Vol 16 No 6, June 2007
1009-1963/2007/16(06)/1615-04
Chinese Physics
© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Multipartite entanglement in the interaction system between a single-mode microwave cavity field and superconducting charge qubits*

Shi Zhen-Gang(施振刚)†, Chen Xiong-Wen(谌雄文), Zhu Xi-Xiang(朱喜香), and Song Ke-Hui(宋克慧)

Department of Physics and Electronic Information Science, Huaihua University, Huaihua 418008, *China*

Research Institute of Information Science, Huaihua University, Huaihua, 418008, China

(Received 18 June 2006; revised manuscript received 21 October 2006)

This paper proposes a method of generating multipartite entanglement through using d.c. superconducting quantum interference devices (SQUID) inside a standing wave cavity. In this scheme, the d.c. SQUID works in the charge region. It is shown that, a large number of important multipartite entangled states can be generated by a controllable interaction between a cavity field and qubits. It is even possible to produce entangled states involving different cavity modes based on the measurement of charge qubits states. After such superpositions states are created, the interaction can be switched off by the classical magnetic field through the SQUID, and there is no information transfer between the cavity field and the charge qubits.

Keywords: superconducting charge qubits, standing wave cavity, Schrödinger cat states, superpositions of mesoscopic quantum states

PACC: 0637, 4250

Two or more quantum systems are entangled when it is impossible to describe their physical properties by means of a direct product of their respective density operators. Entanglement is a natural consequence of linearity of Hilbert spaces, however, it is difficult to create and observe superposed states because the fragile coherence of these states can be easily spoiled by the environment. In recent years, great effort has been put into preparation of the Schrödinger cat states (SCSs) in the laboratory,[1,2] where the extreme cat states have been reduced to superpositions of mesoscopic quantum states (SMSs) with classical counterparts, the so-called coherent states. Decoherence processes, their rate being increased with the size of the system, have been claimed to be the mechanism inhibiting the manifestation of entanglement in macroscopic objects. In this sense, realizing bigger SCSs in the laboratory will let us test decoherence by monitoring their decay.[1,3] Meanwhile, multipartite entangled systems are also considered since they are interesting in connection with quantum information.[4,5]

Superconducting quantum devices[6−10] allow to perform quantum state engineering including the demonstration of SCSs and SMSs. In Ref. [11], a scheme was proposed to generate SCSs and squeezed states for a superconducting quantum interference device (SQUID) ring modelled as an oscillator. Since then, several proposals have been made which focus on superconducting qubits interacting with the nonclassical electromagnetic field.[12−18]

In this contribution, we aim at generating SCSs in the interaction system between a single-mode microwave cavity field and superconducting charge qubits. In contrast to other systems, for example Ref.[19], the interaction between the box and the resonator is not switchable. In our proposal, the interaction between the cavity field and the SQUID can be switched off by a classical magnetic field after the SCSs are generated.

We consider a single d.c. SQUID device. It contains a small superconducting island and two Josephson junctions with the same capacitance $C_{\rm J}$ and Josephson energy $E_{\rm J}$. A controllable gate voltage $V_{\rm g}$ is coupled to the island via the gate capacitor $C_{\rm g}$ with dimensionless gate charge $n_{\rm g} = C_{\rm g}V_{\rm g}/2e$. The

*Project supported by Hunan Provincial Natural Science Foundation of China (Grant No 06jj50014).
†E-mail: shi_zhengang@yahoo.com

qubit is assumed to work in the charge region with $k_{\rm B}T \ll E_{\rm J} \ll E_{\rm C} \ll \Delta$, where $k_{\rm B}, T, E_{\rm C}$, and Δ are the Boltzmann constant, temperature, charge and superconducting gap energies, respectively. When the gate voltage range near a degeneracy point $n_{\rm g} = 1/2$, only two charge states ($|g_k\rangle$ and $|e_k\rangle$) play a leading role. The other charge states with a much higher energy can be neglected, which implies that the d.c. SQUID can be reduced to a two-level system or charge qubit.[20] This superconducting charge qubit can be represented by a spin-$\frac{1}{2}$ notation such that the charge states ($|g_k\rangle$ and $|e_k\rangle$) correspond to eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$ of the spin operator σ_z, respectively. If such a qubit is placed into a single-mode superconducting cavity, the Hamiltonian can be written as [14,21]

