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Synchronization between two different chaotic systems with nonlinear feedback control*

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This paper presents chaos synchronization between two different chaotic systems by using a nonlinear controller, in which the nonlinear functions of the system are used as a nonlinear feedback term. The feedback controller is designed on the basis of stability theory, and the area of feedback gain is determined. The artificial simulation results show that this control method is commendably effective and feasible.

Keywords: nonlinear feedback, the new system, the united system, chaos synchronization
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1. Introduction

Since Pecora and Carroll (PC)[1] introduced a method to synchronize two identical chaotic systems of different initial conditions, chaos synchronization has become a hotspot which attracts a great deal of attention in nonlinearity research. Many methods and techniques for synchronization have been developed, such as the PC method, variable coupling method, adaptive control method, variable feedback method, and so on.[2−15] Most methods are used to synchronize two identical chaotic systems. However, identical chaotic systems are rarely found in practice, such as in physics, autocontrol, information and secure communications. Therefore, it is more important to synchronize two different chaotic systems. Since different chaotic systems have different nonlinear functions, and the number of the equilibrium points, the phase map and the shapes are all different, synchronization of two different chaotic systems is difficult to realize, and it has received less attention.[16,17] The controller in Ref.[16] is constructed by using linear coupling of the state variables of the drive and response systems to realize synchronization. But the state variables involved need to be separated from others and coupled into a linear coupling term to add into the synchronized system. This is difficult to realize in practice. The controller in Ref.[17] is constructed by constructing the Lyapunov function or calculating Lyapunov exponent to realize synchronization, but the calculation is onerous. This paper presents chaos synchronization between two different chaotic systems by using a nonlinear controller, in which the nonlinear function of the system is used as a nonlinear feedback term. So it is unnecessary to separate state variables, and the Lyapunov function or Lyapunov exponent does not need to be constructed or calculated. Then the work of calculation is largely reduced in the chaos synchronization. So it provides a commendably effective and feasible synchronization method.

2. Designing of controller

Let us consider the following two systems:

$$\dot{x} = f(x, \mu), \tag{1}$$

$$\dot{y} = g(y, k), \tag{2}$$

where

$$x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^{n\times 1},$$
$$y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in R^{n\times 1}$$

are state variables of the drive and response systems, μ, k are parameters. The nonlinear controller $u(t)$ is added to the response system

$$\dot{y} = g(y, k) + u(t), \tag{3}$$

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where

$$u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^{\mathrm{T}} \in R^{n\times 1}.$$

Theorem For the response system（3）, the nonlinear controller is designed as

$$u(t) = -F(x, e) + W(t), \tag{4}$$

where x are the state variables, e are errors of the state variables of the two systems, and $W(t) = (W_1(t), W_2(t), \ldots, W_n(t))^{\mathrm{T}} \in R^{n\times 1}$ are the control functions. If the form of $W(t)$ is chosen appropriately, the two systems can be synchronized.

Proof The drive and the response systems are divided into linear terms (constants are included) $f_1(x, \mu)$, $g_1(y, k)$ and nonlinear terms $f_2(x, \mu)$, $g_2(y, k)$

$$\dot{x} = f(x, \mu) = f_1(x, \mu) + f_2(x, \mu), \tag{5}$$

$$\dot{y} = g(y, k) = g_1(y, k) + g_2(y, k). \tag{6}$$

Suppose $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^{\mathrm{T}} \in R^{n\times 1}$ are the errors of the state variables, i.e.

$$e(t) = y(t) - x(t). \tag{7}$$

According to the dynamic equations (1) and (3), we obtain

$$\dot{e}(t) = g_2(y, k) + g_1(y, k) - f_2(x, \mu) - f_1(x, \mu) + u(t). \tag{8}$$

The difference between the two linear terms (constants are included) $f_1(x, \mu)$, $g_1(y, k)$ can be written as

$$g_1(y, k) - f_1(x, \mu) = Ae(t) + f'(x), \tag{9}$$

where $A \in R^{n\times n}$ is the coefficient matrix of $e(t)$, and $f'(x)$ consists of residual terms. The difference between the two nonlinear terms $f_2(x, \mu)$, $g_2(y, k)$ is then written as

$$g_2(y, k) - f_2(x, \mu) = F(x, e) - f'(x). \tag{10}$$

Take

$$u(t) = -F(x, e) + W(t), \tag{11}$$

where $W(t) = (W_1(t), W_2(t), \ldots, W_n(t))^{\mathrm{T}} \in R^{n\times 1}$ are the control functions.

