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Extended speed gradient model for traffic flow on two-lane freeways*

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In this paper, the speed gradient (SG) model is extended to describe the traffic flow on two-lane freeways. Terms related to lane change are added into the continuity equations and velocity dynamic equations. The empirically observed two-lane phenomena, such as lane usage inversion and lane change rate versus density, are reproduced by extended SG model. The local cluster effect is also investigated by numerical simulations.

Keywords: two-lane traffic, speed gradient model, lane usage inversion
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1. Introduction

Recently, traffic flow research has attracted the interests of physicists and engineers.[1−10] Various traffic flow models, including microscopic car-following models, cellular automaton (CA) model, macroscopic continuum model, and mesoscopic gas-kinetic-based model, have been proposed. Most of the models focus on the traffic flow on a single lane road. However, in real traffic, the road usually has more than one lane. Therefore, it is necessary to extend the well-developed single-lane traffic models to multi-lane traffic systems. In this paper, we concentrate on the traffic flow research on two-lane freeways.

The microscopic models are extended to describe traffic flow on two-lane freeways by incorporating the lane changing rules. Both symmetric and asymmetric lane changing rules are studied.[11−18] It is shown that one of the most important two-lane phenomena (i.e., lane usage inversion) can be reproduced if the asymmetric lane changing rules are adopted.

Macroscopic traffic models, which concentrate on the collective behaviour of vehicles, are not so easily extended to two-lane system as compared to microscopic models. In Ref.[19], Daganzo presented a theory of traffic dynamics for freeways with special lanes. The Lighthill–Whitham–Richards model is extended to two lanes which are labelled with a regular lane for all car types and a special lane that only the prior car type is allowed to use. Tang and Huang[20] extended the speed gradient model to two-lane freeways, in which the non-equilibrium phenomena such as stop-and-go waves are reproduced. Nevertheless, the observed phenomena like lane usage inversion and lane changing frequency are not discussed in these papers.

Shvetsov and Helbing presented a gas-kinetic model of multi-lane freeway for mixed traffic,[21] in which spontaneous and interactive lane changes are considered. Nevertheless, this model has so many parameters that it is difficult to carry out the parameter calibration.

In this paper, we extend the speed gradient (SG) model proposed by Jiang *et al*[22] to describe asymmetric two-lane traffic. It is shown that lane usage inversion is reproduced by the extended SG model. We also investigate lane change rate versus density and the local cluster effect in two-lane freeways.

The paper is organized as follows. In Section 2 we summarize the empirical results observed on two-lane freeways where asymmetric lane change rules are adopted. The extended SG model is proposed in Section 3. In Section 4, we show that the lane usage inversion is reproduced by the extended SG model. The lane change rate versus density is also studied. The local cluster effect is discussed in Section 5. Finally, a conclusion is given in Section 6.

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2. Empirical observations

This section summarizes some empirical observations on asymmetric two-lane freeways,[7,9−11,19] which should be reproduced by a realistic model.

(i) On asymmetric two-lane freeways, the legal regulation is the right lane should be used as often as possible, and car's overtaking is only allowed on the left lane. As a result, the lane usage inversion is observed: at small densities, most cars dominate on the right lane. However, with increasing density, most cars drive on the left lane above an inversion point. The density corresponding to the inversion point is smaller than that corresponding to the maximum flow. This inversion vanishes and all cars are evenly populated on both lanes at large densities.[14]

(ii) With lane usage regulation that the left lane is designated for overtaking, the lanes are separated into fast and slow lanes, leading to a negative velocity gradient from left to right. The difference of average velocities between the two lanes decreases approximately linearly with total flow and will be nearly zero at large flows ($Q > 3000$ veh/h).[13]

(iii) It is shown that with the increase of total density from a small value, the flow rate on fast lane reaches the maximum earlier than on slow lane. With further increase of total density, the flow rate on fast lane will decrease while it still increases on slow lane. Therefore, traffic breakdown usually originates from the fast lane.[11,14]