$$H_k = \hbar\omega a^+ a + E_z\sigma_z^{(k)} - E_{\rm J}\sigma_x^{(k)} \times \cos\Big[\frac{\pi}{\Phi_0}(\Phi_{\rm C}I + \Phi_{\rm Q}^{(k)})\Big], \quad (1)$$

where ω is frequency of the single-mode cavity, and $E_z = -2E_{\rm ch}(1-2n_{\rm g})$ with the single-electron charging energy $E_{\rm ch} = e^2/(C_{\rm g}+2C_{\rm J})$. Here Φ_0 is the flux quantum, $\Phi_{\rm C}$ and $\Phi_{\rm Q}^{(k)}$ are the magnetic fluxes of the classical and quantized components of the magnetic field threading the SQUID ring, respectively. I is the identity operator. The Pauli matrices $\big(\sigma_x^{(k)}, \sigma_y^{(k)}, \sigma_z^{(k)}\big)$ are denoted by spin-$\frac{1}{2}$ notation as $\sigma_x^{(k)} = |e_k\rangle\langle g_k| + |g_k\rangle\langle e_k|$, $\sigma_y^{(k)} = -{\rm i}|e_k\rangle\langle g_k| + {\rm i}|g_k\rangle\langle e_k|$, $\sigma_z^{(k)} = |e_k\rangle\langle e_k| - |g_k\rangle\langle g_k|$, and $\sigma_\pm^{(k)} = (\sigma_x^{(k)} \pm {\rm i}\sigma_y^{(k)})/2$.

Provided that the single-mode cavity field is a standing-wave form. Furthermore, we assume that the polarization of the magnetic field is along the y direction, which is perpendicular to the surface of the SQUID ring. The standing-wave can be denoted as[21]

$$B_y = -{\rm i}\sqrt{\frac{\hbar\omega}{\varepsilon_0 V c^2}}(a - a^+)\cos(kz), \quad (2)$$

where ε_0, V, c and k are the dielectric constant of the vacuum, cavity volume, light speed, and wave vector of the cavity-mode, respectively. The standing wave can be formed from pure back reflection because the superposition of a travelling wave and its reflected wave. The travelling wave and its reflected wave propagate along the x-z plane and is parallel to the surface of the SQUID ring. Then the $\Phi_{\rm Q}^{(k)}$ can be represented as

$$\Phi_{\rm Q}^{(k)} = -{\rm i}\sqrt{\frac{\hbar\omega}{\varepsilon_0 V c^2}}(a - a^+)\int \cos(kz)\,{\rm d}S = \eta_k a + \eta_k^* a^+, \quad (3)$$

with parameter

$$\eta_k = -{\rm i}S\sqrt{\frac{\hbar\omega}{\varepsilon_0 V c^2}}\cos(kz),$$

it denotes the strength of the quantum flux inside the cavity.[21]

We now place N identical d.c. SQUID inside the cavity. Assume that each qubit resides in a standing-wave node, and select $z = 0$ in a standing-wave crest. Then $kz = \pm(2{\rm i}+1)\pi, (i = 0, 1, 2, \cdots)$, $\cos(kz) = -1$. So, we have

$$\eta_k = {\rm i}S\sqrt{\frac{\hbar\omega}{\varepsilon_0 V c^2}}.$$

It manifests that the couplings of the N qubits with cavity are identical. In this case, the Hamiltonian of the whole system is

$$H = \hbar\omega a^+ a + E_z\sum_{k=1}^{N}\sigma_z^{(k)} - E_{\rm J}\sum_{k=1}^{N}\sigma_x^{(k)} \times \cos\Big[\frac{\pi}{\Phi_0}(\Phi_{\rm C}I + \eta_k a + \eta_k^* a^+)\Big]. \quad (4)$$