With the nonlinear controller $u(t)$, the error equation of the system variables can be written as

$$\dot{e}(t) = Ae(t) + W(t). \tag{12}$$

If error states $e(t)$ approaches to zero as time lapses, that is

$$\lim_{t\to\infty} e(t) = 0, \tag{13}$$

the synchronization of the two different chaotic systems is achieved.The error equation of the system variables becomes at last an arbitrary linear stable system

$$\dot{e}(t) = Le(t), \tag{14}$$

and the control function $W(t)$ should be

$$W(t) = (L - A)e(t). \tag{15}$$

Matrix $(L-A)$ is the gain matrix of the feedback control.

The eigenvalue equation of the matrix $L \in R^{n\times n}$ is

$$|\lambda I - L| = 0. \tag{16}$$

According to Hurwitz stability theory,[18] if the eigenvalue λ has a negative real part,the synchronization of the two different chaotic systems can be achieved, the area of the gain can be determined by this condition together with Eqs.(15) and (16).

3. Simulation

The new system considered as a drive system is described as follows:[19]

$$\begin{aligned} \dot{x}_1 &= -x_2x_3 + ax_1, \\ \dot{x}_2 &= x_1x_3 - bx_2, \\ \dot{x}_3 &= x_1x_2 - cx_3, \end{aligned} \tag{17}$$

the response system is the united system[20]

$$\begin{aligned} \dot{y}_1 &= (25\alpha + 10)(y_2 - y_1), \\ \dot{y}_2 &= (28 - 35\alpha)y_1 - y_1y_3 + (29\alpha - 1)y_2, \\ \dot{y}_3 &= y_1y_2 - \frac{\alpha + 8}{3}y_3, \end{aligned} \tag{18}$$

where $a = 5$, $b = 10$, $c = 3.4$, $\alpha \in [0, 1]$.

Since the new system (17) and the united system (18) have different nonlinear functions, they are two different chaotic systems. Simulation is made between systems (17) and (18), when the parameters are chosen as $a = 5$, $b = 10$, $c = 3.4$, $\alpha = 0.7$, initial values of the state variables are $x_1(0) = 3$, $x_2(0) = 3$, $x_3(0) = -6$, $y_1(0) = -5$, $y_2(0) = 0$, $y_3(0) = 3$, respectively. As shown in Fig.1 chaotic behaviour appears for both systems, and the traces change of x, y with time are different. As shown in Fig.2 the return maps are also different in shape.

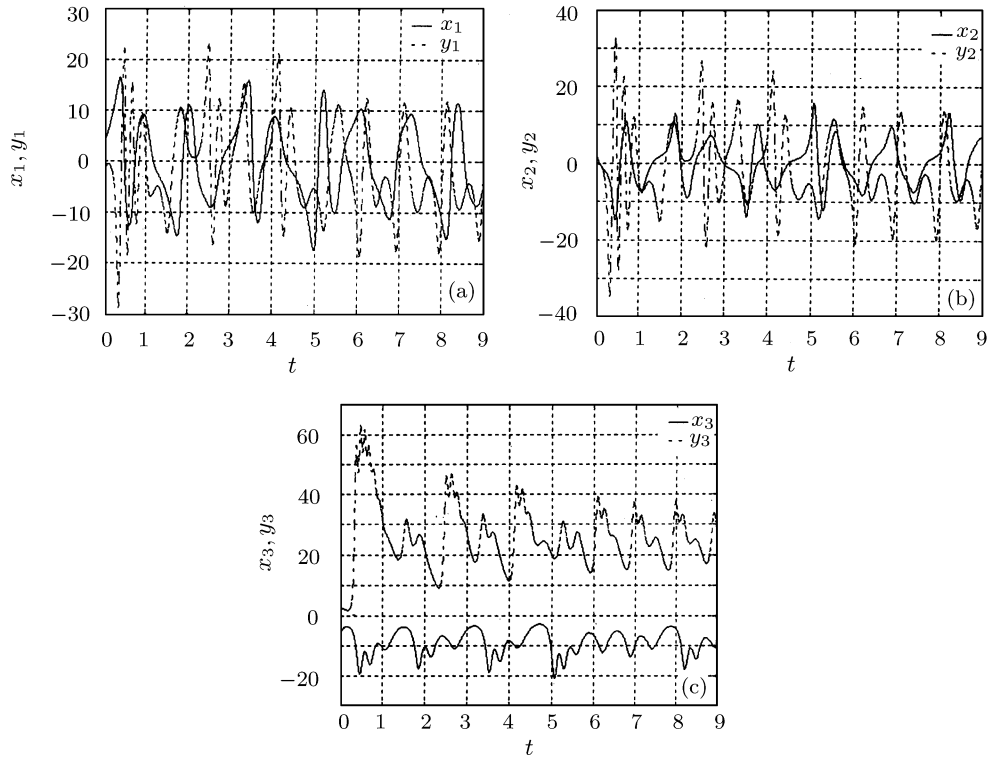


Fig.1. (a) Variation of (x_1, y_1) with time t; (b) Variation of (x_2, y_2) with time t; (c) Variation of (x_3, y_3) with time t.

