Vol 16 No 6, June 2007
1009-1963/2007/16(06)/1559-07

Chinese Physics

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Creating unconventional geometric phase gate using squeeze-like operator assisted by strong driving field*

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(Received 24 August 2006; revised manuscript received 4 January 2007)

Based on the idea that a squeezing process can be thought of as a total cumulative effect of a large number of tiny squeezing processes, we define a squeeze-like operator with a time-dependent squeeze parameter. Applying this operator to and combining with a system which includes a two-photon interaction between two atoms and an initial vacuum cavity field, and resorting to a resonant strong driving classical field, we obtain an unconventional geometric phase gate with a shorter gating time.

Keywords: squeeze-like operator, quantum phase gate, strong driving field
PACC: 0365, 4250

1. Introduction

As a basic computation unit of quantum computer, a quantum logic gate has been investigated extensively in recent years.[1−9] There are two methods to construct a quantum logic gate primarily, one is based on dynamic evolution[5−12] and the other is based on geometric operation.[13−16] The former has a shortcoming that it is sensitive to the parameter fluctuations in operation, while the latter provides a promising approach to achieving built-in fault-tolerant quantum gates.[15] The geometric gate possesses the advantage that the phase is determined only by the path area, insensitive to the starting state distributions, the path shape, and the passage rate to traverse the closed path.[17] Furthermore, the fidelity of the geometric gate might be higher than that of the dynamic gate.[14] Therefore, the geometric gate has become a primary object of the research. So far, two kinds of geometric quantum gates have been investigated which are obtained, respectively, by using adiabatic cyclic evolutions[18,19] and a displacement operator along a closed path conditional on the state of the qubits.[14−16,20] They are called the conventional geometric gate and unconventional geometric gate, respectively. Particularly, the latter has attracted much attention in that it provides a method of fault-tolerant quantum computation. We have noted that in the phase space, a squeeze vacuum state can be achieved by a squeeze operator acting on a vacuum state. From the viewpoint of geometry, this is equivalent to a change from a circle to an ellipse. Moreover, a phase shift can also be obtained during this change. However, most researches about the geometric phase gate are devoted to the displacement operator.[14−16,20] Very recently, Chen and Gao[21] and Joshi and Xiao[22] have proposed schemes to construct an unconventional geometric logic gate in cavity quantum electrodynamics (QED) using a squeeze operator. Their schemes are based on the large-detuning interaction between the atom and the cavity. In this work, by defining a squeeze-like operator, we first give a clearer physical picture for the process of phase gate using a squeeze operator. Then a geometric phase gate is created by using this squeeze-like operator. Different from the situation in Refs.[21,22], in our scheme the atoms resonantly interact with a strong driving classical field (while the situation in the above references is based on the large detuning interaction), thus a shorter gating time is achieved. Moreover, the

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geometric phase gate under the cavity decay is also considered.

The remainder of this paper is organized as follows. In Section 2, we present the definition of the squeeze-like operator and its theoretic elements in quantum optics. In Section 3, we describe the dynamic model in which two atoms interact with the cavity through a two-photon process, by resorting to a resonant strong driving classical field. In Section 4 and Section 5, the two-qubit quantum phase gates under an ideal situation and under cavity decay are obtained, respectively. Finally, a discussion and summary are given in Section 6.

2. Definition of squeeze-like operator

An unitary squeeze operator in quantum optics is written as

$$S(\xi) = \exp\left\{\frac{1}{2}\xi^{\star}a^{2} - \frac{1}{2}\xi a^{\dagger 2}\right\}, \tag{1}$$

where $\xi = r\mathrm{e}^{\mathrm{i}\theta}$ is an arbitrary complex number and $|\xi| = r$ represents the degree of squeezing. In this work, ξ is a time-dependent parameter. The squeeze operator has the property

$$S^{\dagger}(\xi) = S^{-1}(\xi) = S(-\xi). \tag{2}$$

A squeezed vacuum state can be obtained by the squeeze operator $S(\xi)$ acting on a vacuum state. In the phase space of two orthogonal components X_1 and X_2 of the field, the relation between the vacuum state $|\Psi\rangle_{\mathrm{vac}}$ and the squeezed vacuum state $|\Psi\rangle_{\mathrm{sv}}$ is usually described by a figure as shown in Fig.1, where the circle and the ellipse denote the error circles of the vacuum state $|\Psi\rangle_{\mathrm{vac}}$ and of the squeezed vacuum state $|\Psi\rangle_{\mathrm{sv}}$, respectively. The error circle of $|\Psi\rangle_{\mathrm{vac}}$ is squeezed into an error ellipse of the same area. The principal axes of the ellipse lie along Y_1 and Y_2 are rotated at an angle of $\theta/2$ from the original axes X_1 and X_2, respectively.

