Vol 16 No 6, June 2007
1009-1963/2007/16(06)/1526-08

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Propagation of bright femtosecond pulses in a nonlinear optical fibre with the third- and fourth-order dispersions*

Ao Sheng-Mei(敖胜美), Yan Jia-Ren(颜家壬)†, and Yu Hui-You(俞慧友)

College of Physics and Information Science, Hunan normal University, Changsha 410081, *China*

(Received 17 August 2006; revised manuscript received 28 December 2006)

We solve the generalized nonlinear Schrödinger equation describing the propagation of femtosecond pulses in a nonlinear optical fibre with higher-order dispersions by using the direct approach to perturbation for bright solitons, and discuss the combined effects of the third- and fourth-order dispersions on velocity, temporal intensity distribution and peak intensity of femtosecond pulses. It is noticeable that the combined effects of the third- and fourth-order dispersions on an initial propagated soliton can partially compensate each other, which seems to be significant for the stability controlling of soliton propagation features.

Keywords: femtosecond pulse, nonlinear optical fibre, higher-order dispersion
PACC: 0340, 0290

1. Introduction

It is well known that the propagation of bright solitons in nonlinear optical fibres can be described by the nonlinear Schrödinger (NLS) equation taking into account the second-order dispersion and self-phase modulation.[1,2] However, when pulses at a wavelength close to zero group-velocity dispersion, or ultrashort pulses propagate in nonlinear optical fibres, higher-order dispersion terms become more important and should be taken into consideration,[3,4] thus the generalized nonlinear Schrödinger (GNLS) equation is obeyed.

In the recent decades, the effects of the third-order dispersion (TOD) and fourth-order dispersion (FOD) on bright solitons have been studied. Wai *et al*[5−7] have shown in their asymptotic analysis that the action of TOD on bright solitons leads to a nonvanishing asymptotic behaviour in the form of a tail and the tail's amplitude A exponentially decaying as indicated in the TOD coefficient β_3, i.e. $A \sim \exp(-1/\beta_3)$. Nail Akhmediev *et al*[8] have calculated analytically the radiation of the NLS solitons perturbed separately by TOD and FOD. Michel Piché *et al*[9] have studied the fourth-order dispersion, showing that there exists an analytical expression for a bright optical soliton in the absence of the third-order dispersion (where the bright soliton is only a non-propagating soliton). Tang and Yan[10] have studied the propagation of soliton with the first-order dispersion by using a perturbation approach, but the effect of the first-order dispersion on the ultrashort pulse is much smaller than that of TOD or FOD, thus it is negligible.

The analytical study on combined effects of TOD and FOD on bright solitons has been reported very little up to now. For the case in which the duration of optical fibre pulses is shorter than 10 fs, TOD and FOD terms are of the equal importance.[8,9,11−14] Therefore, it is significant to investigate their combined effects on bright solitons.

In this paper we study combined effects of TOD and FOD on bright solitons by using the systematic perturbation theory, from which one can obtain not only the evolution for soliton parameters but also the first-order correction. Using the direct perturbation approach developed in our previous paper,[15,16] we investigate both the adiabatic evolution of the NLS bright soliton perturbed by TOD and FOD terms simultaneously and the first-order correction, and we analyse the combined effects of TOD and FOD on the

*Project supported by the National Natural Science Foundation of China (Grant No 10375022) and the Education Department of Hunan Province (Grant No 05C414).
†Corresponding author. E-mail: jiarenyan@hunnu.edu.cn

soliton as well.

2. The adiabatic soliton solution of GNLS equation and the first-order correction

We start from the following GNLS equation with higher-order dispersive terms:

$$\mathrm{i}\frac{\partial E}{\partial \chi}+\frac{|\beta_2|}{2}\frac{\partial^2 E}{\partial T^2}+\gamma|E|^2E=\mathrm{i}\frac{|\beta_3|}{6}\frac{\partial^3 E}{\partial T^3}+\frac{|\beta_4|}{24}\frac{\partial^4 E}{\partial T^4}, \quad (1)$$

where $E=E(T,\chi)$ is the pulse envelope, χ is the propagation distance, T is the time variable and β_n results from an expansion of the propagation constant $\beta(\omega)$ in a Taylor series (i.e. $\beta_n=\mathrm{d}^n\beta(\omega)/\mathrm{d}\omega^n$, $n=2,3,4$). Here parameter $\gamma(>0)$ is a measure of the strength of self-phase modulation, and it is related to the nonlinearly refractive index as well as both the area and the carrier frequency of the pulse. When the TOD and FOD are negligible (i.e. $\beta_3=\beta_4=0$), Eq.(1) can be reduced to a standard NLS equation and has a bright-soliton solution. In general, GNLS equation (1) is not exactly solvable. If $|\beta_3|$ and $|\beta_4|$ are so small that the two terms on the right-hand side of Eq.(1) can be considered as perturbations, Eq.(1) can be solved by using the systematic perturbation theory.

For convenience, we rewrite Eq.(1) in a dimensionless form

$$\mathrm{i}u_x+u_{2t}+2|u|^2u=\mathrm{i}\epsilon f[u]=\mathrm{i}\epsilon(cu_{ttt}-\mathrm{i}u_{tttt}), \quad (2)$$

in which

$$u=u(t,x)=E/E_0,\ \ t=T/T_0,\ \text{and}\ \ x=\chi/L_\mathrm{d}. \quad (3)$$

Here the variables t and x are the dimensionless time and spatial coordinates, respectively, $\epsilon=|\beta_4|/(12|\beta_2|T_0^2)$ is a small positive constant measuring the weakness of the FOD term, $E_0=\sqrt{|\beta_2|/(\gamma T_0^2)}$, $c=4|\beta_3|T_0/|\beta_4|>0$, T_0 corresponds to the initial pulse duration and $L_\mathrm{d}(=2T_0^2/|\beta_2|)$ is the dispersion length. Letting $u=u^{(\mathrm{s})}+\epsilon w$, introducing the independent variables $x_j=\epsilon^j x$, $j=0,1$ (so $\partial_x=\partial_{x_0}+\epsilon\partial_{x_1}$) and following the lines of Refs.[15,16], we obtain easily the adiabatic (zeroth-order approximate) solution of Eq.(2)