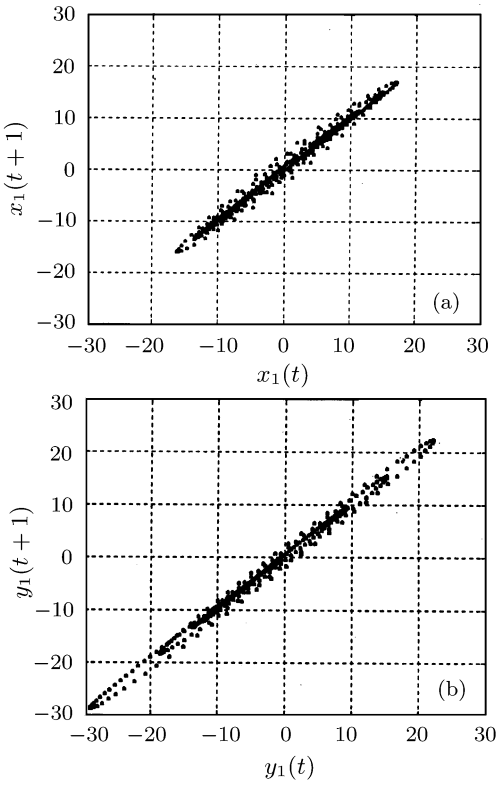


Fig.2. (a) The return maps of phase space in new system; (b) The return maps of phase space in united system.

The controller $u(t) = (u_1(t), u_2(t), u_3(t))^{\mathrm{T}}$ is designed to make the united system (18) synchronized to the new system (17). Thus, the system error is

$$e_1 = y_1 - x_1, \quad e_2 = y_2 - x_2, \quad e_3 = y_3 - x_3, \quad (19)$$

the linear differences between two systems are

$$\begin{aligned} g_{11}(y) - f_{11}(x) &= (25\alpha + 10)(e_2 - e_1) \\ &\quad + (25\alpha + 10)x_2 - (25\alpha + 10 + a)x_1, \\ g_{12}(y) - f_{12}(x) &= (28 - 35\alpha)e_1 + (29\alpha - 1)e_2 \\ &\quad + (28 - 35\alpha)x_1 + (29\alpha - 1 + b)x_2, \\ g_{13}(y) - f_{13}(x) &= -\frac{\alpha + 8}{3}e_3 + (c - \frac{\alpha + 8}{3})x_3, \end{aligned} \quad (20)$$

and the nonlinear differences between two systems are

$$\begin{aligned} g_{21}(y) - f_{21}(x) &= x_2 x_3, \\ g_{22}(y) - f_{22}(x) &= -e_1 e_3 - e_3 x_1 - e_1 x_3 - 2x_1 x_3, \\ g_{23}(y) - f_{23}(x) &= e_1 e_2 + e_2 x_1 + e_1 x_2 \end{aligned} \quad (21)$$

Thus, the nonlinear controller is designed as follows:

$$\begin{aligned} u_1(t) = &-(25\alpha + 10)x_2 + (25\alpha + 10 + a)x_1 \\ &- x_2 x_3 + W_1(t), \\ u_2(t) = &-(28 - 35\alpha)x_1 - (29\alpha - 1 + b)x_2 \\ &+ e_1 e_3 + e_3 x_1 + e_1 x_3 + 2x_1 x_3 + W_2(t), \\ u_3(t) = &-\Big(c - \frac{\alpha + 8}{3}\Big)x_3 - e_1 e_2 - e_2 x_1 \\ &- e_1 x_2 + W_3(t). \end{aligned} \quad (22)$$

The dynamical equation of the error of the two systems can be written as

$$\begin{aligned}\dot{e}_1 &= (25\alpha + 10)(e_2 - e_1) + W_1(t),\\ \dot{e}_2 &= (28 - 35\alpha)e_1 + (29\alpha - 1)e_2 + W_2(t),\\ \dot{e}_3 &= -\frac{\alpha + 8}{3}e_3 + W_3(t).\end{aligned} \tag{23}$$

The control function

$$W(t) = (W_1(t), W_2(t), W_3(t))^{\mathrm{T}}$$

is a function of e_1, e_2, e_3. If $\lim\limits_{t\to\infty} e(t) = 0$ and the error system (23) has three eigenvalues with negative real parts, synchronization can be realized.There are many possible choices for $W(t)$, here we choose

$$\begin{aligned}W_1(t) &= -(25\alpha + 10)e_2,\\ W_2(t) &= -(28 - 35\alpha)e_1 - [(29\alpha - 1) + 1]e_2,\\ W_3(t) &= 0.\end{aligned} \tag{24}$$

In this case, the three eigenvalues of the coefficient matrix are $-(25\alpha+10)$, -1, $-(\alpha+8)/3$ which are all negative, and the two systems are synchronized.

