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# Squeezing properties of a trapped ion in the running-wave laser beyond the Lamb–Dicke limit*

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Beyond the Lamb–Dicke limit, this paper investigates the squeezing properties of the trapped ion in the travelling-wave laser. It shows that the squeezing properties of the trapped ion in the travelling-wave laser are strongly affected by the sideband number $k$, the Lamb–Dicke parameter $\eta$ and the initial average phonon number.

**Keywords:** squeezing properties, trapped ion, travelling-wave laser, beyond Lamb–Dicke limit
**PACC:** 0365, 4250

## 1. Introduction

The investigation of trapped ions manipulated by laser beams has attracted considerable attention in recent years.[1] This is not only due to the fundamental physics involved, but also because of potential applications, such as generations of macroscopic quantum states of a single trapped ion: the even/odd coherent states,[2] precision spectroscopy,[3] quantum computation,[4] and quantum state reconstruction.[5,6]

Some authors have extensively investigated many dynamical properties of the trapped ion system. In Ref.[7] Liu has studied the dynamics of a trapped ion placed at an antinode of the standing wave inside a high finesse cavity with consideration of the second sideband excitation between the ionic internal levels and the light field. Also Svandal and Hansen [8] considered the collapse and revival dynamics of two entangled trapped ions. In addition, many schemes for generation of quantum states have been proposed, for example, the motional thermal, Fock, coherent, and squeezed states of a harmonically bound ion have been generated experimentally.[6,9,10]

In general, what has been achieved so far concerns most of the dynamics of trapped ions and generation of states; less attention has been paid to the squeezing properties of these ions, especially beyond Lamb–Dicke limit. In fact, the squeezing of light and the atom have potential applications in high-precision atomic fountain clocks,[11] high-precision spin polarization measurements,[12] and the generation of squeezed light.[13] Previously, Fang and Liu[14] studied the squeezing properties of trapped ions in a standing-wave laser by transforming the trapped ion system to the Jaynes–Cummings type system under the Lamb–Dicke limit. The results show that the squeezing of a trapped ion depends on its position in the latter, the detuning parameter and the initial average phonon number. In this paper, we investigate the squeezing properties of a trapped ion driven by travelling-wave laser beyond the Lamb–Dicke limit.

## 2. The model

We consider a single two-level ion stored in a coaxial resonator RF ion trap with $\omega_x \ll \omega_{y,z}$. Only the quantized vibrational motion along the $x$ direction is considered for the cooled ion.[15] The trapped ion is driven by a classical travelling-wave light field of frequency $\omega_{\rm L}$ and initial phase $\phi_{\rm L}$. In the rotating-wave approximation, the corresponding Hamiltonian can be described as[16] (assuming $\hbar = 1$)

$$H = \omega\left(\hat{a}^{+}\hat{a} + \frac{1}{2}\right) + \frac{\delta}{2}\hat{\sigma}_z + \frac{\Omega}{2}\{\hat{\sigma}_{+}\exp[\mathrm{i}\eta(\hat{a} + \hat{a}^{+}) - \mathrm{i}\phi_L] + \mathrm{H.c.}\}, \quad (1)$$

where $\eta$ is the Lamb–Dicke parameter, which describes the strength of coupling between the internal and external degrees of freedom of the trapped ion. $\hat{a}^{+}$and $\hat{a}$ are the creation and annihilation operators of the

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atomic external vibrational quanta with frequency $\omega$. $\hat{\sigma}_z$ and $\sigma_\pm$ are the Pauli operators, defining by the internal ground state $|g\rangle$ and excited state $|e\rangle$ of the ion of mass $M$ as $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|, \sigma_+ = |e\rangle\langle g|$, and $\sigma_- = |g\rangle\langle e|$. $\delta = \omega_0 - \omega_L$ is the detuning between the ion and the applied classical laser beam. $\Omega$ is the carrier Rabi frequency which describes the coupling between the classical laser beam and the ion. The frequency $\omega_{\rm L}$ and phase $\phi_{\rm L}$ of the applied classical laser beam are experimentally controllable. Obviously, under the rotating-wave approximation and beyond the Lamb–Dicke regime, we can easily obtain the Hamiltonian in the interaction picture:

$$H_{\rm I} = \frac{\Omega}{2}{\rm e}^{-\eta^2/2}\left[{\rm e}^{-{\rm i}\phi_{\rm L}}\hat{\sigma}_+\left(\sum_{n=0}^{\infty}\frac{({\rm i}\eta)^{2n}\hat{a}^{+n}\hat{a}^{n+k}}{n!(n+k)!}\right) + {\rm H.c}\right], \tag{2}$$

by using laser beams with frequencies $\omega_{\rm L} = \omega_0 - k\omega$, $(k = 0, 1, 2, \cdots)$. Note that the Lamb–Dicke parameter, $\eta = (\omega_0 - k\omega)/\sqrt{\hbar/(2M\omega c^2)}$ is almost unchanged for different excitation, because of $\omega_0 \gg \omega$ (in usual $\omega_0 = 2\pi \times 4.11 \times 10^{11}$ kHz and $\omega = 2\pi \times 135$ kHz). we assume that the initial state of the present system is described by the state vector,

$$\begin{aligned} |\Psi(0)\rangle &= |\psi\rangle_a \otimes |\psi\rangle_b \\ &= |g\rangle_a \otimes {\rm e}^{-\frac{|\alpha|^2}{2}}\sum_{m=0}^{\infty}\frac{\alpha^m}{\sqrt{m!}}|m\rangle_b, \end{aligned} \tag{3}$$

i.e., the ionic internal levels prepared in the ground state $|g\rangle$ and the ionic vibrational centre-of-mass motion prepared in the coherent state. Where $\alpha = \sqrt{\bar{n}}\exp({\rm i}\varphi)$, $\bar{n}$ is the initial average phonon number and $\varphi$ is the phase of the phonon coherent state. The state vector for the system governed by the above Hamiltonian at anytime $t$ is:

$$\begin{aligned} |\Psi(t)\rangle = {\rm e}^{-\frac{|\alpha|^2}{2}}\sum_{m=0}^{k-1}\frac{\alpha^m}{\sqrt{m!}}|m\rangle\otimes|g\rangle + {\rm e}^{-\frac{|\alpha|^2}{2}}\sum_{m=k}^{\infty}\frac{\alpha^m}{\sqrt{m!}}[\cos(\Omega_{m-k,k}t)|m\rangle\otimes|g\rangle \\ +{\rm i}^{k-1}{\rm e}^{-{\rm i}\phi}\sin(\Omega_{m-k,k})|m-k\rangle\otimes|e\rangle], \end{aligned} \tag{4}$$

with the Rabi frequency

$$\Omega_{m-k,k} = \frac{\Omega\eta^k}{2}{\rm e}^{-\eta^2/2}\sqrt{\frac{m!}{(m-k)!}}\sum_{n=0}^{m-k}\frac{({\rm i}\eta)^{2n}(m-k)!}{(n+k)!(m-k-n)!n!}. \tag{5}$$

Here $t$ is the duration of the applied laser beam.

## 3. Squeezing properties of a trapped ion

To analyse the squeezing properties of a trapped ion, we define the two Hermitian quadrature operators

$$S_1 = \frac{1}{2}(S_+ + S_-), \quad S_2 = \frac{1}{2i}(S_+ - S_-), \tag{6}$$

and they obey the commutation relation

$$[S_1, S_2] = {\rm i}S_3. \tag{7}$$

Correspondingly, the Heisenberg uncertainty relation is

$$(\triangle S_1)^2(\triangle S_2)^2 \geq \frac{1}{4}|\langle S_3\rangle|^2, \tag{8}$$

where $(\triangle S_l)^2 = \langle S_l^2\rangle - \langle S_l\rangle^2$ is the variance in the component $S_l (l = 1$ or $2)$ of the ionic dipole. The fluctuations in the component $S_l (l = 1$ or $2)$ of the ionic dipole are said to be squeezed if $S_l$ satisfies the condition

$$F_l = (\triangle S_l)^2 - \frac{1}{2}|\langle S_3\rangle| < 0, (l = 1 \quad \text{or} \quad 2). \tag{9}$$

