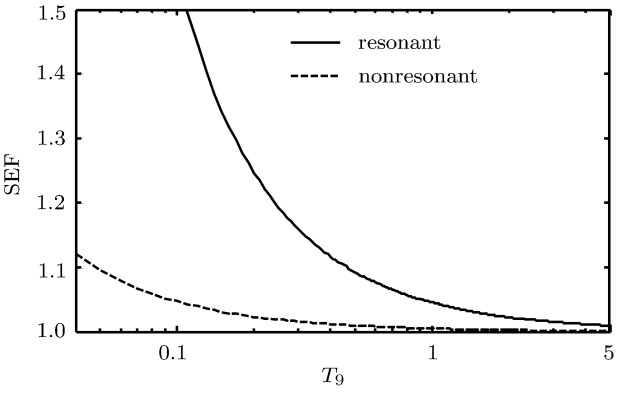


Fig.1. The SEF as a function of temperature (T_9).

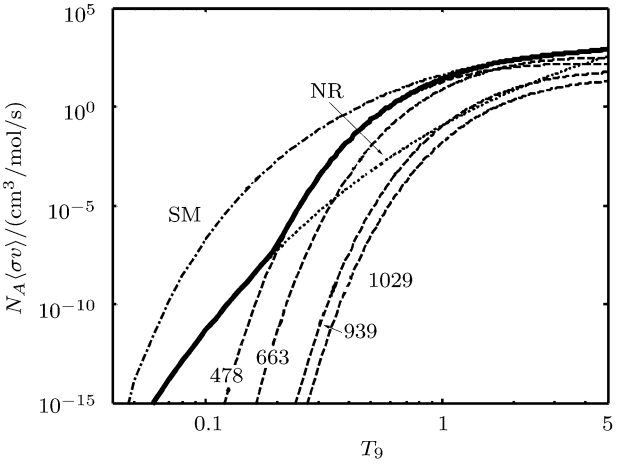


Fig.2. The reaction rates as a function temperature (T_9) when considered the screening effect and the total reaction rates are compared with the results derived by using statistical model (SM). (478, 663, 939 and 1029 are expressed the contribution from resonance energies 478 keV, 663 keV, 939 keV and 1029 keV, respectively. NR: the contribution from non-resonant).

Figure 2 shows the total screening reaction rates of ^{23}Mg(p,γ)^{24}Al including the contributions from resonant and non-resonant reaction, and the total rates are compared with the results derived from the statistical model. The critical temperature at which direct capture process determines the reaction rates is $T_9 = 0.2$. The resonant contribution to the total reaction rate is obvious only at the stellar temperatures above $T_9 = 0.10$. The reaction rate is sensitive to the stellar temperatures, and increases quickly with the rising of the stellar temperatures. The $E_r = 478$ keV resonance dominates the reaction rates at the stellar temperatures $T_9 = 0.2 - 1.5$

range, and the $E_r = 663$ keV resonance is important at high temperatures above $T_9 = 1.5$, while the $E_r = 939$ keV as well as $E_r = 1029$ keV resonance are negligible compared to the former two lower resonance over the whole temperature range. The Fig.2 also shows what the lower resonant energy and the stronger screening effect is. One can come to the conclusion that the effect of electron screening on proton capture reaction rates is prominent in the temperature range of explosive stellar burning of hydrogen. In O–Ne–Mg (massive mass) white dwarfs, the typical peak temperatures are about $T_9 = 0.2 - 0.3$,[19] our work shows that the non-resonant and the corresponding resonant reaction rates would increase around 1.5%–2.3% and 15%–25% respectively compared with the previous results that without the effect of the electron screening. In one word, considering the effect of the screening for analyses the thermonuclear reaction rate of ^{23}Mg(p,γ)^{24}Al in O–Ne–Mg white dwarfs is extremely important. The increase of the reaction rates for ^{23}Mg(p,γ)^{24}Al means that more ^{23}Mg would escape the Ne–Na cycle,[20] and the subsequent reaction ^{24}Al($\beta^+\nu$) convert more ^{24}Al into Mg-Al-cycle and lead to a larger number of ^{26}Al to be synthesized, which may help us to partly solve the problem of ^{26}Al overabundance in the interstellar space.

In addition, at the stellar temperatures below $T_9 = 0.4$, the present results have large difference(so far as to several orders of magnitude) from the result derived by using statistical model, especially around the temperatures $T_9 = 0.1-0.3$. The statistical model, however, is reasonable for the low temperatures. The differences may be resolved by considering contribution from more low energy resonance reactions. Therefore, further experimental work is clearly necessary.

References

[1] Sarriguren P, Álvarez-Rodriguez R and Moya de Guerra E 2005 *EPJA* **24** 193

[2] Wallace R K and Woosley S E 1981 *ApJS* **45** 389

[3] Wiescher M, Gorres J, Thielemann F K and Ritter H 1986 *Astron. Astrophys* **160** 56

[4] Kubono S, Kajino T and Kato S 1995 *Nucl. Phys.* A **588** 521

[5] Herndl H, Fantini M, Iliadis C, Endt P M and Oberhummer H 1998 *Phys. Rev.* C **58** 1798

[6] Bahcall J N, Brown L, Gruzinov A and Sawer R 2002 *Astron. Astrophys.* **383** 291

[7] Salpeter E E 1954 *Aust. J. Phys.* **7** 373

[8] Graboske H C and De Witt H E 1973 *ApJ* **181** 457

[9] Mitler H E 1977 *ApJ* **212** 513

[10] Liolios T E 2001 *Phys. Rev.* C **64** 018801

[11] Liu M Q, Zhang J and Luo Z Q 2006 *Acta Phys. Sin.* **55** 3197 (in Chinese)

[12] Luo Z Q, Liu M Q, Lin L B and Peng Q H 2006 *ChA & A* **30**(6) 19

[13] Luo Z Q, Liu M Q, Lin L B and Peng Q H 2005 *Chin. Phys.* **14**(6) 1272

[14] Liolios T E 2000 *EPJA* **9** 287

[15] Schatz H, Aprahamian A, Goerres J, Wiescher M, Rauscher T. Rembges J F, Thielemann F K, Pfeiffe B, Moeller P and Kratz K L 1998 *Phys. Rep.* **294** 167

[16] Iliadis C, D'Auria J M, Starrfield S, Thompson W J and Wiescher M 2001 *ApJS* **134** 151

[17] Rauscher T and Thielemann F K 2000 *At. Data. Nucl. Data. Tables* **75** 1

[18] Itoh N, Tomizawa N, Wanajo S and Nozawa S 2003 *ApJ* **586** 1436

[19] Rembges J F, Freiburghaus C, Thielemann F K, Schatz H and Wiescher M 1997 *Nucl. Phys.* A **621** 413

[20] Peng Q H 1995 *Progress in Physics* **15**(1) 79 (in Chinese)