(iv) The lane change rate should increase with density at small densities. After reaching a maximum in the vicinity of the density corresponding to the maximum flow, the lane change rate decreases with the density. However, cars can still change lane at large densities.[13,15]

3. Extended SG model for two-Lane traffic

The SG model is proposed independently by several groups of researchers.[22,24−26] It is an improvement over the previous high-order continuum models in that it does not exhibit the characteristic speed larger than macroscopic traffic speed. Therefore, it satisfies the requirement that the vehicles are anisotropic and in response to stimuli from the front, and it will not lead to a wrong-way travel problem.

The SG model consists of the continuity equation (1) and the dynamic equation (2) as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{u_e(\rho) - u}{T} + c_0\frac{\partial u}{\partial x}, \tag{2}$$

where u denotes the average vehicle velocity, ρ is the vehicle density. The right-hand side of the velocity equation (2) contains a relaxation term reflecting the process when the driver adjusts the speed of vehicle to the equilibrium velocity $u_{\rm e}(\rho)$ in the relaxation time interval T, and an anticipation term representing the process when the driver reacts to the traffic ahead with the propagation speed of small disturbance c_0.

On two-lane freeways, the lane change behaviour will occur. To model this effect, a source term is added to the continuity equation to describe the flow rate changing from the other lane, and a sink term is added to the continuity equation to describe the flow rate changing to the other lane. Accordingly; two terms are added in the speed dynamic equation. The extended SG model reads

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial (\rho_i u_i)}{\partial x} = S_{ji} - S_{ij}, \tag{3}$$

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x} = \frac{u_{ei}(\rho_i) - u_i}{T_i} + c_{0i}\frac{\partial u_i}{\partial x} + r_1 S_{ij} - r_2 S_{ji}. \tag{4}$$

Here $i = 1, 2$ denotes the i lane and $j = 1, 2 (j \neq i)$ denotes the other lane. The term S_{ij} represents lane change rate from i lane to j lane; r_1 and r_2 are two constant parameters. We suppose the terms of lane change rate have the form as follows:

$$S_{12} = aQ_{\rm e1}(\bar{\rho})\rho_1(1 - \rho_2/\rho_m), \tag{5}$$

$$S_{21} = a(1 + b(Q_{\rm e1}(\bar{\rho}) - Q_{\rm e2}(\bar{\rho}))) \times Q_{\rm e2}(\bar{\rho})\rho_2(1 - \rho_1/\rho_m), \tag{6}$$

where Q_{ei} is the equilibrium flow ($Q_{ei} = \rho u_{ei}(\rho)$) and $\bar{\rho}$ is the average density of the two lanes ($\bar{\rho} = (\rho_1 + \rho_2)/2$). The special case of $Q_{\rm e1} = Q_{\rm e2}$ corresponds to symmetric lane change rules, while $Q_{\rm e1} \neq Q_{\rm e2}$ corresponds to asymmetric lane change rules; a and b are constant parameters, ρ_m is the jam density, which is assumed to be the same on the two lanes for simplicity (i.e., the lengths of slow vehicles and fast vehicles are assumed to be the same).

In real traffic, the lane change rate, say from lane 1 to 2, should be zero when either $\rho_2 = \rho_m$ (lane 2 is in complete jam) or $\rho_1 = 0$ (there is no vehicle on lane 1). Furthermore, it obviously depends on the equilibrium speed density relationship on the present lane and on

the average density $\bar{\rho}$. These properties are fulfilled in Eqs.(5) and (6).

It is necessary to point out that in previous works,[20,27] usually only the net lane change rate $S_i = S_{ji} - S_{ij}$ is modelled. As a result, the lane change rate versus average density $\bar{\rho}$ cannot be investigated. Namely, when the equilibrium state is reached, one only knows that the net lane change rate is zero, but does not know the exact value of lane change rate. Moreover, S_{12} and S_{21} may have different effect on the velocity dynamic equation, which can be modelled in our model by setting $r_1 \neq r_2$.