$$u^{(\mathrm{s})}=2\beta\mathrm{e}^{-\mathrm{i}\theta}\mathrm{sech}z,\quad z=2\beta(t-\xi),\quad \xi_{x_0}=-4\alpha,$$
$$\theta=\alpha z/\beta+\delta,\quad \text{and}\quad \delta_{x_0}=-4(\alpha^2+\beta^2), \quad (4)$$

which is formally a bright soliton, where z is the variable corresponding to the time coordinate $(t-\xi)$ in the tracing frame of reference (TFR). Here due to the perturbation, the parameters of the soliton α, β, ξ and δ are assumed to be functions of the slow variable x_1 and the soliton's "velocity" $v=\mathrm{d}\xi/\mathrm{d}x=\xi_{x_0}+\epsilon\xi_{x_1}=-4\alpha+\epsilon\xi'$, where the prime denotes the derivative with respect to x_1. Using the method similar to that in Ref.[16], we derive the evolution equations of soliton parameters and the first-order correction as follows:

$$\alpha'=-\frac{1}{2}\int_{-\infty}^{\infty}\mathrm{d}z\ \tanh z\,\mathrm{sech}\,z\,\mathrm{Im}\{f[u^{(\mathrm{s})}(z)]\mathrm{e}^{\mathrm{i}\theta}\}, \quad (5)$$

$$\beta'=\frac{1}{2}\int_{-\infty}^{\infty}\mathrm{d}z\,\mathrm{sech}\,z\,\mathrm{Re}\{f[u^{(\mathrm{s})}(z)]\mathrm{e}^{\mathrm{i}\theta}\}, \quad (6)$$

$$\xi'=\frac{1}{4\beta^2}\int_{-\infty}^{\infty}\mathrm{d}z\,z\,\mathrm{sech}\,z\,\mathrm{Re}\{f[u^{(\mathrm{s})}(z)]\mathrm{e}^{\mathrm{i}\theta}\}, \quad (7)$$

$$\delta'=2\alpha\xi'-\beta'/\beta+\frac{1}{2\beta}\int_{-\infty}^{\infty}\mathrm{d}z\,z\ \tanh z\mathrm{sech}\,z\mathrm{Re}\{f[u^{(\mathrm{s})}(z)]\mathrm{e}^{\mathrm{i}\theta}\}, \quad (8)$$

$$w(t,x)=\frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}\theta}}{4\beta^2}\int_{-\infty}^{\infty}\frac{\mathrm{d}k}{k^2+1}\times\{[1-\mathrm{e}^{4\mathrm{i}\beta^2(k^2+1)x_0}]g^+(k)\phi^+(z,k)-[1-\mathrm{e}^{-4\mathrm{i}\beta^2(k^2+1)x_0}]g^-(k)\phi^-(z,k)\} \quad (9)$$

with

$$g^+(k)=\int_{-\infty}^{\infty}\mathrm{d}z(\overline{\phi^-}(z,k)\overline{r\mathrm{e}^{\mathrm{i}\theta}}-\overline{\phi^+}(z,k)r\mathrm{e}^{\mathrm{i}\theta}),$$
$$g^-(k)=\int_{-\infty}^{\infty}\mathrm{d}z(\overline{\phi^-}(z,k)r\mathrm{e}^{\mathrm{i}\theta}-\overline{\phi^+}(z,k)\overline{r\mathrm{e}^{\mathrm{i}\theta}}), \quad (10)$$

$$\phi^+(z,k)=-\frac{\mathrm{e}^{\mathrm{i}kz}}{\sqrt{2\pi}(k^2+1)}\mathrm{sech}^2z,$$
$$\phi^-(z,k)=\frac{\mathrm{e}^{\mathrm{i}kz}}{\sqrt{2\pi}(k^2+1)}(\tanh z-\mathrm{i}k)^2, \quad (11)$$

where an overbar represents the complex conjugate, and $r\equiv f[u^{(\mathrm{s})}(z)]-\partial u^{(\mathrm{s})}/\partial x_1$. From Eq.(2) it is seen clearly that $f=cu^{(\mathrm{s})}_{ttt}-\mathrm{i}u^{(\mathrm{s})}_{tttt}$. Substituting it into Eqs.(5)–(8), we obtain

$$\alpha'=0,\quad \beta'=0,$$
$$\xi'=4[c(\beta^2+3\alpha^2)-8\alpha(\beta^2+\alpha^2)],$$
$$\delta'=16[c\alpha(\alpha^2-\beta^2)+(7\beta^4+6\alpha^2\beta^2-9\alpha^4)/3] \quad (12)$$

after a series of calculations. We now return to the variable x. With the aid of $\xi_{x_0}=-4\alpha$ and $\delta_{x_0}=-4(\alpha^2+\beta^2)$, we solve Eq.(12) and obtain

$$\alpha=\alpha(x)=\alpha(0),\quad \beta=\beta(x)=\beta(0),$$
$$\xi=-4\{\alpha-\epsilon[c(\beta^2+3\alpha^2)-8\alpha(\beta^2+\alpha^2)]\}x,$$
$$\delta=-4(\alpha^2+\beta^2)x+16\epsilon[c\alpha(\alpha^2-\beta^2)$$

$$+ (7\beta^4 + 6\alpha^2\beta^2 - 9\alpha^4)/3]x. \quad (13)$$

It can be seen from Eq.(12) or Eq.(13) and Eq.(4) that the "velocity" $v(= \mathrm{d}\xi/\mathrm{d}x)$ and phase $\theta(= \alpha z/\beta + \delta)$ of the adiabatic soliton are affected by the TOD and FOD perturbations, but the height (2β) and width $(1/2\beta)$ of adiabatic solitons are not affected.

