Vol 16 No 6, June 2007
1009-1963/2007/16(06)/1566-04
Chinese Physics
© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

A scheme of quantum phase gate for trapped ion*

Cai Jian-Wu(蔡建武)[a)b)], Fang Mao-Fa(方卯发)[a)†]
Zheng Xiao-Juan(郑小娟)[a)c)], and Liao Xiang-Ping(廖湘萍)[a)b)]

[a)] *College of Physics and Information Science, Hunan Normal University, Changsha*, 410081, *China*
[b)] *Department of Physics Science and electronics, Zhuzhou Teacher's College, Zhuzhou* 412007, *China*
[c)] *School of Physics Science and Technology, Central South University, Changsha* 410082, *China*

(Received 30 June 2006; revised manuscript received 28 December 2006)

We propose a scheme to implement two-qubit controlled quantum phase gate(CQPG) via a single trapped two-level ion located in the standing wave field of a quantum cavity, in which the trap works beyond the Lamb–Dicke limit. When the light field is resonant with the atomic transition $|g\rangle \leftrightarrow |e\rangle$ of the ion located at the antinode of the standing wave, we can perform CQPG between the internal and external states of the trapped ion; while the frequency of the light field is chosen to be resonant with the first red sideband of the collective vibrational mode of the ion located at the node of the standing wave, we can perform CQPG between the cavity mode and the collective vibrational mode of the trapped ion. Neither the Lamb–Dicke approximation nor the assistant classical laser is needed. Also we can generate a GHZ state if assisted with a classical laser.

Keywords: quantum phase gate, single trapped ion, single quantum cavity mode, GHZ state
PACC: 0367, 4250, 3280

1. Introduction

The quantum computer (QC) is a device based on the superposition principle of quantum dynamics, and it can solve a large number of factoring and searching problems faster than a conventional one. In the last decade, many schemes have been proposed to realize a quantum computer for different systems, such as cavity quantum electromagnetic dynamics (CQED)[1−3] and trapped ions.[4,5] The scheme for trapping ions is first introduced by Cirac and Zoller,[4] and the system is based on the laser–ion interaction and possesses long qubit coherence time.[6] In the scheme, quantum information is stored in the internal and external states of an array of trapped cold ions, the trapped ion must have three levels, moreover, the trap operates in the Lamb–Dicke limit (LDL).

Recently, a lot of other schemes have been proposed to modify that scheme. Childs and Chuang[7] have proposed a scheme to perform a universal quantum logic operation between ions with two internal levels in a common trap, but the trap was also in the LDL; Wei *et al*[8] have proposed a modified programme to realize a CQPG between the internal and external states of a trapped cold ion beyond the LDL.

We also notice that performing quantum computing with a system combining an ion trap and a cavity has received more and more attention[9−11] in recent years. In the Refs.[9,10] a CQPG is realized by means of large detuning between the cavity mode and the internal states of the trapped ions. As stated in Ref.[4], two special conditions are required: (a) the ions must have three levels and (b) the trap must be workable in the LDL, i.e. the coupling between the internal spin and external vibrational degrees of freedom of the ion must be sufficiently weak, thus the gate operational time is relatively long. While in Ref.[11] a QPG involving the vibration mode and cavity mode for a trapped ion is implemented by means of lasers and quantized cavity field in the LD limit, where the electronic internal state of the ion is only an assistant qubit.

In this paper, we present a simpler alternative scheme to realize the CQPG beyond the LDL via a single trapped two-level cold ion in a high-Q cavity. In our scheme, neither the LD approximation nor the auxiliary atomic level is needed.