4. Steady states

The steady state solutions of the extended SG model can be obtained by setting the derivative terms in Eqs.(3) and (4) to zero. Then we have

$$Q_{e1}(\bar{\rho})\rho_1\Big(1-\frac{\rho_2}{\rho_m}\Big) = (1+b(Q_{e1}(\bar{\rho}) - Q_{e2}(\bar{\rho}))) \times Q_{e2}(\bar{\rho})\rho_2\Big(1-\frac{\rho_1}{\rho_m}\Big), \quad (7)$$

$$\frac{u_{ei}(\rho_i) - u_i}{T_i} + r_1 S_{ij} - r_2 S_{ji} = 0. \quad (8)$$

Moreover, we have

$$\rho_1 + \rho_2 = 2\bar{\rho}. \quad (9)$$

Therefore, when $\bar{\rho}$ is given, we can calculate ρ_1 and ρ_2 from Eqs.(7) and (9). Substituting ρ_1 and ρ_2 into Eq.(8), we can obtain u_1 and u_2. Then the flow rates on lanes 1 and 2 are $q_i = \rho_i u_i$.

On the fast lane, the equilibrium speed density relationship proposed by Lee *et al*[28] is adopted,

$$u_{e1}(\rho) = u_f(1-\rho/\rho_m)/(1+E(\rho/\rho_m)^4). \quad (10)$$

Here u_f is the free flow speed on the fast lane and E is a constant parameter. On the slow lane, the following modified equilibrium speed density relationship is adopted[1)]

$$u_{e2}(\rho) = \begin{cases} \frac{u_f(1-\frac{\rho}{\rho_m})}{(1+E(\frac{\rho}{\rho_m})^4)}(0.5+0.5\frac{\rho}{\rho_c}), & \text{if } \rho < \rho_c; \\ u_f(1-\frac{\rho}{\rho_m})/(1+E(\frac{\rho}{\rho_m})^4) & \text{if } \rho \geq \rho_c. \end{cases} \quad (11)$$

The free flow speed on the slow lane is $u_f/2$, which can be calculated by setting $\rho = 0$ in Eq.(11). We compare the equilibrium speed density relationships (10) and (11). It can be seen that at small densities, the equilibrium speed of the slow lane is smaller than that of the fast lane, while at large densities ($\rho > \rho_c$) they are just the same. This is consistent with the simulated results from CA models.[29] The values of parameters are: $T_1 = T_2 = 10$ s, $u_f = 30$ m/s, $a = 0.01$, $b = 5$ s, $c_{01} = c_{02} = 10$ m/s, $\rho_m = 0.14$ veh/m, $r_1 = 150$ m^2/s, $r_2 = 50$ m^2/s,[2)] $\rho_c = 0.05$ veh/m, $E = 100$.

The steady states on slow lane and fast lane against average density $\bar{\rho}$ are presented in Fig.1. At small densities, the flow on both lanes increases with density. Note that on the fast lane, the flow rate reaches the maximum at a smaller density ρ_{c1}. In the density range $\rho_{c1} < \bar{\rho} < \rho_{c2}$, the flow rate decreases on the fast lane and increases on the slow lane. This is consistent with category (iii) in Section 2. The maximum flow rate on both lanes is reached at $\rho_{c3} \approx 0.033$ veh/m. When the average density $\bar{\rho} > \rho_c$, the flow rates on the two lanes are the same.