From Eq.(1) the squeeze operator satisfies the following relation after the higher-order terms have been neglected in the expansion under the condition of $|\xi_i| \ll 1$:

$$\begin{aligned} & S(\xi_2)S(\xi_1) \\ \simeq\ & S(\xi_2+\xi_1)\exp\left\{\frac{\mathrm{i}}{2}\mathrm{Im}(\xi_2\xi_1^{\star})(aa^{\dagger}+a^{\dagger}a)\right\}. \end{aligned} \tag{3}$$

Now based on the above knowledge, we regard a squeezing process from $|\Psi\rangle_{\mathrm{vac}}$ to $|\Psi\rangle_{\mathrm{sv}}$ as a total result of many small processes of squeezing which occur one by one during some equal intervals Δt_j. Among these small processes of squeezing, either of the orthogonal components with an arbitrary squeezing angle θ has a possibility to be squeezed.

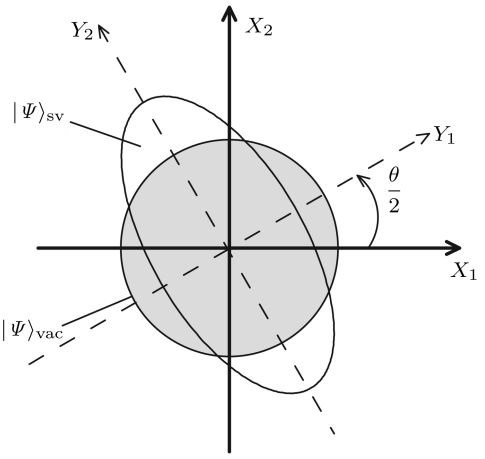


Fig.1. A squeezed vacuum state $|\Psi\rangle_{\mathrm{sv}}$ can be created by a squeeze operator acting on a vacuum state $|\Psi\rangle_{\mathrm{vac}}$.

Such a squeezing process is realized by the following series of operators acting on an initial vacuum state:

$$\begin{aligned} S_t &= S_t(\Delta\xi_N)S_t(\Delta\xi_{N-1})\cdots S_t(\Delta\xi_1) \\ &\simeq S_t\left(\sum_{j=1}^{N}\Delta\xi_j\right)\exp\left\{\frac{\mathrm{i}}{2}\mathrm{Im}\left[\sum_{j=2}^{N}\left(\Delta\xi_j\sum_{k=1}^{j-1}\Delta\xi_k^{\star}\right)\right]\right. \\ &\quad \left.\times(aa^{\dagger}+a^{\dagger}a)\right\}. \end{aligned} \tag{4}$$

In the above equation, Eq.(3) has been used, and $\Delta\xi_j$ denotes the tiny squeezing parameter of the jth small process. For an initial vacuum state of the field, the term corresponding to $a^{\dagger}a$ in Eq.(4) makes no contribution to the evolution of the system. Therefore, when $N \to \infty$ within process p of a definite time, the operator S_t is

$$S_t = S_t\left(\int_p \mathrm{d}\xi\right)\mathrm{e}^{\mathrm{i}\Theta}, \tag{5}$$

where

$$\Theta = \frac{1}{2}\mathrm{Im}\int_p \xi^{\star}\mathrm{d}\xi. \tag{6}$$

If the field evolves back to the initial vacuum state again (which we call an evolution period) after a series of infinitesimal squeezing processes within infinitesimal time intervals, from Eq.(1), Eq.(5) can be written

as

$$S_t = S_t(0)\mathrm{e}^{\mathrm{i}\Theta} = \mathrm{e}^{\mathrm{i}\Theta},$$
$$\Theta = \frac{1}{2}\mathrm{Im}\oint_T \xi^\star \mathrm{d}\xi. \quad (7)$$