2. Model

We consider a system with a single two-level trapped ion located in a finesse cavity which supports

*Project supported by the National Natural Science Foundation of China (Grant No 10374025).
† Corresponding author. E-mail: mffang@hunnu.edu.cn

a single-mode quantized radiation field with frequency ω, so that the cavity mode couples to both the internal electronic state transition and the vibrational degree of freedom of the ion. The dynamical characteristics of this system have been extensively studied in Refs.[12–16]. As discussed in Refs.[10–12] and references therein, the Hamiltonian corresponding to such a system may be written as(assuming $\hbar = 1$)

$$H = \nu a^+ a + \omega b^+ b + \frac{\omega_0}{2}\sigma_z + g(\sigma_+ + \sigma_-) \times (b^+ + b)\sin[\eta(a^+ + a) + \varphi], \quad (1)$$

where $a^+(a)$ is the creation (annihilation) operator of the centre-of-mass of vibrational motion of the ion with frequency ν, $b^+(b)$ denotes the creation (destruction) operator of photons in the cavity mode of frequency ω, g is the ion–field coupling constant, η is the usual Lamb–Dicke parameter, φ represents the relative position of the centre of mass of the ion to the standing wave, ω_0 is the atomic transition frequency between the internal electronic states $|g\rangle$ and $|e\rangle$, σ_+, σ_- and σ_z are the familiar pseudo-spin operators for the ion.

The Hamiltonian in the interaction picture reads

$$H_{\rm I} = \frac{g}{2{\rm i}}(\sigma_+ {\rm e}^{{\rm i}\omega_0 t} + \sigma_- {\rm e}^{{\rm i}\omega_0 t})(b^+ {\rm e}^{{\rm i}\omega t} + b{\rm e}^{-{\rm i}\omega t}) \times [{\rm e}^{{\rm i}\eta(a^+ {\rm e}^{{\rm i}\nu t} + a{\rm e}^{-{\rm i}\nu t})}{\rm e}^{{\rm i}\varphi} - {\rm e}^{-{\rm i}\eta(a^+ {\rm e}^{{\rm i}\nu t} + a{\rm e}^{-{\rm i}\nu t})}{\rm e}^{-{\rm i}\varphi}]. \quad (2)$$

We consider the case in which the light field is resonant with the kth red-shifted vibrational sideband, i.e. the frequency of the light field is chosen to be $\omega = \omega_0 - k\nu$, where k is a positive integer. After a lengthy algebra and under a rotating wave approximation, this Hamiltonian reduces to

$$H_{\rm I} = g{\rm e}^{-\eta^2/2}\eta^k \sin\Big(\varphi + k\frac{\pi}{2}\Big) \times \Big[\sigma_+ b \sum ({\rm i}\eta)^{2n}\frac{a^{+n}a^{n+k}}{n!(n+k)!} + {\rm H.c.}\Big]. \quad (3)$$

With this Hamiltonian, we can study the realization of the controlled quantum phase gate either between the internal and external states of the ion or between the cavity mode and the vibrational mode of the trapped ion; also, we can generate a nonclassical GHZ state if assisted by a classical laser.

3. Implementation of quantum phase gate

With the Hamiltonian defined by Eq.(3), we can obtain

$$\langle m-k, q-1, e|H_{\rm I}|m, q, g\rangle = \sin\Big(\varphi + k\frac{\pi}{2}\Big) g{\rm e}^{-\eta^2/2}\eta^k \sqrt{\frac{q\cdot m!}{(m-k)!}} \sum_{n=0}^{m-k} \frac{({\rm i}\eta)^{2n}}{(k+n)!} C_{m-k}^n = \Omega_{m-k,q,k}, \quad (4)$$

where q and m are the photon number of the cavity mode and the phonon number of the collective vibrational mode of the ion, respectively. Then, the time evolution of the input states $|m\rangle|q\rangle|g\rangle$ and $|m\rangle|q\rangle|e\rangle$ can be expressed, respectively, as

$$|m\rangle|q\rangle|g\rangle \Rightarrow \begin{cases} |m\rangle|q\rangle|g\rangle, & m < k \ \text{ or } \ q = 0, \\ \cos(\Omega_{m-k,q,k}t)|m\rangle|q\rangle|g\rangle - {\rm i}\sin(\Omega_{m-k,q,k}t)|m-k\rangle|q-1\rangle|e\rangle, & m \geq k, \ \ q \geq 1, \end{cases}$$