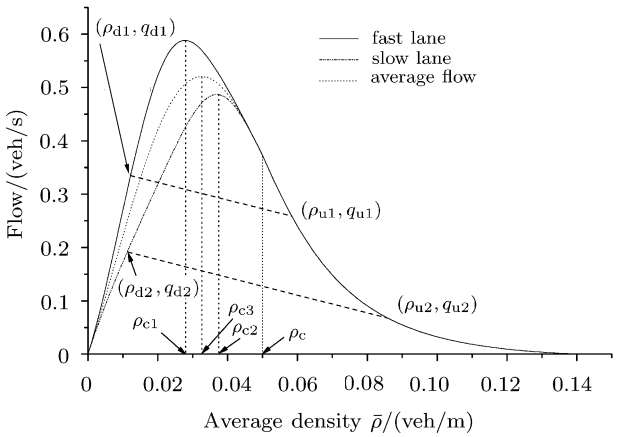


Fig.1. The steady states of asymmetric two-lane traffic. The slope of dashed line denotes the propagation velocity of D-fronts ($v_{gi} = (q_{ui} - q_{di})/(\rho_{ui} - \rho_{di})$).

Figure 2 shows the curve of lane usage. The lane usage inversion phenomenon is reproduced. At small densities most of the cars dominate on the slow lane. With the increase of $\bar{\rho}$, there appears an inversion point ρ_{inv} where cars are evenly distributed on both lanes. When $\bar{\rho} > \rho_{inv}$, more cars prefer to drive on the fast lane rather than to be trapped on the slow lane. When both lanes are congested ($\bar{\rho} > \rho_c$), both lanes are equally used. The density ρ_{inv} is smaller than the

[1)] Theoretically the equilibrium speed density relationships u_{e1} and u_{e2} need to be obtained when there is no lane change. Therefore, strictly speaking, we cannot obtain u_{e1} and u_{e2} from field observations because lane change always exists. Nevertheless, our simulations show that when there is a lane change, the actual speed density relationship is only slightly different from u_{e1} and u_{e2}. Therefore, we may approximate u_{e1} and u_{e2} by actual speed density relationship observed from field.

[2)] Empirical observations show that on a two-lane road, the flow rate per lane is larger than the flow rate on a single lane. This is fulfilled here by setting $r_1 > r_2$.

density ρ_{c3}. This is consistent with empirical results of category (i) in Section 2.

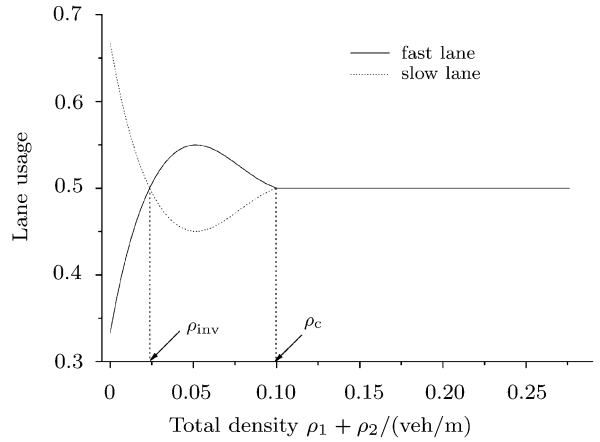


Fig.2. Lane usage in asymmetric two-lane traffic.

Figure 3 shows the density dependence of the lane change rate. It first increases with density, and after reaching the maximum at $\rho_{ch} \approx 0.035$veh/m, it decreases with density. We note that (i) ρ_{ch} is in the vicinity of ρ_{c3} corresponding to the maximum flow; (ii) the lane change behaviour still persists at large densities (when $\bar{\rho} > \rho_c$, the densities are equal on both lanes). These are consistent with category (iv) in Section 2.

