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Evolution of network from node division and generation*

Sun Hui-Jun(孙会君)[a)b)] and Wu Jian-Jun(吴建军)[a)b)]

[a)] *State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing* 100044, *China*

[b)] *School of Traffic and Transportation, Beijing Jiaotong University, Beijing* 100044, *China*

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Aimed at lowering the effect of 'rich get richer' in scale-free networks with the Barabási and Albert model, this paper proposes a new evolving mechanism, which includes dividing and preference attachment for the growth of a network. A broad scale characteristic which is independent of the initial network topology is obtained with the proposed model. By simulating, it is found that preferential attachment causes the appearance of the scale-free characteristic, while the dividing will decrease the power-law behaviour and drive the evolution of broad scale networks.

Keywords: Evolution, dividing, generation, scale-free network
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Since the ground-breaking paper by Barabási and Albert (BA) on scale-free networks[1] and that by Watts and Strogatz on small-world networks[2] the interest in large, growing, and complex networks has soared.[3] BA discover that many networks in nature appear to exhibit scale-free features, and that the distribution $P(k)$ exhibits a power-law behaviour in a range of k-values over several orders of magnitudes: $P(k) \sim k^{-\lambda}$.[1,3] The mechanisms leading to the power-law distribution are argued to be growth and preferential attachment,[1,4,5] where the former means that the size of the network keeps increasing with time and the latter underlies that the relative probability for an already heavily connected node to get new links is proportionally large. A small world network, on the other hand, is characterized by high clustering like regular lattices, yet has small path lengths like random graphs. With the further study of topology and the generation theory of complex networks, many real networks, such as natural networks and artificial networks,[6−19] are analysed, and some exciting results have been drawn in recent years.

There is empirical evidence for the occurrence of three classes of small world networks by studying technologic and economic networks, social networks and natural sciences networks. Three classes of small world networks are described as follows:[20]

(1) scale-free networks, characterized by a connectivity distribution with a tail that decays as a power-law; (2) broad-scale or truncated scale-free networks, characterized by a connectivity distribution that has a power-law regime followed by a sharp cut-off, like an exponential or Gaussian decay of the tail; and (3) single-scale networks, characterized by a connectivity distribution with a fast decaying tail, such as exponential or Gaussian.

The traditional BA model for generating scale-free networks cannot correctly reproduce all characteristics of real world networks.[21,22] First, it produces a temporally correlated network in the sense that older nodes tend to get more edges than younger ones, which was not always a rule. In the real world, when the nodes grow larger and larger (or nodes connect more and more links), they may be divided at some time. In our study, we find that the rule of 'rich get richer' is independent of the initial network topology. And the topology of network has stronger stability in growing process with BA model (See Appendix), which means that the state of 'rich get richer' will not change for ever, and it is not consistent with the practice of the BA model. Additionally, in the original form of BA model, it reproduces only networks with degree distribution characterized by exponent $\lambda = 3$, which is also impractical.

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In this paper, the main contribution is to present a new way to overcome some shortcuts of BA model and generate the broad scale degree distribution. The model describes an evolving network (See Fig.1), in which the main components, represented by nodes, are divided, and at the same time, a new node with links is added into the current network at each time (step) in according with preference attachment. In the growing mechanism, actually, there are two nodes which are added into the network at each time (step).