$$|m\rangle|q\rangle|e\rangle \Rightarrow \cos(\Omega_{m+k,q+1,k}t)|m\rangle|q\rangle|e\rangle - {\rm i}\sin(\Omega_{m+k,q+1,k}t)|m+k\rangle|q+1\rangle|g\rangle, \quad (5)$$

to implement the CQPG defined by the following operations:

$$|0\rangle_a|g\rangle \to |0\rangle_a|g\rangle, \quad |0\rangle_a|e\rangle \to |0\rangle_a|e\rangle,$$
$$|1\rangle_a|g\rangle \to |1\rangle_a|g\rangle, \quad |1\rangle_a|e\rangle \to -|1\rangle_a|e\rangle,$$

where the subscript a denotes the vibrational mode of the trapped ion, and the number m in $|\cdot\rangle_a$ means the phonon number of the collective vibrational mode.

We set $k = 0$, $q = 0$, and $\varphi = \pi/2$, i.e. the light field is resonant with the atomic transition $|e\rangle \leftrightarrow |g\rangle$, the cavity mode is initially in a vacuum state, and the ion is located at the antinode of the standing wave. According Eq.(5), it is easy to find that

$$|0\rangle_b|0\rangle_a|g\rangle \to |0\rangle_b|0\rangle_a|g\rangle, \quad |0\rangle_b|1\rangle_a|g\rangle \to |0\rangle_b|1\rangle_a|g\rangle,$$
$$|0\rangle_b|0\rangle_a|e\rangle \to \cos(\Omega_{0,1,0}t)|0\rangle_b|0\rangle_a|e\rangle$$

$$-\mathrm{i}\sin(\Omega_{0,1,0}t)|1\rangle_b|0\rangle_a|g\rangle,$$
$$|0\rangle_b|1\rangle_a|e\rangle \rightarrow \cos(\Omega_{1,1,0}t)|0\rangle_b|1\rangle_a|e\rangle$$
$$-\mathrm{i}\sin(\Omega_{1,1,0}t)|1\rangle_b|1\rangle_a|g\rangle,$$

where the subscript b denotes the cavity mode, $\Omega_{0,1,0} = ge^{-\eta^2/2}$, and $\Omega_{1,1,0} = g\mathrm{e}^{-\eta^2/2}(1-\eta^2)$.

If the following conditions are satisfied:

$$\Omega_{0,1,0}t = 2p\pi, \quad \Omega_{1,1,0}t = (2l-1)\pi, \tag{7}$$

where p and l are two positive integers, and $p \geq l$, we can have

$$|0\rangle_b|0\rangle_a|e\rangle \rightarrow |0\rangle_b|0\rangle_a|e\rangle,$$
$$|0\rangle_b|1\rangle_a|e\rangle \rightarrow -|0\rangle_b|1\rangle_a|e\rangle; \tag{8}$$

thus the CQPG defined by Eq.(6) is realized. Here we give some values of the suitable η and its corresponding gate operational scale time gt in Table 1.

Table 1. Some of the suitable values of η and its corresponding gate operational scale time.

η	0.2132	0.31623	0.40825	0.5	0.70711	0.86603	0.91287
gt	72.329	34.72	22.268	16.136	10.359	26.603	43.372

We can see from Table 1 that the scale time is not longer than 72.329 (equivalent to 0.5 ms in case of $g = 2\pi \times 24$ kHz) in each case. Also we depict the relations between the gate operational scale time and the Lamb–Dicke parameter in Figs.1 and 2, in the cases of $l \leq 13$ and $l \geq 12$, respectively. In Figs.1 and 2, those points marked by the same symbol and connected by the same (dotted) line is corresponding to the same value of l, and the value of l increases from right to left. In Fig.1, those points corresponding to $gt > 100$ are not depicted.

