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Suppression of decoherence by bath ordering*

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The dynamics of two coupled spins-1/2 coupled to a spin-bath is studied as an extended model of the Tessieri–Wilkie Hamiltonian. The pair of spins served as an open subsystem is prepared in one of the Bell states and the bath consisting of some spins-1/2 is in a thermal equilibrium state from the very beginning. It is found that with increasing coupling strength of the bath spins, the bath forms a resonant antiferromagnetic order. The polarization correlation between the two spins of the subsystem and the concurrence of it are recovered to some extent in the isolated subsystem. This suppression of the subsystem decoherence may be used to control the quantum devices in practical applications.

Keywords: decoherence, open subsystem, Bell states, thermal equilibrium, polarization correlation

PACC: 0155, 0365, 0530

1. Introduction

Quantum decoherence is a common and inevitable phenomenon in an open quantum system due to its interaction with the surrounding bath or environment. An initial superposition state of the system, $\rho_{\rm s}(t=0)=|\psi_0\rangle\langle\psi_0|$, has to decay into a classical mixture of states, $\rho_{\rm s}=\sum_i p_i\rho_i$, $\sum_i p_i=1$, after a decoherence time $\tau_{\rm d}$. During the same period of time, some of the information in the initial state of the system might irreversibly lose into the bath.[1−5] A real-world system, for instance a quantum device or qubit, cannot be completely isolated from the environment. In the last two decades, much interest was focused on the search for realization of quantum computation as well as quantum communications. Such technologies rely on the possibilities that quantum devices can be manufactured with negligible decoherence during the period of operation. Since decoherence is an intrinsic property of open quantum systems, the problem is transformed to the one of how to reduce or eliminate the decoherence of an open quantum system. Many works have been devoted in the research of the influences caused by the subsystem-bath coupling and the decoherence dynamics of open quantum systems,[6,7] for the possible realization of quantum communication and quantum computation.

A very important part of any theoretical research of decoherence is the modelling of the bath or environment. There have been mainly two important schemes for the bath: (i) the bath consisting of harmonic oscillators, e.g., the Caldeira–Leggett model;[8] (ii) the bath made up of spins-1/2, e.g., the model used in Ref.[9]. For both types of bath model, there are typically two kinds of approximations, Markovian[10] or non-Markovian,[11] used in the studies. The focus of most of the previous researches was on the elimination of the destructive effects of the environment on the system. However, Tessieri and Wilkie proposed a new idea to reduce the decoherence[12] of an open subsystem, which is a spin-1/2 coupled to a low-temperature bath of spins. They introduced the coupling terms between bath spins into their spin-bath Hamiltonian, which is a simplification of the vibronic Hamiltonian of the impurity and crystal[13] using spin-1/2 modes. One of the most important results they found is that the strong antiferromagnetic intra-bath interactions could be utilized to make the dynamics of the central spin almost independent of the surrounding bath. Dawson *et al*[14] also found that the decoherence effect of the bath can be suppressed by increasing the intra-bath coupling in the Tessieri–Wilkie model.

Inspired by their works and the papers of Xiang *et al*[15] and Liu *et al*[16] the aim of this paper is to study the dynamics of correlation characteristics between the two coupled centre spins, which constitute our subsystem. The bath in our study is the same model as used in Refs.[12,14], which is prepared in a

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thermal equilibrium state at time $t = 0$. We study the evolution of the spatial correlation between polarizations of the two subsystem spins along three directions x, y and z. And we also discuss the entanglement between the two system spins,[17] which is an essential ingredient in the quantum communication[18,19] and has no classical counterpart. The concurrence of the subsystem is chosen to be a measure of the entanglement between the two spins within the subsystem.[20,21] It will be demonstrated that in our extended model, both the polarization correlation and entanglement of the subsystem can be restored to a great extent to the isolated case by increasing pairwise couplings between the bath spins. And we also try to clarify the physics behind this revival function of intra-bath coupling. The rest of this paper is organized as follows. In Section 2, we introduce the Hamiltonian for our two-centre-spins spin-bath model. In Section 3, we explain the computation procedures about the evolution of the correlation and of the concurrence for the subsystem. Detailed results and discussions are given in Section 4. The conclusion of our study is given in Section 5.