After a straightforward calculation, the expectation values of the Hermitian operations $S_l (l = 1, 2$ or $3)$ and $S_l^2 (l = 1$ or $2)$ are given by

$$\langle S_1\rangle = \begin{cases} A\cos(\phi - k\varphi), & (k = 2n+1, \ \ n = 0, 1, 2...), \\ A\sin(\phi - k\varphi), & (k = 2n, \ \ n = 0, 1, 2...). \end{cases} \tag{10}$$

$$\langle S_2\rangle = \begin{cases} -A\sin(\phi - k\varphi), & (k = 2n+1,\ \ n = 0,1,2...), \\ -A\cos(\phi - k\varphi), & (k = 2n,\ \ n = 0,1,2...). \end{cases} \tag{11}$$

$$\langle S_3\rangle = -\frac{1}{2}\sum_{m=0}^{k-1} \mathrm{e}^{-|\alpha|^2}\frac{|\alpha|^{2m}}{m!} - \frac{1}{2}\sum_{m=k}^{\infty} \mathrm{e}^{-|\alpha|^2}\frac{|\alpha|^{2m}}{m!}\cos(2\Omega_{m-k,k}t), \tag{12}$$

$$\langle S_1^2\rangle = \langle S_2^2\rangle = \frac{1}{4}, \tag{13}$$

where A is

$$A = \mathrm{e}^{-|\alpha|^2}\sum_{m=0}^{k-1}\frac{|\alpha|^{2m+k}}{\sqrt{m!(m+k)!}}\sin(\Omega_{m,k}t) + \mathrm{e}^{-|\alpha|^2}\sum_{m=k}^{\infty}\frac{|\alpha|^{2m+k}}{\sqrt{m!(m+k)!}}\sin(\Omega_{m,k}t)\cos(\Omega_{m-k,k}t). \tag{14}$$

The corresponding squeezing function $F(S_i)(i = 1,2)$ can be expressed in the form:

$$F(S_1) = \frac{1}{4} - \langle S_1\rangle^2 - \frac{1}{2}|S_3|,$$
$$F(S_2) = \frac{1}{4} - \langle S_2\rangle^2 - \frac{1}{2}|S_3|. \tag{15}$$

If $F(S_i) < 0$ $(i = 1,2)$, the squeezing is said to exist.

In the following, we will numerically discuss the properties of squeezing for the trapped ion in the different conditions. Figures 1 and 2 exhibit the numerical results of time evolution of the function $F(S_i)$ for different values of the Lamb–Dicke parameter $\eta$ and the initial average photon number $\bar{n}$, respectively. The result shows that the squeezing properties strongly depend on the parameters $k, \eta$ and $\bar{n}$. (For convenience, we assume $\phi = \frac{\pi}{4}, \varphi = 0$.)