With Eqs.(4) and (9)–(12) and after a series of calculations, we obtain the first-order correction

$$w(t,x) = \beta \mathrm{e}^{-\mathrm{i}\theta} \int_{\infty}^{\infty} \mathrm{d}k \mathrm{e}^{\mathrm{i}kz} \{g^+(k,x)\mathrm{sech}^2 z + g^-(k,x)(\tanh z - \mathrm{i}k)^2\}, \quad (14)$$

in which

$$g^{\pm}(k,x) = \frac{1 - \mathrm{e}^{\pm 4\mathrm{i}\beta^2(k^2+1)x}}{(k^2+1)\cosh\frac{\pi k}{2}} \Big[c(k\beta \pm 3\alpha) \mp 4\Big(3\alpha^2 + \beta^2 \pm 2k\alpha\beta + \frac{2}{3}k^2\beta^2\Big)\Big]. \quad (15)$$

3. Analysis and discussion

To investigate further the effects of TOD and FOD on the pulse propagation, we make analytic calculations, and give simulation results in two cases corresponding non-propagating and propagating solitons, separately. In this section, we return to the laboratory time and spatial coordinates (T, χ).

3.1. Velocity and phase of an adiabatic soliton

First, we investigate the effects of TOD and FOD on the velocity and the phase of the NLS soliton. When $\beta_3 = \beta_4 = 0$, Eq.(1) has the bright soliton

$$E^{(\mathrm{nls})}(T,\chi) = 2\beta E_0 \,\mathrm{sech}\frac{2\beta(T - V_0\chi)}{T_0}\mathrm{e}^{-\mathrm{i}\theta_0},$$
$$V_0 = -\frac{4\alpha T_0}{L_\mathrm{d}}, \text{ and } \theta_0 = \frac{2\alpha T}{T_0} + \frac{4(\alpha^2 - \beta^2)\chi}{L_\mathrm{d}}. \quad (16)$$

In particular, if $\alpha = 0$ and $2\beta = 1$, $V_0 = 0$, $\theta_0 = -\chi/L_\mathrm{d}$ and $E^{(\mathrm{nls})}(T,\chi) = E_0 \,\mathrm{sech}(T/T_0)\mathrm{e}^{\mathrm{i}\chi/L_\mathrm{d}}$, which describes a non-propagating soliton.

When $\beta_3 \neq 0$ and $\beta_4 \neq 0$, we can calculate the adiabatic soliton of GNLS equation as

$$E^{(\mathrm{ad})}(T,\chi) = 2\beta E_0 \,\mathrm{sech}\frac{2\beta(T - V\chi)}{T_0}\mathrm{e}^{-\mathrm{i}\theta}, \quad (17)$$

where the velocity V and the phase θ are given by

$$V = -\frac{4\alpha T_0}{L_\mathrm{d}} + 4\epsilon[c(\beta^2 + 3\alpha^2) - 8\alpha(\beta^2 + \alpha^2)]\frac{T_0}{L_\mathrm{d}}, \quad (18)$$
$$\theta = \frac{2\alpha T}{T_0} + \frac{4(\alpha^2 - \beta^2)\chi}{L_\mathrm{d}} + \frac{16\epsilon[15\alpha^4 + 30\alpha^2\beta^2 + 7\beta^4 - 3c\alpha(\alpha^2 + \beta^2)]\chi}{3L_\mathrm{d}}, \quad (19)$$

respectively. If $\alpha = 0$ and $2\beta = 1$, we have $V = \epsilon c T_0/L_\mathrm{d} = |\beta_3|/(6T_0^2)$, and $\theta = 7\epsilon\chi/(3L_\mathrm{d}) - \chi/L_\mathrm{d}$ from Eqs. (18) and (19). That is, the TOD turns a non-propagating soliton into a propagating one without an additional phase-shift, while the FOD leads only to an additional phase-shift of the non-propagating soliton.

The additional velocity of adiabatic GNLS soliton, $V_\mathrm{add} = V - V_0$, is plotted as a function of the parameter 2α in Fig.1. We have chosen the following parameters: $\gamma = 2.36 \times 10^{-6}\,\mathrm{W^{-1}cm^{-1}}$, $\beta_2 = -100\ \mathrm{fs^2/cm}$ (which are representative of a femtosecond Ti:sapphire laser,[11]) $T_0 = 9.13$ fs, $2\beta = 1$, and then $E_0^2 = 0.51$ MW. Figure 1(a) shows the effects of TOD on additional velocity V_add with $\beta_4 = -5000\,\mathrm{fs^4/cm}$, $\epsilon = 0.05$ for $c = 0$, 1.0, 3.65, 7.3 corresponding to $\beta_3 = 0$, $-137, \mathrm{fs^3/cm}$, $-500, \mathrm{fs^3/cm}$, $-1000\,\mathrm{fs^3/cm}$, while Fig.1(b) shows the effects of FOD on additional velocity V_add with $\beta_3 = -500\,\mathrm{fs^3/cm}$ for $\epsilon = 0.01, 0.025, 0.5, 0.10$ corresponding to $\beta_4 = -1000, \mathrm{fs^4/cm}$, $-2500, \mathrm{fs^4/cm}$, $-5000, \mathrm{fs^4/cm}$, $-10000\,\mathrm{fs^4/cm}$. It is shown in Fig.1 that for a certain value of $\alpha(\neq 0)$, the greater the TOD (FOD) parameter $|\beta_3|$ ($|\beta_4|$) is, the greater (smaller) the additional velocity will be. As long as β_3 and β_4 are chosen appropriately, the effect of FOD on NLS soliton velocity can be compensated by that of TOD, and then the intensity distributions and the velocity of adiabatic GNLS solitons are not affected by TOD and FOD although the phase is still affected by them.