Now we extend the above idea to defining a squeeze-like operator S_t^{L}. Because the squeeze operator has the property described by Eq.(2), we define a squeeze-like operator as

$$S^{\mathrm{L}}(\xi) = \exp\left\{\frac{1}{2}\xi(-t)a^2 - \frac{1}{2}\xi(t)a^{\dagger 2}\right\}, \quad (8)$$

where $\xi(t)$ is a time-dependent complex parameter, and $\xi(-t) = \xi^*$ in order to satisfy Eq.(2). We obtain the following total operator for a squeezing process consisting of N small processes of squeezing (we have assumed that the field is initially in a vacuum state), which is similar to Eq.(4):

$$\begin{aligned} S_t^{\mathrm{L}} &= S^{\mathrm{L}}(\Delta\xi_N)S^{\mathrm{L}}(\Delta\xi_{N-1})\cdots S^1(\Delta\xi_1) \\ &\simeq S^{\mathrm{L}}\left(\sum_{j=1}^{N}\Delta\xi_j\right)\exp\left\{\frac{1}{4}\left[\sum_{j=2}^{N}\Delta\xi_j(t)\right.\right. \\ &\quad \left.\left.\times\sum_{k=1}^{j-1}\Delta\xi_k(-t) - \sum_{j=2}^{N}\Delta\xi_j(-t)\sum_{k=1}^{j-1}\Delta\xi_k(t)\right]\right\}. \quad (9) \end{aligned}$$

In the limit of $N \to \infty$, we have

$$\begin{aligned} S_t^{\mathrm{L}} &= S^{\mathrm{L}}\left(\int_p \mathrm{d}\xi(t)\right)\mathrm{e}^{-A}\mathrm{e}^{\mathrm{i}\Theta}, \\ \Theta &= \frac{1}{4}\mathrm{Im}\left[\int_p \xi(-t)\mathrm{d}\xi(t) - \int_p \xi(t)\mathrm{d}\xi(-t)\right], \\ A &= -\frac{1}{4}\mathrm{Re}\left[\int_p \xi(-t)\mathrm{d}\xi(t) - \int_p \xi(t)\mathrm{d}\xi(-t)\right]. \quad (10) \end{aligned}$$

For an evolution period T, the above equation turns into the following form:

$$\begin{aligned} S_t^{\mathrm{L}} &= S^{\mathrm{L}}(0)\mathrm{e}^{-A}\mathrm{e}^{\mathrm{i}\Theta} = \mathrm{e}^{-A}\mathrm{e}^{\mathrm{i}\Theta}, \\ \Theta &= \frac{1}{4}\mathrm{Im}\left[\oint_T \xi(-t)\mathrm{d}\xi(t) - \oint_T \xi(t)\mathrm{d}\xi(-t)\right], \\ A &= -\frac{1}{4}\mathrm{Re}\left[\oint_T \xi(-t)\mathrm{d}\xi(t) - \oint_T \xi(t)\mathrm{d}\xi(-t)\right]. \quad (11) \end{aligned}$$

The operator S_t^{L} defined here and Eqs.(10) and (11) are used to create a quantum phase gate. It will be seen that this method is applicable not only to a system including a cavity without loss, but also to one including a cavity with loss.

3. Dynamic model of a cavity QED system consisting of two-photon process and a strong driving field

Consider a system in which two identical three-level atoms each with an excited state $|i\rangle$ and two ground states $|e\rangle$ and $|g\rangle$, interact with a single-mode cavity which is in the vacuum state initially, through a two-photon process, assisted by an external classical driving field as shown in Fig.2.

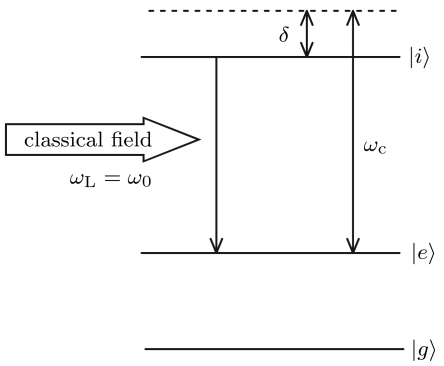


Fig.2. A model of two three-level atoms interacting with a cavity field through a two-photon process, assisted by a classical driving field.