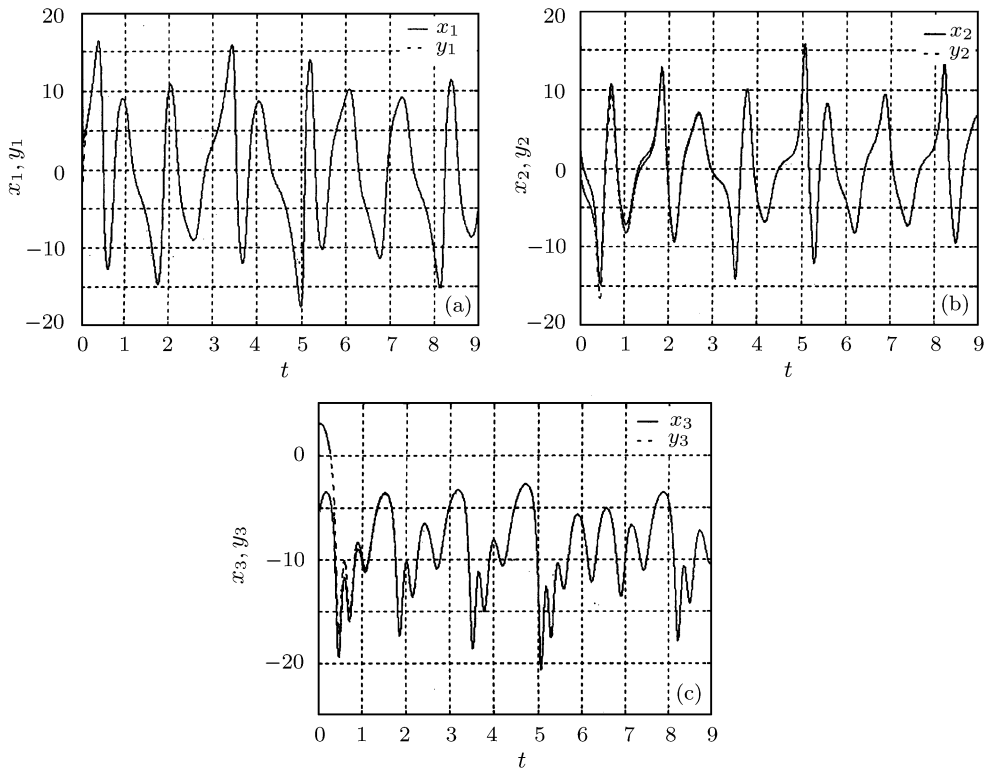


Fig.3. (a) Variation of (x_1, y_1) with time t; (b) Variation of (x_2, y_2) with time t; (c) Variation of (x_3, y_3) with time t.

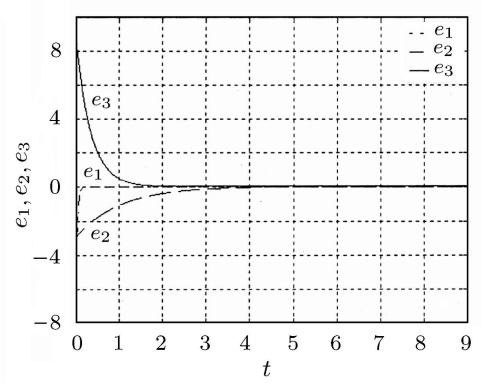


Fig.4. Synchronization errors with time t.

Simulation of synchronization is made, while the system parameters and state variables of the original systems keep constant during the simulation. The states of the systems synchronized are shown in Fig.3, and the error is shown in Fig.4. As shown in Figs.3 and 4, the three variables of the united system are synchronized with those of the new system by the function of the nonlinear controller $u(t)$, and the orbit of the united system in the return maps is exactly the same as that of the new system (shown in Figs.5 and 6). Because the nonlinear functions of the systems are used as the nonlinear feedback terms to the controller, the nonlinearity of the controller proceeds harmoniously with that of the systems, which enhances the strength of the control and there is not a sudden rise or fury surge in the error signal even when the systems have stronger nonlinearity. The error signal approaches to zero smoothly, which made the controller more effi-

cient.