The evolutions of intensity distribution of an adiabatic GNLS soliton are shown in Figs.2(a), 2(b) and 2(c) for different values of β_3 and β_4. Figure 2(d) shows the temporal intensity distributions of the adiabatic GNLS soliton at a distance of $\chi = 3L_\mathrm{d}$ for comparison with that of the NLS soliton: the dashed, dash-dotted, solid and circle curves correspond to Fig.2(a), Fig.2(b), Fig.2(c) and the NLS soliton, respectively. Here 2α is chosen to be 1, all other parameters are the same as those given above. As shown in Fig.2, the temporal shape of an adiabatic GNLS soliton is not affected by TOD and FOD; the main action of higher-order dispersions is to advance or to delay the adiabatic GNLS soliton, e.g. under the parameters given above, the TOD increases the soliton velocity in the absence of FOD, on the contrary, the FOD decreases the soliton velocity in the absence of TOD, while the combined effect of TOD and FOD on soliton velocity can be nil.

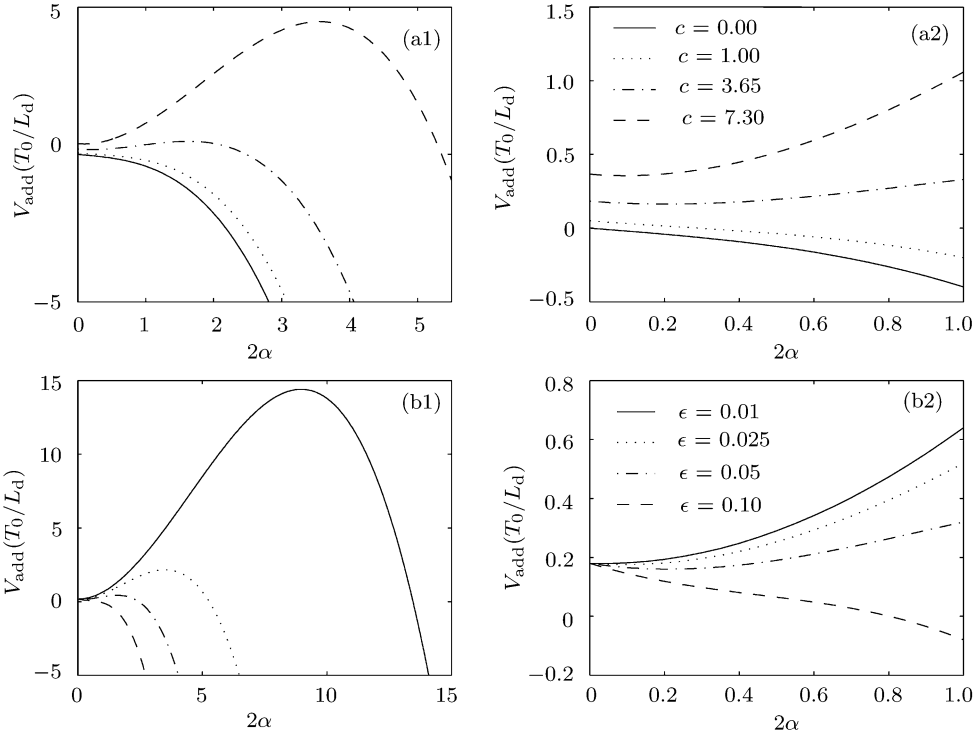


Fig.1. Curves of the additional velocity $V_{\rm add}$ of adiabatic GNLS soliton plotted as a function of parameter 2α.

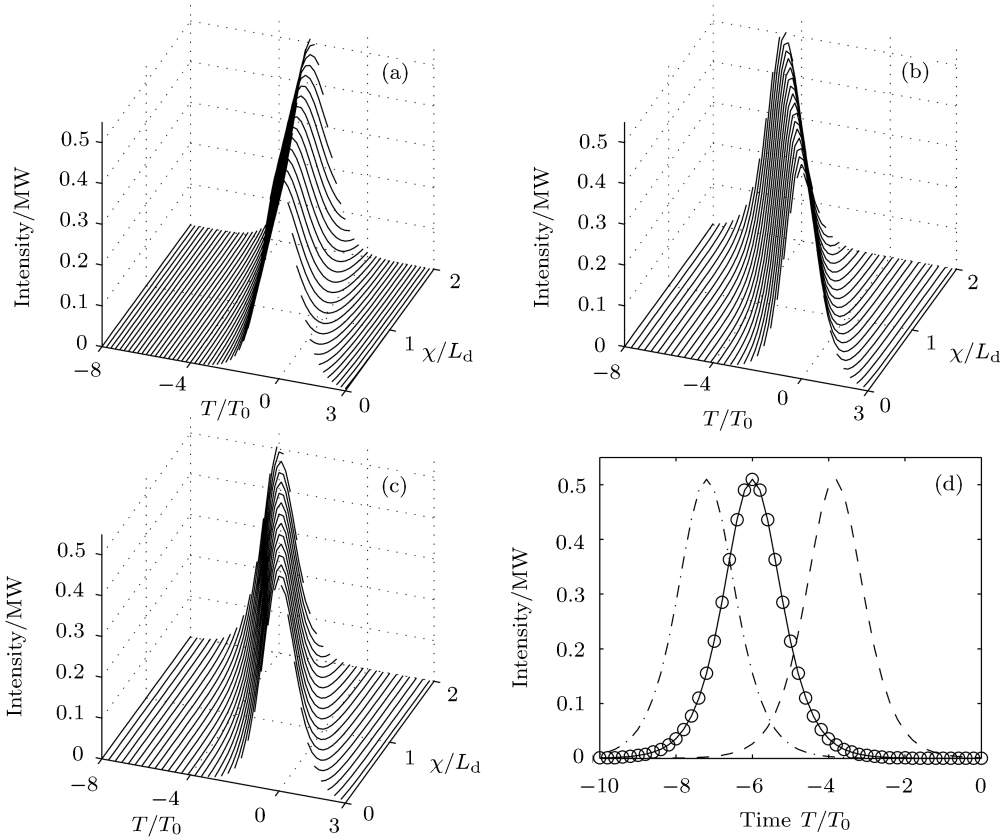


Fig.2. The evolutions of intensity distributions of adiabatic GNLS solitons. (a) and the dashed curve in (d): $\beta_3 = -500\,\mathrm{fs}^3/\mathrm{cm}$, $\beta_4 = 0$. (b) and the dash-dotted curve in (d): $\beta_3 = 0$, $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$. (c) and the solid curve in (d): $\beta_3 = -274\,\mathrm{fs}^3/\mathrm{cm}$, $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$.