We further assume that

$$\frac{\pi|\eta_k|}{\Phi_0}\sqrt{\bar{n}+1} \ll 1,$$

where $\bar{n}$ is the mean photon number. In this case, all higher orders of $\pi|\eta_k|/\Phi_0$ can be neglected in the Eq.(4). The Hamiltonian (4) reduces to

$$H = \hbar\omega a^+ a + E_z\sum_{k=1}^{N}\sigma_z^{(k)} - E_{\rm J}\sum_{k=1}^{N}\sigma_x^{(k)} \times [\cos\theta - \sin\theta(\xi_k a + \xi_k^* a^+)], \quad (5)$$

where $\theta = \pi(\Phi_{\rm C}/\Phi_0)$, and $\xi_k = \xi = \pi(\eta_k/\Phi_0)$. In the following, we introduce collective spin operator $J_\alpha = \sum_{k=1}^{N}\sigma_\alpha^{(k)}$, $\alpha = x, y, z$, and further assume that $n_{\rm g} = 1/2$, then $E_z = 0$. In this case, the Hamiltonian (5) reduces to

$$H = \hbar\omega a^+ a - E_{\rm J}[\cos\theta - \sin\theta(\xi_k a + \xi_k^* a^+)]J_x. \quad (6)$$

The exact time evolution operator of the Hamiltonian (6) is

$$U(t) = \exp\Big[{\rm i}J_x^2\Big(\frac{E_{\rm J}^2\sin^2\theta|\xi|^2}{\hbar^2\omega}t - \frac{E_{\rm J}^2\sin^2\theta|\xi|^2}{\hbar^2\omega^2}\sin\omega t\Big)\Big]\exp(-{\rm i}\omega a^+ a t) \times \exp\Big[-\frac{E_{\rm J}\sin\theta}{\hbar\omega}J_x(\xi^* a^+({\rm e}^{{\rm i}\omega t}-1) - \xi a({\rm e}^{-{\rm i}\omega t}-1))\Big]\exp\Big({\rm i}J_x\frac{E_{\rm J}\cos\theta}{\hbar}t\Big). \quad (7)$$

Some examples of the possible applications of the interaction described in Eq.(6) are given. If at

$t = 0$, $N = 1$, the 1-qubit-field state is $|g\rangle|0\rangle = (|+\rangle + |-\rangle)|0\rangle/2$, where $|\pm_k\rangle = (|g_k\rangle \pm |e_k\rangle)/\sqrt{2}$ are eigenstates of $\sigma_x^{(k)} = \sigma_+^{(k)} + \sigma_-^{(k)}$ with eigenvalues ± 1, respectively. The evolved state after a time t will be

$$|\psi(t)\rangle = \frac{1}{2}[|g\rangle(\mathrm{e}^{\mathrm{i}\Omega t}|\alpha\rangle + \mathrm{e}^{-\mathrm{i}\Omega t}|-\alpha\rangle)] + \frac{1}{2}[|e\rangle(\mathrm{e}^{\mathrm{i}\Omega t}|\alpha\rangle - \mathrm{e}^{-\mathrm{i}\Omega t}|-\alpha\rangle)], \quad (8)$$

where

$$\Omega = \frac{E_\mathrm{J}\cos\theta}{\hbar},$$

and a global phase factor $\exp(\mathrm{i}\varphi)$ has been neglected, with

$$\varphi = \Big(\frac{E_\mathrm{J}^2\sin^2\theta|\xi|^2}{\hbar^2\omega}t - \frac{E_\mathrm{J}^2\sin^2\theta|\xi|^2}{\hbar^2\omega^2}\sin\omega t\Big).$$