The transition $|e\rangle \leftrightarrow |i\rangle$ of the atom is driven by the single-mode cavity field with a coupling constant g and detuning δ, and the driving field is resonant to this transition with Rabi frequency Ω. The level $|g\rangle$ is an auxiliary state. The quantum information is encoded in the states $|g\rangle$ and the dress state $|-\rangle = 1/\sqrt{2}(|e\rangle - |i\rangle)$. The Hamiltonian of the system is expressed as

$$\begin{aligned} H = \ & \omega_0\sum_{j=1}^{2}S_{z,j} + \omega_{\mathrm{c}}a^\dagger a + g\sum_{j=1}^{2}\left(a^{\dagger 2}S_j^- + a^2 S_j^+\right) \\ & + \Omega\sum_{j=1}^{2}\left(\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{L}}t}S_j^+ + \mathrm{e}^{\mathrm{i}\omega_{\mathrm{L}}t}S_j^-\right), \ (\hbar = 1), \quad (12) \end{aligned}$$

where $S_{z,j} = \frac{1}{2}(|i\rangle_{jj}\langle i| - |e\rangle_{jj}\langle e|)$, $S_j^+ = |i\rangle_{jj}\langle e|$, and $S_j^- = |e\rangle_{jj}\langle i|$ are the atomic spin operators for population, rasing and lowering, respectively; $a^\dagger(a)$ is the creation (annihilation) operator of the cavity field; ω_0, ω_{c} and ω_{L} are the frequencies of the transition $|e\rangle \leftrightarrow |i\rangle$ of the atoms, cavity mode and classical field, respectively. Performing a rotation with respect to the classical field ω_{L}, the Hamiltonian of Eq.(12) can

be written as

$$H_{\rm r}=\delta a^{\dagger}a+g\sum_{j=1}^{2}\left(a^{\dagger 2}S_j^{-}+a^2S_j^{+}\right)+\Omega\sum_{j=1}^{2}\left(S_j^{+}+S_j^{-}\right), \tag{13}$$

where $\delta=\omega_{\rm c}-\omega_0$. In order to acquire the effective Hamiltonian, we define

$$\begin{aligned} H_{\rm r}&=H_{\rm r0}+H_{{\rm r}i},\\ H_{\rm r0}&=\delta a^{\dagger}a+\Omega\sum_{j=1}^{2}\left(S_j^{+}+S_j^{-}\right),\\ H_{{\rm r}i}&=g\sum_{j=1}^{2}\left(a^{\dagger 2}S_j^{-}+a^2S_j^{+}\right), \end{aligned} \tag{14}$$

and perform the following transformation:

$$H_i=\mathrm{e}^{\mathrm{i}H_{\rm r0}t}H_{{\rm r}i}\mathrm{e}^{-\mathrm{i}H_{\rm r0}t}. \tag{15}$$

Then the effective Hamiltonian of the system in the strong driving regime $\Omega\gg g,\delta$ and the rotating-wave approximation is obtained as

$$\begin{aligned} H_{\rm eff}=\ & \frac{g}{2}\sum_{j=1}^{2}\left(|+\rangle_{jj}\langle+|-|-\rangle_{jj}\langle-|\right)\\ &\times\left(a^2\mathrm{e}^{-2\mathrm{i}\delta t}+a^{\dagger 2}\mathrm{e}^{2\mathrm{i}\delta t}\right), \end{aligned} \tag{16}$$

where $|\pm\rangle_j=1/\sqrt{2}(|e\rangle_j\pm|i\rangle_j)$. In Eq.(16) the term corresponding to the state $|+\rangle_j$ makes no contribution to the phase gate to be built because the state $|+\rangle_j$ is orthogonal to both $|-\rangle_j$ and $|g\rangle_j$. Hence, we write the Hamiltonian related to the phase gate as

$$H_{\rm effg}=-\frac{g}{2}\sum_{j=1}^{2}|-\rangle_{jj}\langle-|\left(a^2\mathrm{e}^{-2\mathrm{i}\delta t}+a^{\dagger 2}\mathrm{e}^{2\mathrm{i}\delta t}\right). \tag{17}$$