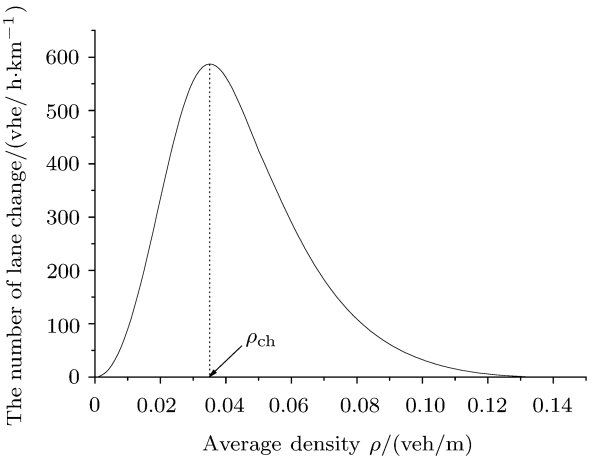


Fig.3. Lane change rate in asymmetric two-lane traffic.

Figure 4 shows the difference of average velocity between the two lanes against total flow. When the total flow Q is very small, cars drive at free flow speeds on both lanes. The difference between the two lanes decreases nearly linearly with increasing Q for not very large flow rate and becomes nearly zero at the maximum flow. This is consistent with category (ii) in Section 2.

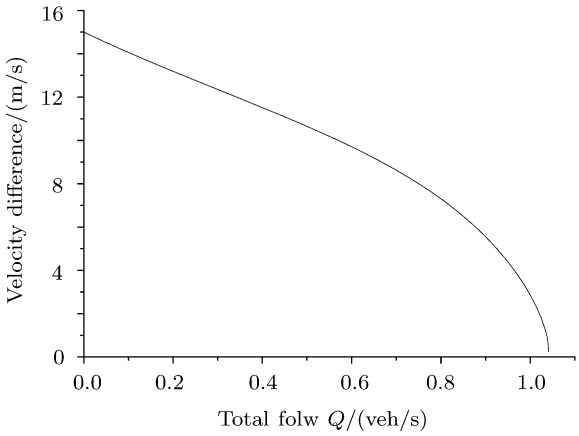


Fig.4. The difference of average velocity between two lanes versus the total flow in asymmetric two-lane traffic.

5. Local cluster effect

In this section we investigate the local cluster effect in the extended SG model with respect to a localized perturbation either on fast lane or on slow lane. We start with the homogeneous conditions on a circular road with a length $L = 32.2$ km. The following initial variation of the average density ρ_h as proposed in Ref.[30] is used on either fast lane or slow lane:

$$\rho(x,0) = \rho_h + \Delta\rho_h \Big\{ \cosh^{-2}\Big[\frac{160}{L}\Big(x - \frac{5L}{16}\Big)\Big] - \frac{1}{4}\cosh^{-2}\Big[\frac{40}{L}\Big(x - \frac{11L}{32}\Big)\Big]\Big\}. \quad (12)$$

The numerical scheme developed in Ref.[18] is adopted here. Moreover, we have

$$\rho_1 + \rho_2 = 2\bar{\rho}.$$

Our simulations show that, at low average density $\bar{\rho}$, perturbations on either the fast lane or the slow lane will dissipate and the traffic flow is stable. With the increase of $\bar{\rho}$, traffic breakdown may occur if a perturbation is exerted on the fast lane. As a result, the local cluster appears on the fast lane. Due to the lane change behaviour, a small hump appears at the corresponding location on the slow lane (Fig.5). In contrast, if the perturbation is exerted on the slow lane, it will dissipate and stable traffic flow is maintained on both lanes (Fig.6). This is because (i) the