3.2. Effect of higher-order dispersions on an femtosecond pulse envelope

Next, we study the combined effect of TOD and FOD on a femtosecond pulse envelope through calculating the first-order correction solution of Eq.(1). From Eq.(3), we have the approximate solution of GNLS equation (1) to the first-order correction

$$E(T,\chi) = E^{(\mathrm{ad})}(T,\chi) + \epsilon E(1)(T,\chi) = E_0 u(t,x) = E_0(u^{(\mathrm{s})} + \epsilon w). \tag{20}$$

Owing to the integration difficulty in Eq.(14), we perform numerical calculation in the following two cases.

Case 1 $2\alpha = 0$ and $2\beta = 1$. In this case, the unperturbed NLS soliton is a non-propagating soliton. Combining Eqs.(4) and (13) with Eqs.(14) and (15), we obtain the first-order correction as follows:

$$\begin{aligned} w(T,\chi) = \frac{\mathrm{e}^{-\mathrm{i}\theta}}{12}\int_{\infty}^{\infty} \frac{\mathrm{d}k \mathrm{e}^{\mathrm{i}kz}}{(k^2+1)\cosh(\pi k/2)}) \\ \times \{(3ck + 6 + 4k^2)[1 - \mathrm{e}^{-\mathrm{i}(k^2+1)\chi/L_{\mathrm{d}}}] \\ \times (\tanh z - \mathrm{i}k)^2 + (3ck - 6 - 4k^2) \\ \times [1 - \mathrm{e}^{\mathrm{i}(k^2+1)\chi/L_{\mathrm{d}}}]\mathrm{sech}^2 z\} \end{aligned} \tag{21}$$

with

$$z = \frac{T}{T_0} - \epsilon c\frac{\chi}{L_{\mathrm{d}}}, \quad \text{and} \quad \theta = \Big(\frac{7}{3}\epsilon - 1\Big)\frac{\chi}{L_{\mathrm{d}}}. \tag{22}$$

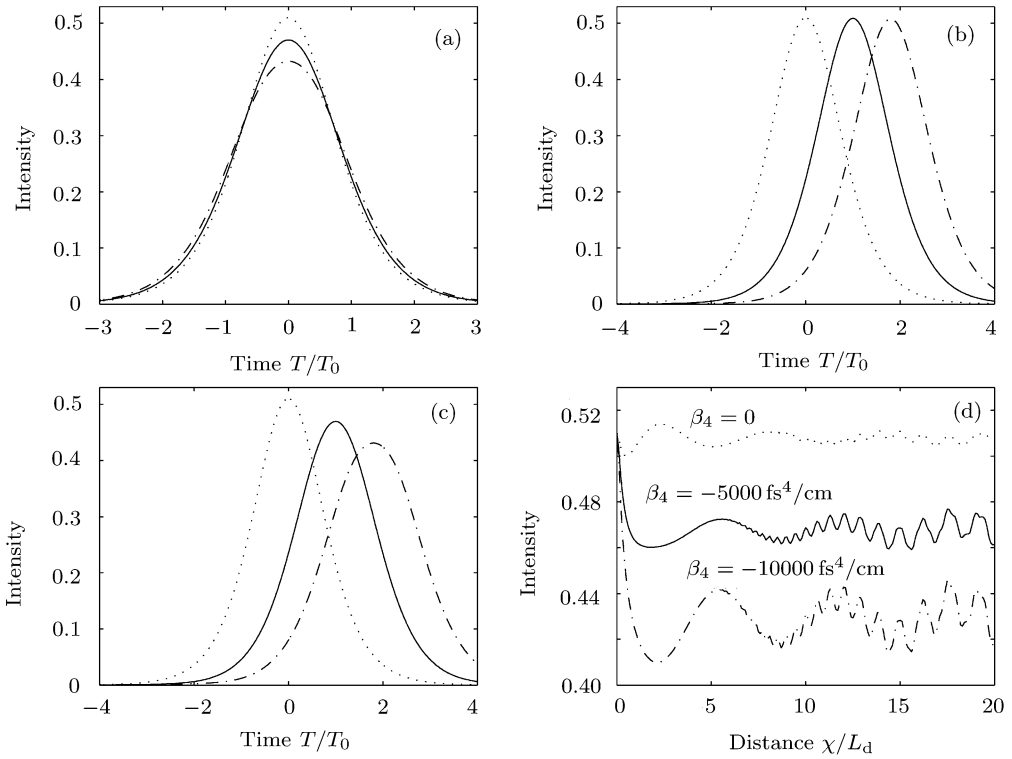


Fig.3. Intensity distributions of femtosecond pulses. (a) $\beta_3 = 0$, the dotted, solid and dash-dotted curves corresponding to $\beta_4 = 0$, $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$ and $\beta_4 = -10000\,\mathrm{fs}^4/\mathrm{cm}$, respectively; (b) $\beta_4 = 0$, the dotted, solid and dash-dotted curves corresponding to $\beta_3 = 0$, $\beta_3 = -274\,\mathrm{fs}^3/\mathrm{cm}$ and $\beta_3 = -500\,\mathrm{fs}^3/\mathrm{cm}$, respectively; (c) the dotted, solid and dash-dotted curves corresponding to $\beta_3 = \beta_4 = 0$, $\beta_3 = -274\,\mathrm{fs}^3/\mathrm{cm}$, $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$ and $\beta_3 = -500\,\mathrm{fs}^3/\mathrm{cm}$, $\beta_4 = -10000\,\mathrm{fs}^4/\mathrm{cm}$, respectively; (d) the variation of peak intensity of pulse with propagating distance χ for $\beta_3 = -500\,\mathrm{fs}^3/\mathrm{cm}$ and the values of β_4 as shown in the figure.