4. Ideal two-qubit unconventional quantum phase gate created by the squeeze-like operator

Now we calculate the time-dependant squeezing parameter $\xi(t)$ according to this model. Considering the definition of the squeeze-like operator in Eq.(8), during an infinitesimal interval t–$t+\mathrm{d}t$, the corresponding state evolution is determined by the operator

$$S^{\rm L}(\mathrm{d}\xi)=\mathrm{e}^{\frac{1}{2}\left[\mathrm{d}\xi(-t)a^2-\mathrm{d}\xi(t)a^{\dagger 2}\right]}=\mathrm{e}^{-\mathrm{i}H_{\rm effg}\mathrm{d}t}. \tag{18}$$

It can be seen from Eq.(18) that the state of the system, which includes only a single state $|-\rangle_j$, will evolve according to $S^{\rm L}(\mathrm{d}\xi)$, while those that include l states $|-\rangle_j$ $(1\leq l\leq N)$ will evolve according to the operator $S^{\rm L}(l\mathrm{d}\xi)$. From Eqs.(17) and (18), we obtain the expression for the system state including a single state $|-\rangle_j$

$$\mathrm{d}\xi(t)=-\mathrm{i}g\mathrm{e}^{2\mathrm{i}\delta t}\mathrm{d}t, \tag{19}$$

and the expression for those including l states $|-\rangle_j$

$$\mathrm{d}\xi_l(t)=-\mathrm{i}gl\mathrm{e}^{2\mathrm{i}\delta t}\mathrm{d}t. \tag{20}$$

After an interaction time t, the time-dependant squeezing parameters $\xi(t)$ and $\xi_l(t)$ are obtained as follows:

$$\xi(t)=\int_0^t(-\mathrm{i}g)\mathrm{e}^{2i\delta\tau}\mathrm{d}\tau=\frac{g}{2\delta}\left(1-\mathrm{e}^{2\mathrm{i}\delta t}\right), \tag{21}$$

and

$$\xi_l(t)=l\xi(t). \tag{22}$$

Hence, in terms of Eqs.(10), (21) and (22), the phase shift Θ (for single qubit state $|-\rangle_j$) and Θ_l (for l states $|-\rangle_j$) are solved as

$$\Theta=\frac{g^2}{4\delta^2}\left(\delta t-\frac{1}{2}\sin 2\delta t\right), \tag{23}$$

$$\Theta_l=l^2\Theta. \tag{24}$$

In addition, $A=0$ and $A_l=0$ are obtained. We have noted from Eqs.(21) and (22) that the condition for the system to undergo a periodical evolution is

$$\delta t=m\pi,\qquad (m=1,2,3,\cdots). \tag{25}$$

The time $t=m\pi/\delta$ is just the period of the quantum phase gate. Considering Eqs.(10) and (14), the total evolution operator is as follows:

$$U(t)=\mathrm{e}^{-\mathrm{i}H_{\rm r0}t}S_t^{\rm L}. \tag{26}$$

According to Eqs.(11), (14) and (26), we obtain the following evolutions after an evolution period $t=m\pi/\delta$:

$$\begin{aligned} |g\rangle_1|g\rangle_2|0\rangle\ &\longrightarrow|g\rangle_1|g\rangle_2|0\rangle,\\ |g\rangle_1|-\rangle_2|0\rangle\ &\longrightarrow\mathrm{e}^{\mathrm{i}\Omega t}S^{\rm L}(0)\mathrm{e}^{\mathrm{i}\Theta}|g\rangle_1|-\rangle_2|0\rangle\\ &=\mathrm{e}^{\mathrm{i}(\Theta+\Omega t)}|g\rangle_1|-\rangle_2|0\rangle,\\ |-\rangle_1|g\rangle_2|0\rangle\ &\longrightarrow\mathrm{e}^{\mathrm{i}\Omega t}S^{\rm L}(0)\mathrm{e}^{\mathrm{i}\Theta}|-\rangle_1|g\rangle_2|0\rangle\\ &=\mathrm{e}^{\mathrm{i}(\Theta+\Omega t)}|-\rangle_1|g\rangle_2|0\rangle,\\ |-\rangle_1|-\rangle_2|0\rangle\ &\longrightarrow\mathrm{e}^{2\mathrm{i}\Omega t}S^{\rm L}(0)\mathrm{e}^{\mathrm{i}\Theta_2}|-\rangle_1|-\rangle_2|0\rangle\\ &=\mathrm{e}^{\mathrm{i}(4\Theta+2\Omega t)}|-\rangle_1|-\rangle_2|0\rangle. \end{aligned} \tag{27}$$