$|\pm\alpha\rangle$ denotes coherent state

$$|\pm\alpha\rangle = \mathrm{e}^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{(\pm\alpha)^n}{\sqrt{n!}}|n\rangle, \quad (9)$$

where

$$|\alpha\rangle = \frac{\xi^* E_\mathrm{J}\sin\theta}{\hbar\omega}(\mathrm{e}^{-\mathrm{i}\omega t} - 1). \quad (10)$$

After the time interval t, we impose $\Phi_\mathrm{C} = 0$ by adjusting the classical magnetic field, thus the interaction between the charge qubit and the cavity field is switched off (e.g., the last term in Eq.(6) vanishes). Eq.(8) shows that entanglement of the qubit and the microwave cavity field can be prepared for an evolution time $t \neq 2m\pi$ with the integer number m, then the SCSs can be created. And measurement of the qubit state will produce the so-called even or odd coherent states (one of the SMSs) in the cavity field denoted by

$$|\mathrm{SMS}\rangle_\pm = \mathrm{N}_\pm(\mathrm{e}^{\mathrm{i}\Omega t}|\alpha\rangle \pm \mathrm{e}^{-\mathrm{i}\Omega t}|-\alpha\rangle), \quad (11)$$

$+$ or $-$ depending on whether $|g\rangle$ or $|e\rangle$ was found.

If at $t = 0$, $N = 2$, the 2-qubit-field state is $|g_1g_2\rangle \otimes |0\rangle$, the evolved state after a time t will be

$$|\psi(t)\rangle = \frac{1}{2}[\mathrm{e}^{\mathrm{i}(4\varphi+2\Omega t)}|\phi_1\rangle|2\alpha\rangle + \mathrm{e}^{\mathrm{i}(4\varphi-2\Omega t)}|\phi_2\rangle|-2\alpha\rangle + (|\phi_3\rangle + |\phi_4\rangle)|0\rangle], \quad (12)$$

where $|\phi_i\rangle$ are the eigenstates of the qubit operator $\sum_{k=1}^{2}(\sigma_+^{(k)} + \sigma_-^{(k)})$ with eigenvalues $\gamma_{1,2} = \pm 2$ and $\gamma_{3,4} = 0$. The state of Eq.(12) is a bigger and more elaborate microscopic-mesoscopic 2-qubit-field entangled state. After a time interval t, we can switch off the interactions between the charge qubits and the cavity field by setting $\Phi_\mathrm{C} = 0$ and $n_\mathrm{g} = 1/2$. Measuring the two qubits in the state $|g_1g_2\rangle$ will produce the field state

$$N'[\mathrm{e}^{\mathrm{i}(4\varphi+2\Omega t)}|2\alpha\rangle + \mathrm{e}^{\mathrm{i}(4\varphi-2\Omega t)}|-2\alpha\rangle + 2|0\rangle], \quad (13)$$

which is a triple mesoscopic field superposition state with not only an 'alive' or 'dead' cat, but also and primarily an 'absent' one.

The interaction of Eq.(5) lets us create more sophisticated and bigger microscopic-mesoscopic N-qubit-field entangled states and field superposition states, which will not be described here. Some of these new states,[22] in fact, present large quantum inference regions when displayed in phase-space, showing distinctly their quantum nature.

It is well worth mentioning another application to implement a quantum controlled phase gate in our system as discussed in Ref.[23].

Below, we briefly discuss the experimental feasibility of the current scheme. For the used charge qubit in our scheme, the relaxation time T_1 of the charge qubit is longer than its dephasing time T_2, the switch time t_q during a single-bit operation can be enhanced through choosing the shorter microwave wavelengths and smaller volume cavity. In order to obtain SMSs, the readout time τ_m of the charge qubit should be less than the dephasing time T_2 of the charge qubit and the lifetime time t_d of the cavity field. For example, in Ref.[19] with a set of given parameters, the estimated time, $\tau_\mathrm{m} = 4$ ns, is less than $T_2 = 5$ ns.[7] For a good cavity, the quality factor Q can reach very high values, such as $Q = 3 \times 10^8$, and then the lifetimes of the microwave field would be in the range 0.001 s $\leq 2\pi t_\mathrm{d} \leq 0.15$ s, which implies $t_\mathrm{m} \ll t_\mathrm{d}$. So the readout is possible within current technology.