2. The extended-spin spin-bath model

The subsystem we study here consists of two spins antiferromagnetically coupled in the x direction, and aligned in x and z directions respectively by applied fields. The bath consists of $N-2$ spins, in which every pair of the spins are also coupled antiferromagnetically in the x direction, and aligned in both x and z directions. The subsystem interacts with the bath by the antiferromagnetic coupling in the x direction. This model is an extension of the Tessieri–Wilkie model,[12] which comes from the simplification of an experimentally realizable model system. The original model is based on the system of an interstitial He atom in an otherwise pure silicon or diamond cluster, the He atom causing small lattice distortion in the cluster,[22] thus inducing vibronic coupling between the electronic states and phonon states of the cluster. This mechanism is further simplified by considering only the ground state and the first excited state of the interstitial He atom, representing the states with a spin-1/2 Pauli matrix, and representing the phonon states as spin-1/2 modes, which result in the Tessieri–Wilkie model. This model system is regarded as a potential realization for quantum computing devices. Our extension is that instead of one centre spin, we have two centre spins coupled together by an antiferromagnetic coupling. In this extension we merely regard the Tessieri–Wilkie model as a model system and do not go into details of the physical realization. So we will only focus our attention to the extended model itself in the following discussions.

The Hamiltonian of our system can be written as

$$H = H_{\rm S} + H_{\rm B} + H_{\rm SB}, \tag{1}$$

$$H_{\rm S} = \frac{\omega_{\rm s}}{2}\sigma_z^{(1)} + \beta\sigma_x^{(1)} + \frac{\omega_{\rm s}}{2}\sigma_z^{(2)} + \beta\sigma_x^{(2)} + \lambda_{\rm ss}\sigma_x^{(1)}\sigma_x^{(2)}, \tag{2}$$

$$H_{\rm B} = \sum_{i=3}^{N}\frac{\omega_{\rm b}}{2}\sigma_z^{(i)} + \beta\sum_{i=3}^{N}\sigma_x^{(i)} + \lambda_{\rm bb}\sum_{i=3}^{N-1}\sum_{j=i+1}^{N}\sigma_x^{(i)}\sigma_x^{(j)}, \tag{3}$$

$$H_{\rm SB} = \lambda_{\rm sb}\sum_{i=3}^{N}(\sigma_x^{(1)}\sigma_x^{(i)} + \sigma_x^{(2)}\sigma_x^{(i)}), \tag{4}$$

where $H_{\rm S}$ is the Hamiltonian of the subsystem with two interacting spins (labelled as 1, 2) coupled in x direction with coupling strength $\lambda_{\rm ss}$; β, $\omega_s/2$ and $\omega_{\rm b}$ may be regarded as applied fields acting in x and z directions to the sub-system and bath spins. The σ_x and σ_z are the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5}$$

The bath consists of $N-2$ spins labelled by 3 to N, coupled each other in the x direction with coupling strength $\lambda_{\rm bb}$, denoted as $H_{\rm B}$. The coupling between subsystem and bath is given by $H_{\rm SB}$, which is also the coupling of each subsystem spin with each bath spin in the x direction, with coupling strength $\lambda_{\rm sb}$. All the interactions are antiferromagnetic, that is

$$\lambda_{\rm ss} \geq 0, \quad \lambda_{\rm bb} \geq 0, \quad \lambda_{\rm sb} \geq 0. \tag{6}$$

In the following calculation, we use the system of units where the Boltzmann constant $k_{\rm B} = 1$ and Planck constant $\hbar = 1$. Other parameters in the Hamiltonian take the following values in the subsequent computations:

$$\omega_{\rm s} = 0.8, \quad \beta = 0.1, \quad \omega_{\rm b} = 1.0.$$

3. Calculation procedures

In this section, we describe in detail the algorithms for the calculation of the dynamics of the subsystem modelled by the Hamiltonian (1). The initial state of the subsystem is chosen to be one of the four Bell states:

$$|\psi_{\rm S}(0)\rangle^1 = 1/\sqrt{2}(|11\rangle + |00\rangle), \quad (7)$$

$$|\psi_{\rm S}(0)\rangle^2 = 1/\sqrt{2}(|10\rangle + |01\rangle), \quad (8)$$

$$|\psi_{\rm S}(0)\rangle^3 = 1/\sqrt{2}(|11\rangle - |00\rangle), \quad (9)$$

$$|\psi_{\rm S}(0)\rangle^4 = 1/\sqrt{2}(|10\rangle - |01\rangle), \quad (10)$$

where $|0\rangle$ and $|1\rangle$ refer to as the spin 'down' and spin 'up' in the z direction, respectively. The reason that we use them as the initial states is that among all the quantum states for a pair of coupled spins-1/2, Bell bases have the largest degree of entanglement and many other interesting characteristics.[23] However, it should be noted that the last Bell state given in Eq.(10) is a state free from the bath, i.e. the action of the interaction part of the Hamiltonian, $H_{\rm SB}$, on the state gives zero. So the state will evolve with time just as an isolated system. Furthermore, the state is also an eigenstate of the subsystem Hamiltonian $H_{\rm s}$:

$$H_{\rm s}(1/\sqrt{2}(|10\rangle - |01\rangle)) = -1/\sqrt{2}\lambda_{\rm ss}(|10\rangle - |01\rangle). \quad (11)$$

The time evolution of the state is simple with an exponential factor $\mathrm{e}^{\mathrm{i}\omega t}$, with $\omega = -\lambda_{\rm ss}$. Hence the physical quantities will not vary with time in this state and they will not be considered in the following calculations. The initial state of the bath is taken to be the thermal equilibrium state: $\rho_{\rm B}(0) = Z^{-1}\exp(-H_{\rm B}/T)$, where Z is the partition function of the environment expressed by $Z = {\rm Tr}(\exp(-H_{\rm B}/k_{\rm B}T))$. The density matrix $\rho(t)$ of the whole system can formally be represented as

$$\rho(t) = \exp(-\mathrm{i}Ht)\rho(0)\exp(\mathrm{i}Ht), \quad (12)$$

$$\rho(0) = \rho_{\rm S}(0) \otimes \rho_{\rm B}(0), \quad (13)$$

$$\rho_{\rm S}(0) = |\psi_{\rm S}(0)\rangle\langle\psi_{\rm S}(0)|. \quad (14)$$

In order to find the density matrix $\rho(t)$, we follow the method suggested by Tessieri *et al.*[12] The thermal bath state $\rho_{\rm B}(0)$ can be expanded with the eigenstates of the environment Hamiltonian,

$$\rho_{\rm B}(0) = \sum_{n=1}^{2^m} |\phi_n^{\rm (B)}\rangle\omega_n\langle\phi_n^{\rm (B)}|, \quad (15)$$

$$\omega_{\rm n} = \frac{\mathrm{e}^{-E_n/T}}{Z}, \quad (16)$$

$$Z = \sum_{n=1}^{2^m} \mathrm{e}^{-E_n/T}. \quad (17)$$

Here $|\phi_n^{\rm (B)}\rangle$, $n = 1, 2, 3, \cdots, 2^m$, $m = N-2$, are the eigenstates of the environment Hamiltonian $H_{\rm B}$, and E_n the corresponding eigen energies. With this expansion, the density matrix $\rho(t)$ can be written as

$$\rho(t) = \sum_{n=1}^{2^m} \omega_n |\,\Psi_n(t)\rangle\langle\,\Psi_n(t)|, \quad (18)$$

where

$$|\,\Psi_n(t)\rangle = \exp(-\mathrm{i}Ht)|\,\Psi_n(0)\rangle = U(t)|\,\Psi_n(0)\rangle. \quad (19)$$

The initial state is

$$|\,\Psi_n(0)\rangle = |\psi_{\rm S}(0)\rangle|\phi_n^{\rm (B)}\rangle.$$

The evolution operator $U(t)$ can be evaluated by the efficient algorithm of polynomial schemes.[24–26] The method used in this calculation is the Laguerre polynomial expansion we proposed in Ref.[26], which is pretty well suited to this problem and can give accurate result in a comparatively smaller computation load. More precisely, the evolution operator $U(t)$ is expanded in terms of the Laguerre polynomial of the Hamiltonian as

$$U(t) = \mathrm{e}^{-\mathrm{i}Ht} = \left(\frac{1}{1+\mathrm{i}t}\right)^{\alpha+1} \sum_{k=0}^{\infty} \left(\frac{\mathrm{i}t}{1+\mathrm{i}t}\right)^k L_k^{\alpha}(H),$$

where α distinguishes the different types of Laguerre polynomials,[27] k is the order of the Laguerre polynomial. In practical calculations the expansion has to be cut at some value of $k_{\rm max}$, which is taken to be 24 in this study. With the largest order of the expansion fixed, the time step t is restricted to some value in order to obtain accurate results of the evolution operator. For longer times the evolution can be achieved with more steps. The action of the Laguerre polynomial of Hamiltonian to the states is calculated by recurrence relations of the Laguerre polynomial. The efficiency of this polynomial scheme[26] is about 8 times as that of the Runge-Kutta algorithm used in Ref.[12]. When the states $|\,\Psi_n(t)\rangle$ are obtained, the density matrix can be obtained by performing the summation in Eq.(18).