By performing a single-qubit operation $|-\rangle_j \longrightarrow \mathrm{e}^{-\mathrm{i}(\Theta+\Omega t)}|-\rangle_j$, we obtain the following results:

$$\begin{aligned}
|g\rangle_1|g\rangle_2|0\rangle &\longrightarrow |g\rangle_1|g\rangle_2|0\rangle,\\
|g\rangle_1|-\rangle_2|0\rangle &\longrightarrow |g\rangle_1|-\rangle_2|0\rangle,\\
|-\rangle_1|g\rangle_2|0\rangle &\longrightarrow |-\rangle_1|g\rangle_2|0\rangle,\\
|-\rangle_1|-\rangle_2|0\rangle &\longrightarrow \mathrm{e}^{\mathrm{i}2\Theta}|-\rangle_1|-\rangle_2|0\rangle,
\end{aligned} \tag{28}$$

which is a 2Θ-phase gate, of which the gating time is determined by Eqs.(23) and (25). For $2\Theta=\pi$, we obtain a π-phase gate, the gating time is obtained from Eqs.(23) and (25) as

$$t_{\mathrm{ideal}} = \frac{\sqrt{2m}\pi}{g}. \tag{29}$$

5. Two-qubit quantum phase gate under the cavity decay

In this section we consider the effect of the cavity decay on the quantum phase gate by use of the squeeze-like operator.

The Hamiltonian of the system when the cavity decay is considered is as follows:

$$\begin{aligned}
H' =\ & \omega_0\sum_{j=1}^{2}S_{z,j}+\omega_{\mathrm{c}}a^{\dagger}a+g\sum_{j=1}^{2}\left(a^{\dagger 2}S_j^{-}+a^2S_j^{+}\right)\\
&+\Omega\sum_{j=1}^{2}\left(\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{L}}t}S_j^{+}+\mathrm{e}^{\mathrm{i}\omega_{\mathrm{L}}t}S_j^{-}\right)-\mathrm{i}\frac{\kappa}{2}a^{\dagger}a,
\end{aligned} \tag{30}$$

where κ is the cavity decay rate. By the same steps as those in Section 3, the effective Hamiltonian under the strong driving regime $\Omega \gg g,\delta$ and the rotating-wave approximation is obtained as

$$\begin{aligned}
H'_{\mathrm{eff}} =\ & \frac{g}{2}\sum_{j=1}^{2}\left(|+\rangle_{jj}\langle+|-|-\rangle_{jj}\langle-|\right)\left(a^2\mathrm{e}^{-2\mathrm{i}\delta t}\mathrm{e}^{-\kappa t}\right.\\
&\left.+a^{\dagger 2}\mathrm{e}^{2\mathrm{i}\delta t}\mathrm{e}^{\kappa t}\right),
\end{aligned} \tag{31}$$

in which the terms contributing an unconventional phase shift to the evolution of the qubit states are

$$\begin{aligned}
H'_{\mathrm{effg}} =\ & -\frac{g}{2}\sum_{j=1}^{2}|-\rangle_{jj}\langle-|\left(a^2\mathrm{e}^{-2\mathrm{i}\delta t}\mathrm{e}^{-\kappa t}\right.\\
&\left.+a^{\dagger 2}\mathrm{e}^{2\mathrm{i}\delta t}\mathrm{e}^{\kappa t}\right).
\end{aligned} \tag{32}$$

So by use of the same method as that for Eqs.(18)–(22), we obtain the time-dependent squeezing parameter $\xi'(t)(\xi'_l(t))$ under the cavity decay as

$$\xi'(t) = \frac{2g\delta+\mathrm{i}g\kappa}{4\delta^2+\kappa^2}\left(1-\mathrm{e}^{2\mathrm{i}\delta t}\mathrm{e}^{\kappa t}\right), \tag{33}$$

and

$$\xi'_l(t) = l\xi'(t). \tag{34}$$

According to Eq.(10), we have the following expressions:

$$\begin{aligned}
\Theta' =\ & \frac{g^2}{(4\delta^2+\kappa^2)^2}\left[\left(4\delta^2-\kappa^2\right)\left(\delta t-\frac{1}{2}\sin 2\delta t\cosh\kappa t\right)\right.\\
&\left.+2\delta\kappa\left(\kappa t-\cos 2\delta t\sinh\kappa t\right)\right],
\end{aligned}$$