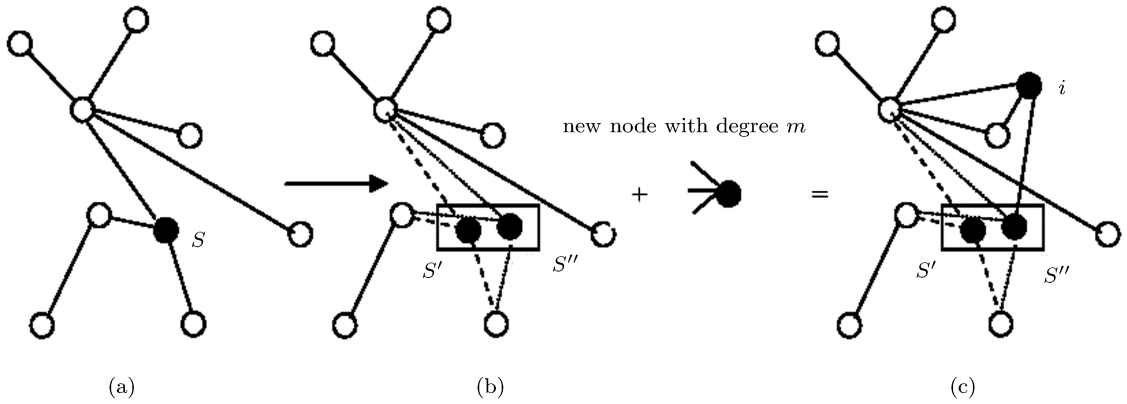


Fig.1. Illustration of the update rules and the growth of network. (a) A node S is chosen randomly. (b) Two nodes S' and S'' are the result of node dividing, and the links are redistributed with probability ρ and $1-\rho$ respectively. A new node is added with degree m connected to other nodes with preferential attachment and the network is updated (c).

The detailed growing mechanism in our paper is described as follows:

(1) At each time (step), we choose the node S with degree k_s randomly.

(2) The node S is divided into two nodes S' and S''. The probability of nodes S' and S'' connecting to the original nodes linked by the node S is ρ and $1-\rho$ respectively. It means that the node S' has the degree $k_s\rho$, and the node S'' has the degree $k_s(1-\rho)$.

(3) At the same time, a new node with degree m is added into the current network. m-value can be a constant or variable. Choosing the nodes to which the new node connects, we assume that the new node is connected to the old nodes with the preferential attachment $m(k_i/\sum_j k_j)$, where k_i is the degree of node i.

In Fig.2, we give three initial network topologies with 16 nodes: star, random and regular networks respectively. All simulation results and analyses presented in this paper are based on these three initial topologies. Throughout the paper, for the topological characteristics of each network, more than 25 realizations are averaged.

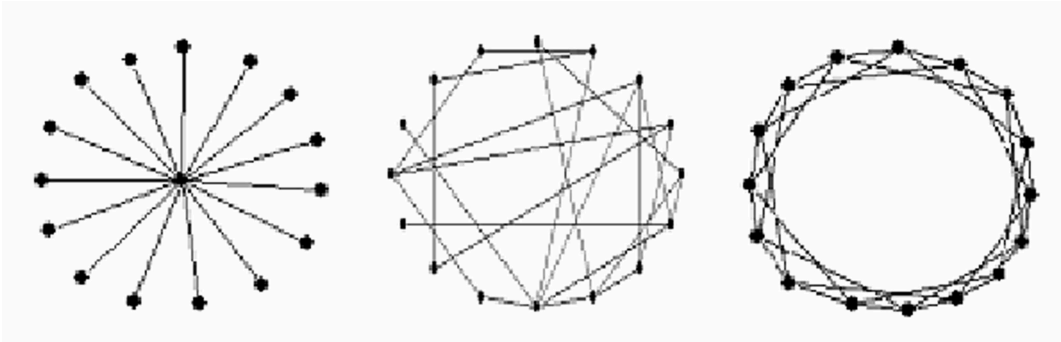


Fig.2. Simple examples of different initial network topologies with 16 nodes.

From Fig.3, we can observe the similar results for the growth of different initial network topologies with sizes $N = 500$, $N = 2000$ and $N = 5000$. Clearly, the network will grow into the broad scale, that is, it has a distribution of connectivities that decays with a fast decaying tail, and the exponent of degree distribution ranges from $0.35 - 0.44$. Moreover, we can see that the final network topology is independent of the initial status. From the perspective of the BA model, the network will have the scale-free characteristic with power-law tail distribution because of the third mechanism, in which the exponent of degree distribution is equal to 3. A simple dividing process decreases the scale-free property as expected. Therefore, two results can be drawn safely: 1) preference attachment is the important cause to prompt the scale-free characteristics; and 2) the division of node decreases the power-law property, and even for different initial network topologies, it always can make the network grow into the broad scale with the increase of system size.