In conclusion, we have analysed the generation of SCSs by using superconducting charge qubits coupling to a cavity with a standing wave. Based on our scheme, the SMSs can be created by using one quantum operation together with the quantum measurements on the charge qubits. After the SCSs or SMSs are created, the coupling between the charge qubits and the cavity field can be switched off, in principle. Therefore, our proposal should be experimentally accessible in the near future.

References

[1] Brune M, Hagley E, Dreyer J, Maître X, Maali A, Wunderlich C, Raimond J M and Haroche S 1996 *Phys. Rev. Lett.* **77** 4887

[2] Monroe C, Meekhof D M, King B E and Wineland D J 1996 *Science* **272** 1131

[3] Myatt C J, King B E, Turchette Q A, Sackett C A, Kielpinske D, Itano W M, Monroe C and Wineland D J 2000 *Nature* **403** 269

[4] Sackett C A, Kielpinski D, King B E, Langer C, Meyer V, Myatt C J, Rowe M, Turchette Q A, Itano W M, Wineland D J and Monroe C 2000 *Nature* **404** 256

[5] Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565

[6] Mooij J E, Orlando T P, Levitov L, Tian L, van der Wal C H and Lloyd S 1999 *Science* **285** 1036;
Yu Y, Han S, Chu X, Chu S I and Wang Z 2002 *Science* **296** 889

[7] Nakamura Y, Pashkin Y A and Tsai J S 1999 *Nature* **398** 786

[8] Vion D, Aassime A, Cottet A, Joyez P, Pothier H, Urbina C, Esteve D and Devoret M H 2002 *Science* **296** 886

[9] You J Q, Tsai J S and Nori F 2002 *Phys. Rev. Lett.* **89** 197902

[10] Wei L F, Liu Yu-Xi and Nori F 2004 *Europhys. Lett.* **67** 1004

[11] Everitt M J, Clark T D, Stiffell P B, Vourdas A, Ralph J F, Prance R J and Prance H 2004 *Phys. Rev.* A **69** 043804

[12] Al-Saidi W A and Stroud D 2002 *Phys. Rev.* B **65** 224512

[13] Zhu S L, Wang Z D and Yang K 2003 *Phys. Rev.* A **68** 034303

[14] You J Q and Nori F 2003 *Phys. Rev.* B **68** 064509
You J Q, Tsai J S and Nori F 2003 *Phys. Rev.* **68** 024510

[15] Vourdas A 1994 *Phys. Rev.* B **49** 12040
Yang C P, Chu Shih-I and Han S 2003 *Phys. Rev.* A **67** 042311

[16] Blais A, Huang R S, Wallraff A, Girvin S M and Schoelkopf R J 2004 *Phys. Rev.* A **69** 062320

[17] Song K H, Zhou Z W and Guo G C 2005 *Phys. Rev.* A **71** 052310

[18] Song K H 2006 *Chin. Phys.* **15** 286

[19] Armour A D, Blencowe M P and Schwab K C 2002 *Phys. Rev. Lett.* **88** 148301

[20] Makhlin Y, Schön G and Shnirman A 2001 *Rev. Mod. Phys.* **73** 357

[21] Liu Yu-Xi, Wei L F and Nori F 2004 *Europhys. Lett.* **67** 941

[22] Solano E, de Matos Filho R L and Zagury N 2001 *Phys. Rev. Lett.* **87** 060402

[23] Zou X, Pahlke K and Mathis W 2002 *Phys. Rev.* A **66** 044307