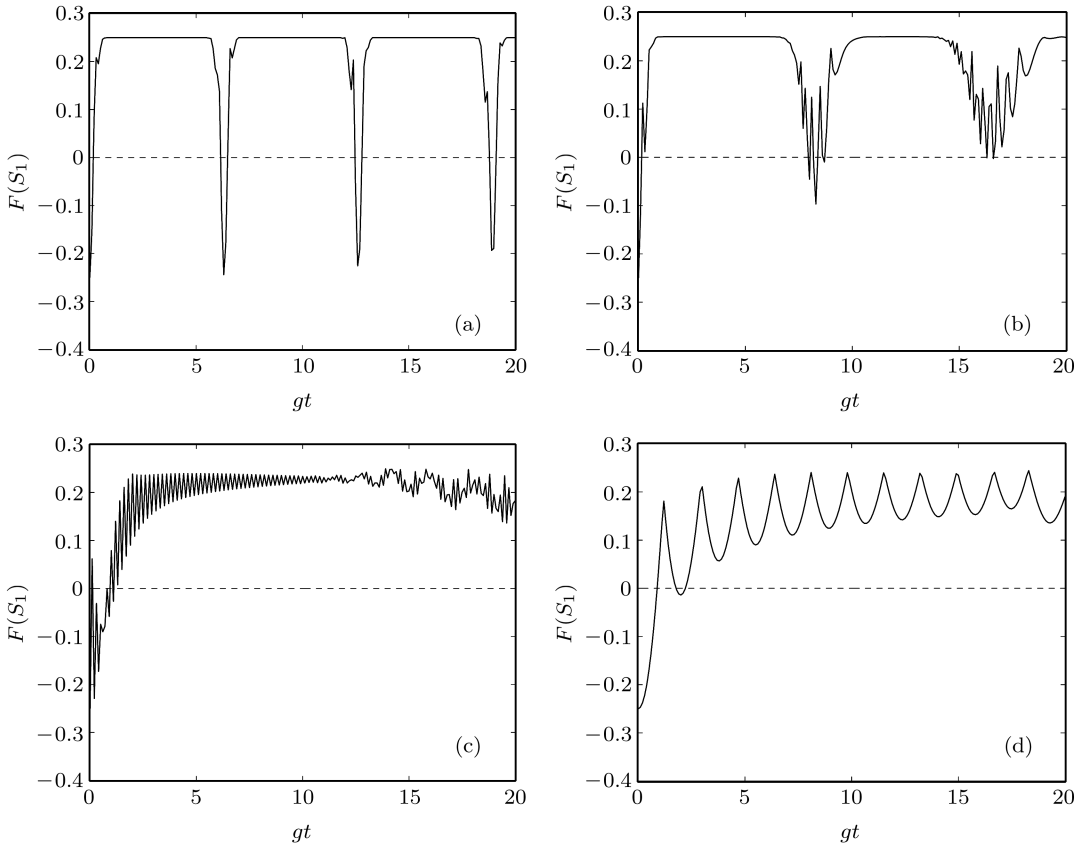



**Fig.1.** Time evolution of the squeezing factor $F(S_1)$ of a trapped ion. The ion is initially in the ground state $|g\rangle$ and the phonons in the coherent state. $\bar{n} = 25$, $k = 0$. (a) $\eta = 0.01$; (b) $\eta = 0.1$; (c) $\eta = 0.2$; (d) $\eta = 0.8$.

Firstly, we study the effect of the Lamb–Dicke parameter $\eta$. The time evolution of function $F(S_1)$ is shown in Fig.1, with $\bar{n} = 25, k = 0$ and different $\eta$. It is observed that, for the small value of the $\eta$, the trapped ion shows the periodical squeezing effects in $S_1$, with the increasing of $\eta$, the periodicity of squeezing is dismissed, and the amplitude of the squeezing is decreased. When $\eta$ is large enough, it can be seen that at the early stage of the time evolution, the quantum fluctuation in $S_1$ of the trapped ion can be squeezed. However, the squeezing properties disappear after an interval, as shown in Fig.1.

Secondly, we discuss the influence of the mean photon number $\bar{n}$ which can be reached in the current experiment.[17] Beyond the Lamb–Dicke limit, we take $\eta = 0.8$ and $k = 0$. When $\bar{n}$ is small, the squeezing effects still display periodically, while for large average phonon numbers the squeezing in $S_1$ shows a chaotic behaviour, as shown in Fig.2(a) and (b). Contrarily, for the small Lamb–Dicke parameter $\eta = 0.01$, i.e., in the Lamb–Dicke limit, we notice that the squeezing properties of trapped ion display periodicity for larger $\bar{n}$ too, and the period is invariable. But the squeezing time becomes short with the increasing of $\bar{n}$, as shown in Fig.2(c), (d), (e) and (f).