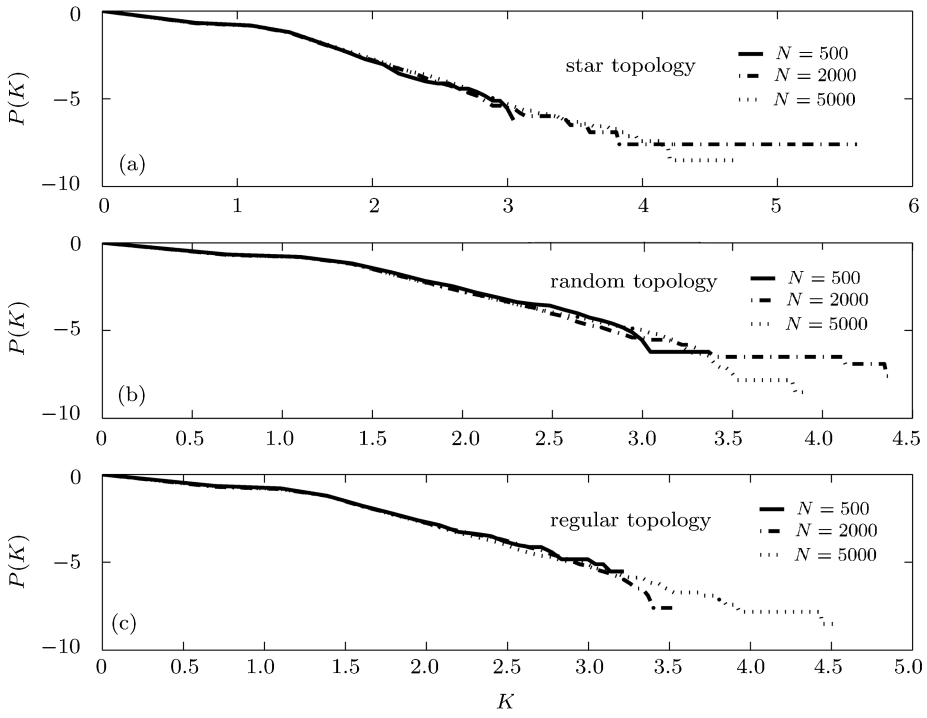


Fig.3. Double logarithmic degree distribution for networks resulting from our growing mechanisms with different initial topologies of sizes $N = 500$, $N = 2000$ and $N = 5000$. Each curve corresponds to an average over 25 independent realizations of the network.

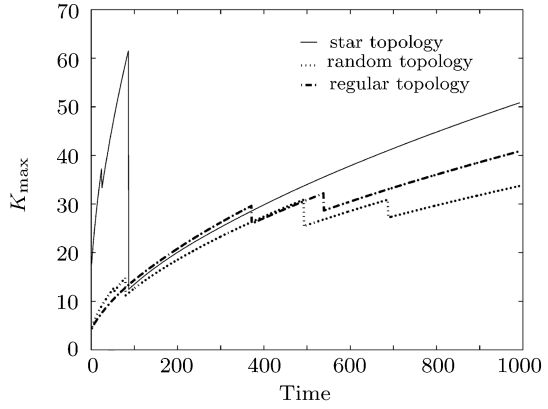


Fig.4. The time evolvement of the maximum degree for different topologies. When the node is chosen to be divided, the maximum degree of the node shows a sudden decrease. Each curve corresponds to an average over 25 independent realizations of the network.

To prove the aforementioned results, we plot Fig.4 which shows the variation of the maximum degree for different topologies with the time. Clearly, there are several obvious phase transitions, which indicate that the node with the maximum degree is chosen and divided.

From Fig.5, we can see that the degree of a given node varies with time. There is a tendency and a clearly sharp variation in the growth process, which is stronger for the star topology network than other two topologies. Similar to the analysis for Fig.4, the phase transition shows that the node with maximum degree is divided, and a sudden change appears. The results can be used to explain why the node with most links not always has the maximum degree in the growth of networks. Additionally, it indicates that the degree of

node may increase with time, but does not exhibit the rule of 'rich get richer' for ever.

So far we have investigated a growing broad scale degree distribution network which is generated with probability ρ. In addition, one might argue that the networks are in a steady state in some cases. Our model can be extended to include a mechanism, i.e. removing one node but not adding a new node at each time. This will be discussed in a further study.

In this paper, we construct a growing mechanism which includes node dividing and preference attachment for the growth of network. The broad scale characteristic is obtained by our mechanism, which is independent of the initial network topology. By experimental simulation, the conclusion is that preferential attachment will produce a tendency of the scale-free characteristic, while the division of node may decrease the state of 'rich get richer' and drive the network to take on the broad scale property.