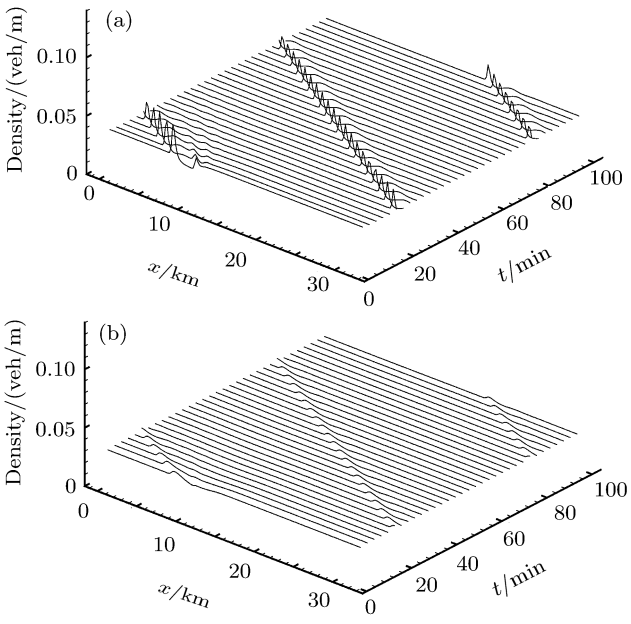


Fig.5. Dynamics of an asymmetric two-lane system with an initial perturbation in the fast lane at $\rho_1 = 0.047$ veh/m, $\rho_2 = 0.021$ veh/m, $\Delta\rho_{\rm h} = 0.06$ veh/m (a)fast lane, (b) slow lane.

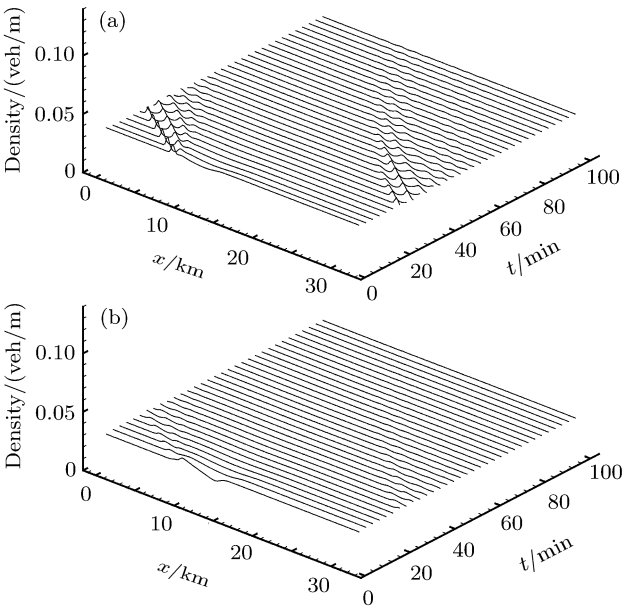


Fig.6. Dynamics of an asymmetric two-lane system with an initial perturbation in the slow lane at $\rho_1 = 0.047$ veh/m, $\rho_2 = 0.021$ veh/m, $\Delta\rho_{\rm h} = 0.06$ veh/m, (a)fast lane, (b) slow lane.

density on slow lane is smaller than that on fast lane due to lane usage inversion; (ii) the traffic is more stable on the slow lane due to its smaller free flow speed. With further increase of density, perturbations on either the fast lane or the slow lane will induce traffic breakdown and stop-and-go waves will appear on both lanes (Fig.7).

We compare the density profiles on the fast and slow lanes in Fig.8, where stop-and-go waves appear on both lanes. One can see that the downstream fronts of the clusters (D-fronts) on both lanes are nearly at the same location. The width of the clusters on the fast lane is usually larger than that on the slow lane. This is easy to understand because drivers would rather follow a fast vehicle on the fast lane than be trapped behind the slow vehicles on slow lane despite that the slow vehicle is ahead of the fast vehicle.