Figure 3 displays the intensity distributions of non-propagating solitons with $\gamma = 2.36 \times 10^{-6}\,\mathrm{W}^{-1}\mathrm{cm}^{-1}$ and $T_0 = 9.13\,\mathrm{fs}$ under perturbations of TOD or/and FOD. Figures 3(a), 3(b) and 3(c) show the temporal shapes of an optical pulse at a distance of $\chi = 10L_{\mathrm{d}}$ respectively in the presence of FOD but in the absence of TOD; in the presence of TOD but in the absence of FOD; in the presence of both TOD and FOD. Figure 3(d) shows the effects of TOD and FOD on the peak intensity of an optical pulse. It is seen in Figs.3(a)–(c) that (1) the TOD turns a non-propagating soliton into a propagating one without

any changes in its shape; (2) the FOD changes only the shape of non-propagating soliton without affecting its velocity; (3) the combined effect of TOD and FOD leads to the changes in both shape and velocity of the pulse. It is seen in Fig.3(d) that the FOD makes the peak value of pulse not only reduce with $|\beta_4|$ but also oscillate with propagating distance χ, and the TOD leads mainly to an oscillation of peak intensity of pulse if the unperturbed pulse is a non-propagating soliton, which implies that the temporal distribution of soliton will evolve into a main pulse with a considerable number of weaker substructures.

According to the above analysis, we know that those two effects of TOD and FOD on an initial non-propagating soliton cannot compensate each other at all.

Case 2 $2\alpha = 2\beta = 1$. In this case, the unperturbed NLS soliton is a propagating soliton with a velocity of $V_0 = -2T_0/L_\mathrm{d}$, and similarly, we have

$$w(T,\chi) = \frac{\mathrm{e}^{-\mathrm{i}\theta}}{12}\int_\infty^\infty \frac{\mathrm{d}k\mathrm{e}^{\mathrm{i}kz}}{(k^2+1)\cosh\frac{\pi k}{2}} \times \{[3c(k+3)-4(6+3k+k^2)] \times [1-\mathrm{e}^{-\mathrm{i}(k^2+1)\chi/L_\mathrm{d}}](\tanh z-\mathrm{i}k)^2 + [3c(k+3)+4(6-3k+k^2)] \times [1-\mathrm{e}^{\mathrm{i}(k^2+1)\chi/L_\mathrm{d}}]\mathrm{sech}^2 z\} \tag{23}$$

with

$$z = \frac{T}{T_0} + 2(1+4\epsilon-2\epsilon c)\frac{\chi}{L_\mathrm{d}}, \qquad \theta = \frac{T}{T_0} + 4\epsilon(13-3c)\frac{\chi}{3L_\mathrm{d}}. \tag{24}$$

Figure 4 shows the evolution of intensity distribution of a propagating optical soliton under perturbations of both TOD and FOD. Here all other parameters are the same as those in Fig.2. Comparing Fig.4 with Figs.1–3, we find that for an unperturbed-propagating soliton, the TOD and the FOD each affect not only the velocity but also the temporal shape of pulse, but the two effects on pulse temporal intensity are opposed against each other. Hence, we can predict that although the combined effects of TOD and FOD on propagation features of femtosecond pulses cannot completely compensate each other, they could be minimized as long as β_3 and β_4 are chosen appropriately.

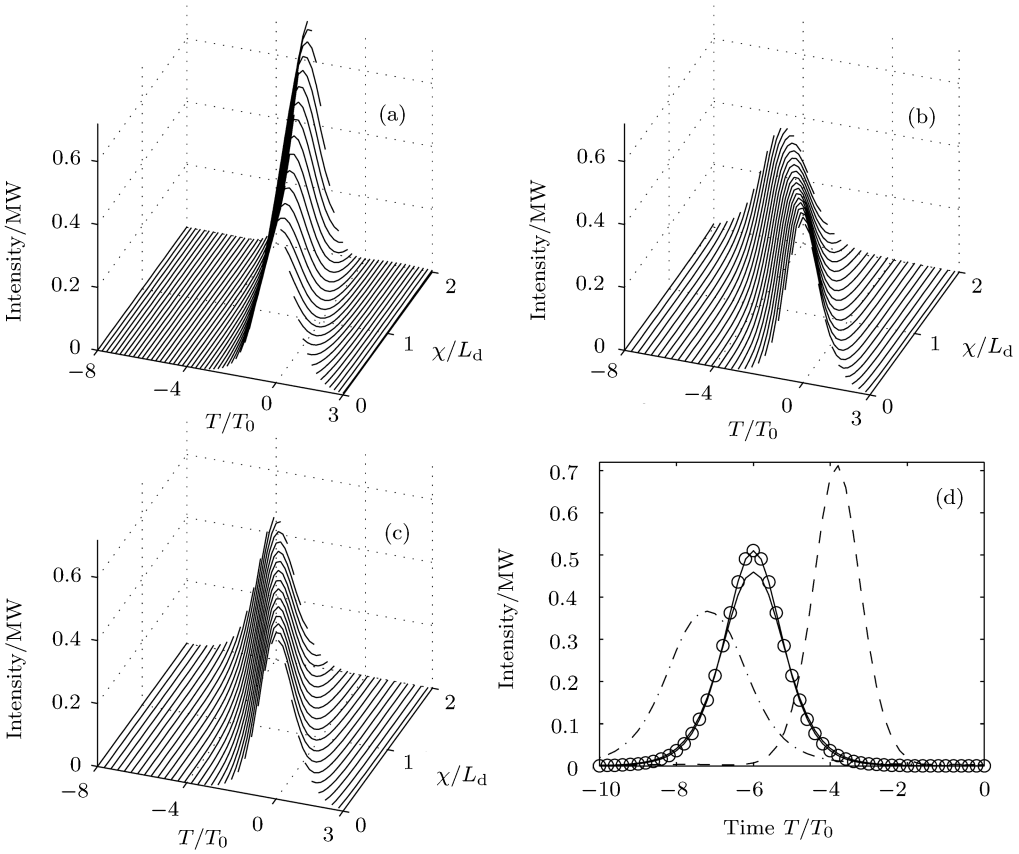


Fig.4. The evolutions of intensity distributions of femtosecond optical pulses, where all parameters are the same as those in Fig.2.