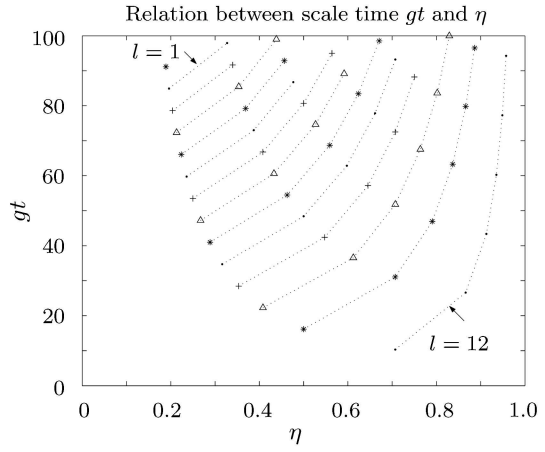


Fig.1. Relation between the gate operational scale time and the parameter η when $l \leq 13$ and $gt \leq 100$.

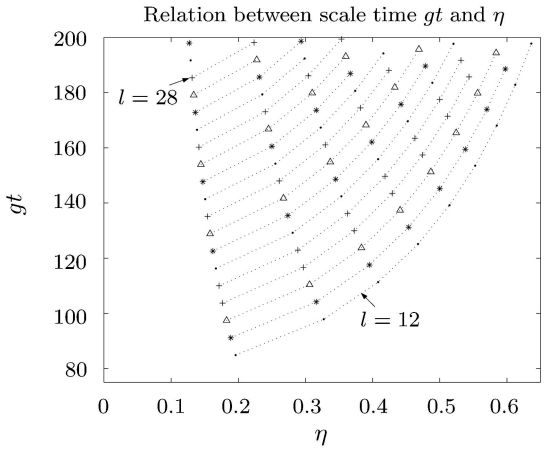


Fig.2. Relation between the gate operational scale time and the parameter η when $l \geq 12$ and $gt \leq 200$.

We can see, from Fig.1 and Fig.2, for a certain value of l, the gate operational scale time increases with the increasing of η, so there is a shortest gate operational scale time for each l, but it increases with the decreasing of η(increasing of l). The scale time 10.359, corresponding to $l = 1$ and $\eta = 0.70711$, is the shortest one among all possible gate operational scale times. Each value of gt in Table 1 is the shortest one for a given η.

When the frequency of the light field is chosen to be resonant with the first red sideband of the collective vibrational mode ($k = 1$ in Eq.(5)) and the ion is located at the node of the standing wave ($\varphi = 0$), we can perform another CQPG between the cavity mode and the vibrational mode of the ion, which is defined as follows:

$$|0\rangle_a|0\rangle_b| \rightarrow |0\rangle_a|0\rangle_b, \quad |0\rangle_a|1\rangle_b| \rightarrow |0\rangle_a|1\rangle_b,$$
$$|1\rangle_a|0\rangle_b \rightarrow |1\rangle_a|0\rangle_b, \quad |1\rangle_a|1\rangle_b \rightarrow -|1\rangle_a|1\rangle_b. \tag{9}$$

According to the time evolution of the input state $|m\rangle|q\rangle|g\rangle$ in Eq.(5), it is easy to find that

$$|0\rangle_a|0\rangle_b|g\rangle \rightarrow |0\rangle_a|0\rangle_b|g\rangle,$$
$$|0\rangle_a|1\rangle_b|g\rangle \rightarrow |0\rangle_a|1\rangle_b|g\rangle,$$
$$|1\rangle_a|0\rangle_b|g\rangle \rightarrow |1\rangle_a|0\rangle_b|g\rangle,$$
$$|1\rangle_a|1\rangle_b|g\rangle \rightarrow \cos(\Omega_{0,1,1}t)|1\rangle_a|1\rangle_b|g\rangle$$
$$-\mathrm{i}\sin(\Omega_{0,1,1}t)|1\rangle_a|0\rangle_b|e\rangle, \tag{10}$$

where $\Omega_{0,1,1} = g\eta\mathrm{e}^{-\eta^2/2}$. At the time defined by

$$\cos(\Omega_{0,1,1}t) = -1, \tag{11}$$

the output state corresponding to the input state $|1\rangle_a|1\rangle_b|g\rangle$ will be $-|1\rangle_a|0\rangle_b|g\rangle$, and the CQPG between the cavity mode and the collective vibrational mode of the ion is then realized.