$$\Theta'_l = l^2\Theta, \tag{35}$$

$$\begin{aligned}
A' =\ & \frac{g^2}{(4\delta^2+\kappa^2)^2}\left[\left(4\delta^2-\kappa^2\right)\left(\frac{1}{2}\cos 2\delta t\sinh\kappa t\right.\right.\\
&\left.\left.-\frac{\kappa t}{2}\right)+2\delta\kappa\left(2\delta t-\sin 2\delta t\cosh\kappa t\right)\right],
\end{aligned}$$

$$A'_l = l^2A'. \tag{36}$$

It is seen from Eqs.(33) and (34) that the time of a periodical evolution for the system satisfies the condition

$$\delta t = \pi m \qquad (\kappa \ll \delta, \quad \text{and} \quad m=1,2,3,\cdots). \tag{37}$$

In a way similar to that in Sec.4, we obtain the following evolutions after the time $t=\pi m/\delta$:

$$\begin{aligned}
|g\rangle_1|g\rangle_2|0\rangle &\longrightarrow |g\rangle_1|g\rangle_2|0\rangle,\\
|g\rangle_1|-\rangle_2|0\rangle &\longrightarrow \mathrm{e}^{-\mathrm{i}H'_{\mathrm{r}0}t}S_t^{\mathrm{L}}(\xi')|g\rangle_1|-\rangle_2|0\rangle\\
&= \mathrm{e}^{-A'}\mathrm{e}^{\mathrm{i}(\Theta'+\Omega t)}|g\rangle_1|-\rangle_2|0\rangle,\\
|-\rangle_1|g\rangle_2|0\rangle &\longrightarrow \mathrm{e}^{-\mathrm{i}H'_{\mathrm{r}0}t}S_t^{\mathrm{L}}(\xi')|-\rangle_1|g\rangle_2|0\rangle\\
&= \mathrm{e}^{-A'}\mathrm{e}^{\mathrm{i}(\Theta'+\Omega t)}|-\rangle_1|g\rangle_2|0\rangle,\\
|-\rangle_1|-\rangle_2|0\rangle &\longrightarrow \mathrm{e}^{-\mathrm{i}H'_{\mathrm{r}0}t}S_t^{\mathrm{L}}(\xi')|-\rangle_1|-\rangle_2|0\rangle\\
&= \mathrm{e}^{-A'_2}\mathrm{e}^{\mathrm{i}(\Theta'_2+2\Omega t)}|-\rangle_1|-\rangle_2|0\rangle\\
&= \mathrm{e}^{-4A'}\mathrm{e}^{\mathrm{i}(4\Theta'+2\Omega t)}|-\rangle_1|-\rangle_2|0\rangle.
\end{aligned} \tag{38}$$

After a single-qubit operation $|-\rangle_j \longrightarrow \mathrm{e}^{-\mathrm{i}(\Theta'+\Omega t)}|-\rangle_j$, we obtain a $2\Theta'$-phase gate under the cavity decay

$$\begin{aligned}
|g\rangle_1|g\rangle_2|0\rangle &\longrightarrow |g\rangle_1|g\rangle_2|0\rangle,\\
|g\rangle_1|-\rangle_2|0\rangle &\longrightarrow \mathrm{e}^{-A'}|g\rangle_1|-\rangle_2|0\rangle,\\
|-\rangle_1|g\rangle_2|0\rangle &\longrightarrow \mathrm{e}^{-A'}|-\rangle_1|g\rangle_2|0\rangle,\\
|-\rangle_1|-\rangle_2|0\rangle &\longrightarrow \mathrm{e}^{-4A'}\mathrm{e}^{2\mathrm{i}\Theta'}|-\rangle_1|-\rangle_2|0\rangle.
\end{aligned} \tag{39}$$

In the above equation, $2\Theta'=\pi$ yields a π-phase gate under the cavity decay, and the corresponding gating time can be calculated from Eq.(37) and the following condition:

$$\begin{aligned}
&\frac{g^2}{(4\delta^2+\kappa^2)^2}\left[\left(4\delta^2-\kappa^2\right)\left(\delta t-\frac{1}{2}\sin 2\delta t\cosh\kappa t\right)\right.\\
&\left.+2\delta\kappa\left(\kappa t-\cos 2\delta t\sinh\kappa t\right)\right]\\
=\ & \frac{\pi}{2}.
\end{aligned} \tag{40}$$

Obviously, Eqs.(35), (36), (39) and (40) will return to the corresponding equations for the ideal situation in which $\kappa = 0$.