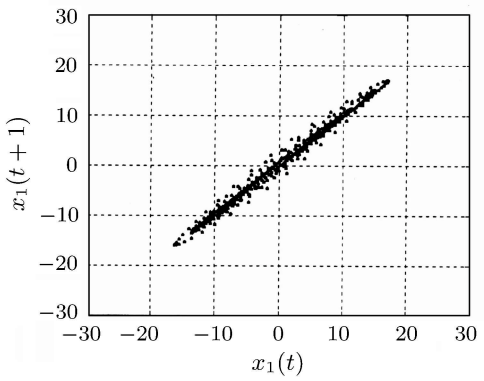


Fig.5. Synchronization return maps in new system.

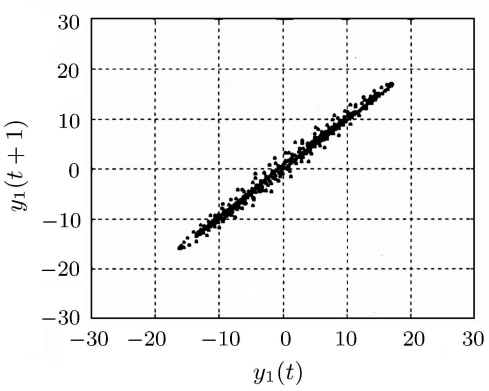


Fig.6. Synchronization return maps in united system.

4. Conclusion

Synchronization between the new system and the united system is studied in this paper. A nonlinear controller is designed with the nonlinear functions of the systems, which are used as a nonlinear feedback term of the controller. The structure and the area of feedback gain are chosen according to the stability theory, and the artificial simulation results show that this control method is commendably effective and feasible. Comparing with the other controllers, the one in this paper has the following advantages.

First, since the nonlinear functions of the systems are used as a nonlinear feedback term to the controller, it is unnecessary to get detailed information of the systems. Second, the nonlinearity of the controller is enhanced according to the nonlinearity of the system, which enhances the strength of the control. Therefore there is not a sudden rise or fury surge in the error signal when the systems have stronger nonlinearity, and the error signal goes to zero smoothly, which makes the controller more efficient. Thirdly, the Lyapunov function or Lyapunov exponent is unnecessary to be constructed or calculated, thus the work of calculation in the Chaos synchronization is largely reduced.

References

[1] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[2] Lu J G 2006 *Chin. Phys.* **15** 83
[3] Awad E G 2006 *Chaos, Solitons and Fractals* **27** 345
[4] Yu H J and Liu Y Z 2005 *Acta Phys. Sin.* **54** 3029 (in Chinese)
[5] Wang Y W,GuanZ H and Wang H O 2005 *Phys. Lett.* A **339** 325
[6] Tsimring L S, Rulkov N F, Larsen M L and Gabbay M 2005 *Phys. Rev. Lett.* **95** 14101
[7] Ma J, Liao G H, Mo X H, Li W X and Zhang P W 2005 *Acta Phys. Sin.* **54** 5585 (in Chinese)
[8] Yan W W,Zhi H G and Hua O W 2005 *Phys. Lett.* A **339** 325
[9] Yue L J and Shen K 2005 *Chin. Phys.* **14** 1760
[10] Wang X Y and Shi Q J 2005 *Acta Phys. Sin.* **54** 5591 (in Chinese)
[11] Lu J, Wu X, Han X and Lü J 2004 *Phys. Lett.* A **329** 327
[12] Huang L,Feng R and Wang M 2004 *Phys. Lett.* A **320** 271
[13] Li S H and Cai H X 2004 *Acta Phys. Sin.* **53** 1687 (in Chinese)
[14] Li Y N, Chen L, Cai Z S and Zhao X Z 2003 *J. Chem. Phys.* **16** 14 (in Chinese)
[15] Cheng L, Zhang R Y and Peng J H 2003 *Acta Phys. Sin.* **52** 536 (in Chinese)
[16] Ge Z M, Chang C M and Chen Y S 2006 *Chaos, Solitons and Fractals* **27** 1298
[17] Yan Z Y 2005 *Phys. Lett.* A **334** 406
[18] Lü L 2000 *Nonlinear Dynamics and Chaos* (Dalian: Dalian Publishing House) p130 (in Chinese)
[19] Liu W B and Chen G R 2004 *Int. J. Bifur. Chaos* **14** 1395
[20] Lü J H, Chen G R, Cheng D Z and Celikovsky S 2002 *Int. J. Bifur. Chaos* **12** 2917