We also notice that a GHZ state

$$(|m\rangle_a|q\rangle_b|g\rangle + \mathrm{i}|m+k\rangle_a|q+1\rangle_b|e\rangle)/\sqrt{2}$$

can be generated via this system. When the system is initially in the state $|m\rangle_a|q\rangle_b|e\rangle$, at the time defined by $\cos(\Omega_{m+k,q+1,m}t) = 1/\sqrt{2}$, the system will be in the state

$$(|m\rangle_a|q\rangle_b|e\rangle - \mathrm{i}|m+k\rangle_a|q+1\rangle_b|g\rangle)/\sqrt{2}.$$

And then we apply a classical laser coupled to the atomic transition $|g\rangle \leftrightarrow |e\rangle$ with a Rabi frequency $\Omega \gg \Omega_{m+k,q+1,k}$, and after a short time $\tau = \pi/2\Omega$, the system will be in the state $(|m\rangle_a|q\rangle_b|g\rangle + \mathrm{i}|m+k\rangle_a|q+1\rangle_b|e\rangle)/\sqrt{2}$. This is a typical GHZ state.

4. Summary

An easy way of realizing quantum computation in small scale is of particular interest for experimentalists, and also very important for developing a real quantum computer. In this paper, we have proposed a simple scheme of CQPG via a single trapped two-level ion in a single mode high-Q cavity beyond the LDL. When the frequency of the light field is chosen to be resonant with the atomic transition or with the first red sideband of the vibrational mode of the ion, we can achieve a CQPG between the external and internal states of the ion or between the cavity mode and the vibrational mode of the ion beyond the LDL. Neither the Lamb–Dicke approximation nor the assistant classical laser is needed, and the gate operational speed is relatively high. Since the techniques of coupling between the cavity mode and a single trapped ion have been developed,[17,18] our scheme can hopefully be realized.

References

[1] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
[2] Domokos P, Raimond J M, Brune M and Haroche S 1995 *Phys. Rev.* A **52** 3554
[3] Cai J W, Fang M F, Liao X P and Zheng X J 2006 *Chin. Phys.* **15** 2518
[4] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74** 4091
[5] Li L X and Guo G C 1999 *Phys. Rev.* A **60** 696
[6] Schmit K F 2000 *J. Mod. Opt.* **47** 2573
[7] Childs A M and Chuang Isaac L 2001 *Phys. Rev.* A **63** 012306
[8] Wei L F, Liu S Y and Lei X L 2002 *Phys. Rev.* A **65** 062316
[9] Jane E, Plenio M B and D Jonathan, 2002 *Phys. Rev.* A **65** 050302
[10] Pachos J and Walther H 2001 e-print quant-ph/0111088
[11] Zou X B, Pahlke K and Mathis W 2002 *Phys. Rev.* A **65** 064303
[12] Wu Y and Yang X X 1997 *Phys. Rev. Lett.* **78** 3086
[13] Semião F L, Vidiella-Barranco A and Roversi J A 2001 *Phys. Rev.* A **64** 024305
[14] Liu X and Fang M F 2002 *Chin. Phys.* **11** 635
[15] Liu X and Fang M F 2003 *Chin. Phys.* **12** 753
[16] Fang M F, Swain S and Zhou P 2001 *Phys. Rev.* A **63** 013812
[17] Mundt A B, Kreuter A, Becher C, Leibfried D, Eschner J, Schmidt-kaler F and Blatt R 2002 *Phys. Rev. Lett.* **89** 103001
[18] Guthohrlein G R, Keller M, Lange W and Walther H 2001 *Nature* (London) **414** 49