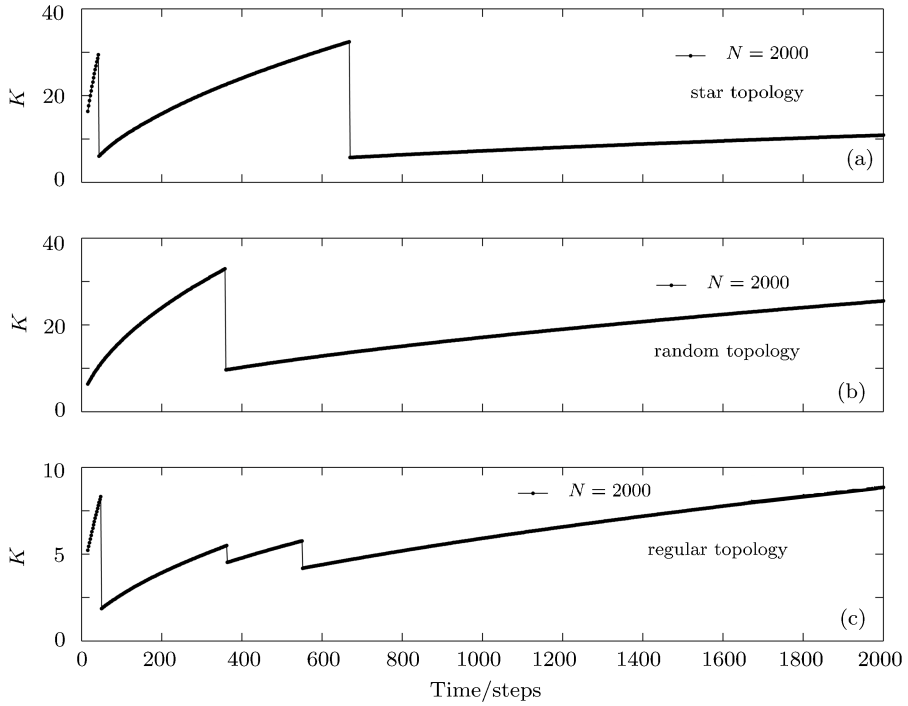


Fig.5. Degree of a given node with the maximum degree in initial status as a function of time (steps) for three different initial network topologies. Each curve corresponds to an average over 25 independent realizations of the network.

Appendix

Assume an initial network with m_0 nodes (no isolate nodes) and m_1 edges; we study the effects caused by the number of initial nodes on characteristics of the final network. It is known that the equations

$$\min(m_1) = \left[\frac{m_0+1}{2}\right]$$

and $\max(m_1 = C_{m_0}^2)$ are held, where $[\cdot]$ represents round numbers. At time t, the degree distribution of node i in the BA model can be described by the following partial differential equation:

$$\frac{\partial k_i}{\partial t} = m\frac{k_i}{2mt - m + 2m_1}.$$

The network will arrive at stable status when $t \to \infty$. We discuss whether the differential initial network topologies and the different m-values will affect the final network. According to the second method of Lyapunov, we construct the function

$$V(k_i) = \frac{k_i^2}{(2mt - m + 2m_1)^2}.$$

$$\begin{aligned} V'(k_i) = & -2\frac{2mk_i^2}{(2mt - m + 2m_1)^3} \\ & + \frac{2k_i}{(2mt - m + 2m_1)^2} \cdot \frac{\partial k_i}{\partial t} \\ = & -2\frac{2mk_i^2}{(2mt - m + 2m_1)^3} \\ & + \frac{2k_i}{(2mt - m + 2m_1)^2} \cdot \frac{mk_i}{2mt - m + 2m_1} \\ = & -\frac{mk_i^2}{(2mt - m + 2m_1)^3} \le 0. \end{aligned}$$

Uniform stability can be drawn in terms of two conditions mentioned above. Because of $k_i > 0$, the following equation is obtained:

$$V'(k_i) = -\frac{mk_i^2}{(2mt - m + 2m_1)^3} < 0.$$

Therefore, the equilibrium status of systems is asymptotic stability. And the result shows that the equilibrium of the network is independent of its initial topology (the number of nodes m_0 and edges m_1) and the variation of m-value. This result also proves that the degree distribution exponent γ in BA model is independent of the initial parameters m,m_0 and m_1.

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