The evolutions of the intensity difference

$$\Delta I(T,\chi) = |E(T,\chi)|^2 - |E^{(\mathrm{nls})}(T,\chi)|^2$$

between a perturbed pulse and a NLS soliton are shown in Fig.5 which are grey-scaled plots. We find from Fig.5 that in the presence of TOD (or FOD) alone, a pair of positive and negative pulses is generated, and the two pulses propagate at different velocities: the dark pulse in Fig.5(a) or Fig.5(b) has the same velocity as that of a NLS soliton, and the bright pulse in Fig.5(a) (or Fig.5(b)) has a smaller (or greater) velocity than that of a NLS soliton.

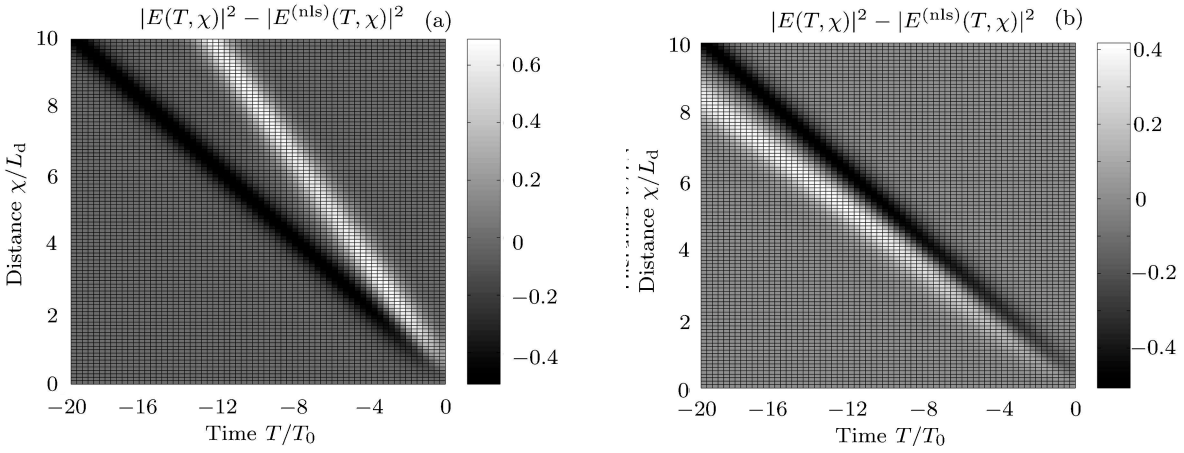


Fig.5. Evolution of intensity difference $\Delta I(T,\chi)$ between a perturbed optical pulse and a NLS optical soliton: grey-scaled plot (a)with $\beta_3 = -500\,\mathrm{fs}^3/\mathrm{cm}$ and $\beta_4 = 0$, (b) with $\beta_3 = 0$ and $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$.

The temporal shapes of $\Delta I(T,\chi)$ at different propagating distances are plotted in Fig.6, in which the dash curve corresponds to $\beta_3 = -500\,\mathrm{fs}^3/\mathrm{cm}$ and $\beta_4 = 0$, the dash-dotted curve corresponds to $\beta_3 = 0$ and $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$, the solid curve corresponds to $\beta_3 = -274\,\mathrm{fs}^3/\mathrm{cm}$ and $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$ and the dotted curve corresponds to $\beta_3 = -425\,\mathrm{fs}^3/\mathrm{cm}$ and $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$.

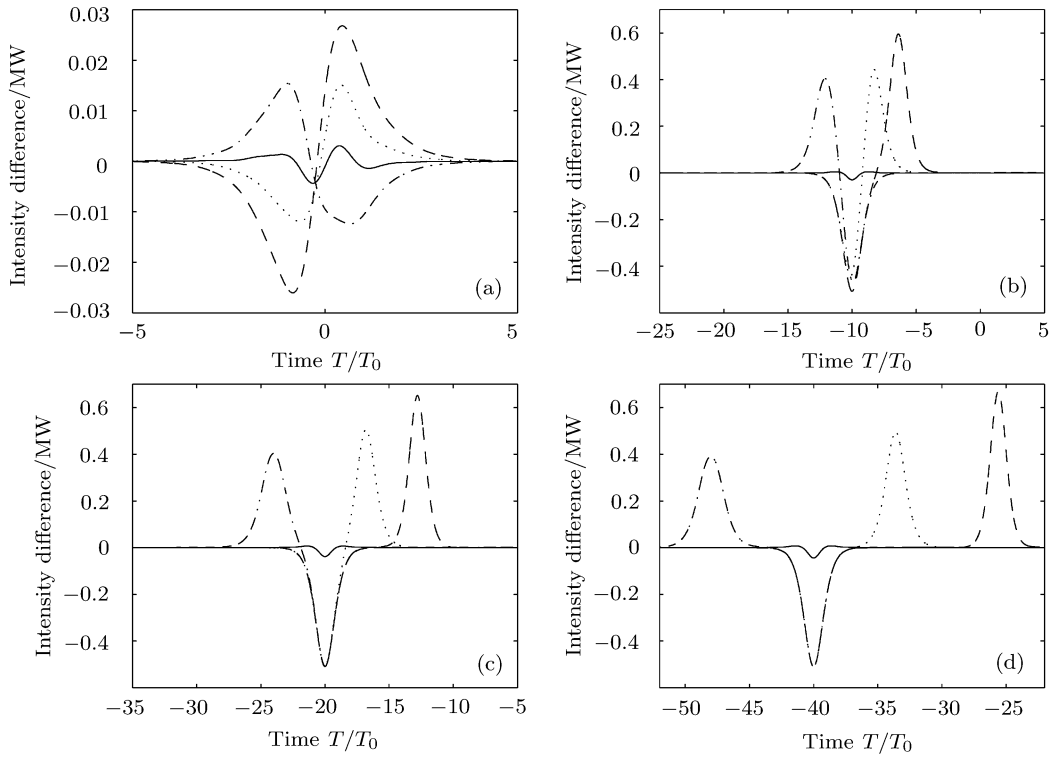


Fig.6. The temporal shapes of $\Delta I(T,\chi)$ at different propagating distances (a) $\chi = 0.1L_\mathrm{d}$, (b) $\chi = 5L_\mathrm{d}$, (c) $\chi = 10L_\mathrm{d}$ and (d) $\chi = 20L_\mathrm{d}$, respectively. the other parameters are given in the text.