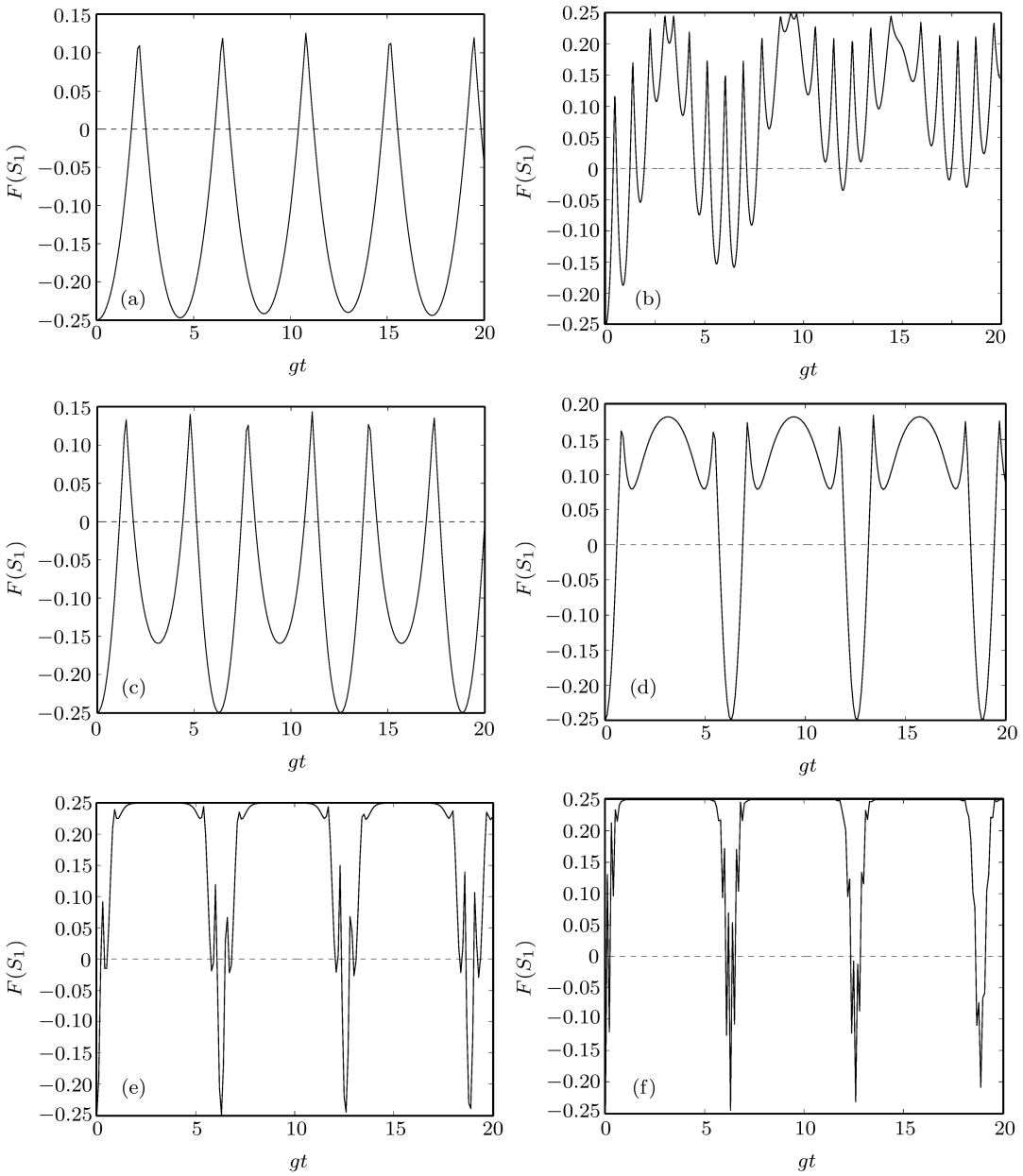



**Fig.2.** Time evolution of the squeezing factor $F(S_1)$ of a trapped ion in the running-wave laser. The ion is initially in the ground state $|g\rangle$, the phonons in the coherent state and $k = 0$ (a) $\eta = 0.8$, $\bar{n} = 0.01$; (b) $\eta = 0.8$, $\bar{n} = 1$; (c) $\eta = 0.01$,$\bar{n} = 0.1$; (d)$\eta = 0.01$, $\bar{n} = 1$; (e)$\eta = 0.01$, $\bar{n} = 5$; (f)$\eta = 0.01$, $\bar{n} = 15$.

On the other hand, when $k$ which can be experimentally controlled in the applied classical laser beam is large enough, even if the other parameters remain unchanged, our results show the squeezing effects vanish.

In conclusion, we have investigated the squeezing properties of a trapped ion in the running-wave laser beyond the Lamb–Dicke limit. In our calculations, we see that the squeezing effect can be optimized by properly adjusting the parameters $\eta$, $\bar{n}$ and $k$. In fact, under the Lamb–Dicke regime, the spatial dimension of the motion in the ground state of the trapped ion is much smaller than the effective wavelength of the applied laser field. That is to say, the trapped ions must almost be cooled utterly, the experimental conditions is very rigorous. However, beyond the Lamb–Dicke regime, the conditions of the experiment can be easily realized and the present system can also provide periodic squeezing characteristic for proper $\bar{n}$ and $k$. So our investigation has realizable meaning and is feasible in the current technology.

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