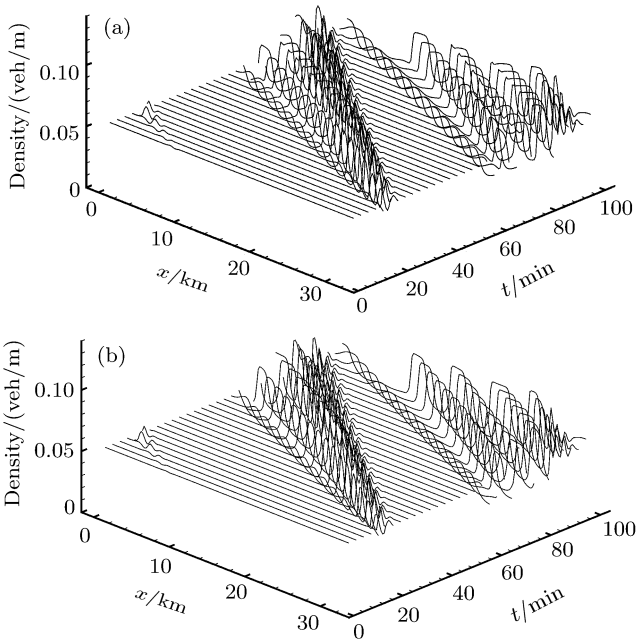


Fig.7. Dynamic traffic of an asymmetric two-lane system at $\rho_1 = 0.053$ veh/m, $\rho_2 = 0.053$ veh/m, (a) fast lane, (b) slow lane.

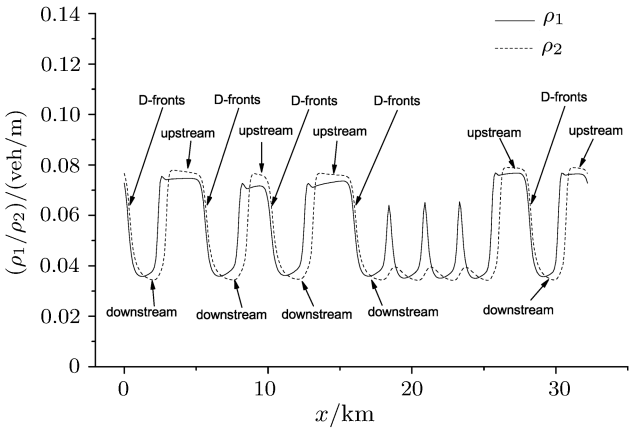


Fig.8. The density profiles in the fast and slow lanes at $t = 20000$ s.

In addition, the density downstream of D-fronts is usually larger on the fast lane because of lane usage inversion. As a result, the density upstream of D-fronts is usually larger on the slow lane. This is explained as follows. The propagation velocity of D-fronts is calculated using

$$v_{gi} = \frac{q_{ui} - q_{di}}{\rho_{ui} - \rho_{di}},$$

where u and d denote the traffic flows upstream and downstream of the front, respectively; v_{gi} should be a constant and does not depend on i. Since $\rho_{d1} > \rho_{d2}$, we have $q_{d1} > q_{d2}$ from Fig.1. Therefore, $\rho_{u1} < \rho_{u2}$ has to be met to enable $v_{g1} = v_{g2} = v_g$. Accordingly, we obtain $q_{u1} > q_{u2}$.

6. Conclusion

In this paper, the speed gradient (SG) traffic model has been extended to describe a two-lane traffic flow. The lane change behaviour is incorporated into the extended SG model by introducing a source term and a sink term in the continuity equation and two corresponding terms in the speed dynamic equations. Our model is different from previous ones in that the source term and the sink term are modelled separately. In this way, the lane change rate can be investigated.

The steady state analysis shows that the lane usage inversion phenomenon, the fundamental diagram, density dependence of lane change rate as well as the variation of velocity difference with the total flow are satisfactorily reproduced.

In our numerical simulation of local cluster effect, it is shown that when the density is small, the perturbations will dissipate. However, with the increase of density, a perturbation on the fast lane will induce a local cluster. In contrast, a perturbation on the slow lane will dissipate. When the density further increases, the perturbation on either the fast lane or slow lane will induce stop-and-go waves. The density profiles on both lanes are also studied.

In our future work, we will introduce the bottleneck effect, such as on-ramp, into the extended SG model. Moreover, how to extend the SG model to describe multi-lane (three lanes or more) traffic is also an interesting work because more and more multi-lane freeways appear in China and all over the world.

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