Figure 6 proves that after a NLS optical soliton has propagated a sufficient distance in the fibre with higher-order dispersions, the induced bright pulse can completely separate from the initial soliton, while the induced dark pulse can almost compensate the initial soliton, and then a new soliton–like pulse, whose amplitude and velocity are different from those of the initial soliton, is formed. The solid curves shown in Fig.6 also confirm that the combined effects of TOD and FOD almost compensate each other, provided that the higher-order dispersion parameters are chosen as $\beta_3 = -274\,\mathrm{fs}^3/\mathrm{cm}$ and $\beta_4 = -5000\,\mathrm{fs}^4/\mathrm{cm}$.

The intensity at pulse centre, $I_{\max}$, versus propagating distance is plotted in Fig.7. It is seen clearly in Fig.7 that the TOD (or FOD) can increase (or decrease) the peak value of pulse, and both also lead to the peak intensity oscillating. Comparing Fig.7 with Fig.3(d), we find that although the effect of TOD (or FOD) on the peak value of pulse in case 2 is greater than that in case 1, the combined effects of TOD and FOD on the peak value of pulse in case 2 can almost compensate each other, provided that the parameters are chosen appropriately (see the dotted curve in Fig.7). Besides, the peak intensity oscillating shown in Fig.7 is much slower than that shown in Fig.3(d).

According to the above analysis, we know that those two effects of TOD and FOD on an initial propagating soliton could almost compensate each other, provided that the parameters are chosen appropriately. We believe that these results are also very significant in practical applications. For example, one can try to control the stability of shape or velocity of femtosecond pulses propagating in a nonlinear optical fibre by adjusting soliton parameters and choosing dispersion parameters, and then obtain a soliton–like pulse propagating at a velocity with a small additive velocity, almost without any change in initial soliton shape (see the dotted curves in Fig.6 and Fig.7), or a soliton–like pulse propagating at an initial velocity with a small change in initial soliton shape (see the solid curves in Fig.6 and Fig.7).

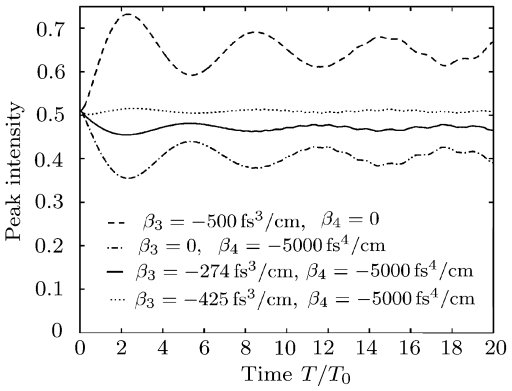


Fig.7. Peak intensity versus propagating distance χ for the different values of β_3 and β_4 as shown in the figure.

4. Conclusions

In conclusion, we have obtained the perturbed solution for GNLS equation describing the propagation of femtosecond pulses in a higher-order dispersion optical fibre by using the direct approach to perturbation, and discussed the combined effects of TOD and FOD on velocity, temporal intensity distribution and peak intensity of pulses in two cases in which an unperturbed pulse is a non-propagating or propagating soliton, separately. It is noticeable that the combined effects of TOD and FOD on an initial propagating soliton can partially compensate each other, which is believed to be significant for the stability controlling of soliton propagation features.

References

[1] Hasegawa A and Tappert F 1973 *Appl. Phys. Lett.* **23** 171
[2] Zakharov V E and Shabat A B 1973 *Zh. Eksp. Teor. Fiz.* **64** 1627
Zakharov V E and Shabat A B 1973 *Sov. Phys. JETP* **37** 823
[3] Agrawal G P 1995 *Nonlinear Fiber Optics* 2nd edn (New York: Academic)
[4] Cheng Ch F, Wang X F and Lu B 2004 *Acta Phys. Sin.* **53** 1826 (in Chinese)
[5] Wai P K A, Menyuk C R, Lee Y C and Chen H H 1986 *Opt. Lett.* **11** 464
Wai P K A, Menyuk C R, Lee Y C and Chen H H 1987 *Opt. Lett.* **12** 628
[6] Menyuk C R and Wai P K A 1992 *Optical Soliton: Theory and Experiment* J. R. Taylor, ed. (Cambridge: Cambridge University Press)
[7] Wai P K A, Chen H H and Lee Y C 1990 *Phys. Rev.* A **41** 426
[8] Akhmediev N and Karlsson M 1995 *Phys. Rev.* A **51** 2602
[9] Piché M, Cormier J F and Zhu X N 1996 *Opt. Lett.* **21** 845
[10] Tang Y and Yan X H 2000 *Chin. Phys.* **9** 565
[11] Stingl A, Spielmann C, Krausz F and Szipöcs R 1994 *Opt. Lett.* **19** 204
[12] Zhou J P, Taft G, Huang C P, Murnane M M and Turitsyn S K 1994 *Opt. Lett.* **19** 1149
[13] Christov I P, Murnane M M, Kapteyn H C, Zhou J P and Huang C P 1994 *Opt. Lett.* **19** 1465
[14] Kasper A and Witte K J 1996 *Opt. Lett.* **21** 360
[15] Yan J R, Tang Y and Zhou G H 1998 *Phys. Rev.* E **58** 1064
[16] Yan J R, Ao Sh M and Yu H Y 2005 *Chin. Phys.* **14** 28