6. Discussion and summary

Firstly, it is necessary to account for the restrictive condition of our scheme. We can give the squeezing degree $|\xi'(t)|$ vs both t and the ratio of κ/δ according to Eq.(33) as shown in Fig.3 where we have assumed $g/\delta = 1$. From this figure we can see that in an ideal situation in which $\kappa = 0$ the squeeze parameter exhibits a standard periodical evolution. However, with the increase of ratio κ/δ, the evolution of $|\xi'|$ gradually deviates from the periodical variation. This indicates that after a period of $m\pi/\delta$, the cavity is unable to return to the initial state precisely because the decay of the cavity increases, i.e. the quantum jump has occurred. In order to avoid this situation, our scheme is restricted to the condition of $\kappa \ll \delta$. In addition, we should point out that an initial vacuum cavity field is required in our present scheme because of the feature of the squeeze operator in Eq.(4).

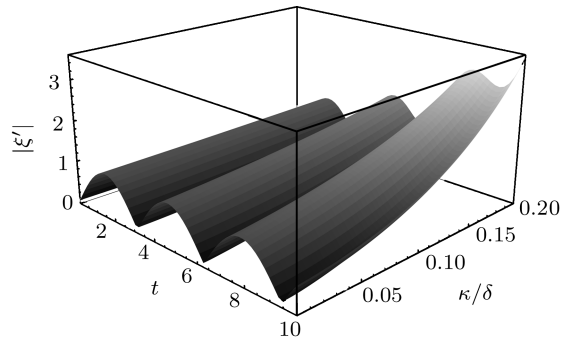


Fig.3. Evolution of the squeeze parameter with both time t and ratio κ/δ, with $g/\delta = 1$ assumed.

Secondly, we should discuss the experimental feasibility of our scheme. The classical driving field in our scheme must be restricted to being a microwave field. Then the levels $|i\rangle, |e\rangle$ and $|g\rangle$ correspond to the levels of a Rydberg atom with the quantum numbers 51, 50 and 49, respectively. According to the data from Ref.[2], the lifetime of the excited level of a Rydberg atom is $T_{\rm r} \simeq 3\times10^{-2}\,$s, the photon lifetime in the cavity is $T_{\rm c} \simeq 3\times10^{-3}\,$s, and the coupling strength of a two-photon process can be taken as $g \simeq 2\pi KH$ (about 1/50 of the coupling strength of the single-photon process). So the gating time for $m = 1$ of our scheme from Eq.(29) is $t_{\rm ideal} = \sqrt{2}\pi/g \simeq 7.0711\times10^{-4}\,$s. It is much shorter than $T_{\rm r}$ and $T_{\rm c}$. If the cavity decay is considered, we can take $\kappa = g/27$.[2] In this case we have $\kappa/\delta \simeq 0.0524$ from Eqs.(37) and (40), so the condition $\kappa \ll \delta$ is satisfied approximately. Then the gating time is calculated as $t_\kappa \simeq 1.41568\pi/g \simeq 7.0784\times10^{-4}\,$s, which is also much shorter than $T_{\rm r}$ and $T_{\rm c}$. The gating time is prolonged by only about 0.104% because of the cavity decay. It can be seen that this time is much shorter than that of the situation of large-detuning.[21,22] But the disadvantage of the present scheme is that it does not work if an optical cavity of laser is used according to relevant experimental data.[3]

In summary, we have given a clear physical description of the geometric phase gate using a squeeze-like operator. This squeeze-like operator possesses the property similar to that of the squeeze operator, but the squeeze parameter is time-dependent. Furthermore, by this operator acting on a QED system which includes a two-photon interaction between the atoms and an initial vacuum cavity field assisted by a resonant strong driving field, an unconventional geometric phase-gate is obtained. By applying a resonant classical driving field, we have obtained a shorter gating time even if in the case of cavity decay. To ensure the validity of the scheme, the condition $\kappa \ll \delta$ must be satisfied. We believe that this scheme provides a new way of exploring an unconventional geometric phase gate.

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