Table 1. Number of the bath states, M, needed in the calculation for different temperatures and thresholds of $\omega_{\rm th}$.

Temperature T	≤ 0.04	0.05	0.08	0.10	0.15	0.20
$M(\omega_n > 0.00001)$	8	8	28	70	70	70
$M(\omega_n > 0.0001)$	1	8	8	28	28	70
$M(\omega_n > 0.001)$	1	1	8	8	28	70

When the temperature is not very high, the weights for high-energy states will be very small so that only a few lowest states need be considered. The maximum states to be included in the summation can be determined by setting a threshold of weight $\omega_{\rm th}$, and keep only those states whose weight $\omega_n \geq \omega_{\rm th}$. As an example, Table 1 gives the number of states to be used for different temperatures and threshold. In this case the number of environment spins $m = 8$, and couplings among the environment spins $\lambda_{\rm bb} = 4.0$. As can be seen from the table, the number of states needed is much less than the total number of states, $2^8 = 256$. In practical calculations the up bound of the summation in Eq.(18) will be $M << 2^m$, chosen according to the criterion specified for the accuracy of results. That is to say, Eq.(18) can be changed into the following equation:

$$\rho(t) = \sum_{n=1}^{M} \omega_n |\,\Psi_n(t)\rangle\langle\,\Psi_n(t)|. \tag{20}$$

After obtaining the density matrix of the whole system, the reduced density matrix is calculated using a partial trace operation to $\rho(t)$, which traces out the degrees of freedom of the environment,

$$\rho_S(t) = {\rm Tr}_{\rm B}(\rho(t)). \tag{21}$$

In the model of this paper, $\rho_{\rm S}(t)$ is the density matrix of the open subsystem consisting of two centre spins, which can be expressed as a 4×4 matrix in the Hilbert space of the subsystem spanned by the orthonormal vectors $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$.

After the reduced density matrix is obtained, any physical quantities of the subsystem can be obtained easily. In the following we concentrate on two important physical quantities of the subsystem, which reflect the decoherence and entanglement degree of the subsystem state. These two quantities are the spatial polarization correlation and the concurrence.

3.1. Polarization correlation

The polarization of either spin in the subsystem is defined as

$$\boldsymbol{P}^{(i)}(t) = {\rm Tr}(\rho^{(i)}(t)\boldsymbol{\sigma}^{(i)}), \quad i = 1, 2, \tag{22}$$

where $\boldsymbol{\sigma} = \sigma_x\boldsymbol{i} + \sigma_y\boldsymbol{j} + \sigma_z\boldsymbol{k}$. To simplify the calculation, we may also trace out one of the spin degrees of freedom to obtain the 2×2 sub-reduced density matrix for each spin:

$$\rho^{(i)}(t) = {\rm Tr}_{\bar{i}}(\rho_S(t)) = \begin{pmatrix} \rho^{(i)}(t)_{11} & \rho^{(i)}(t)_{10} \\ \rho^{(i)}(t)_{01} & \rho^{(i)}(t)_{00} \end{pmatrix}.$$

Here ${\rm Tr}_{\bar{i}}$ means to trace out the other degrees of freedom of i, i.e. trace out 2 when $i = 1$ and trace out 1 when $i = 2$. Then the three components of $\boldsymbol{P}^{(i)}(t)$ are expressed as

$$\langle\sigma_x^i\rangle = {\rm Tr}\left(\rho^{(i)}(t)\sigma_x\right) = \rho^{(i)}(t)_{10} + \rho^{(i)}(t)_{01}, \tag{23}$$

$$\langle\sigma_y^i\rangle = {\rm Tr}\left(\rho^{(i)}(t)\sigma_y\right) = i\left(\rho^{(i)}(t)_{10} - \rho^{(i)}(t)_{01}\right), \tag{24}$$

$$\langle\sigma_z^i\rangle = {\rm Tr}\left(\rho^{(i)}(t)\sigma_z\right) = \rho^{(i)}(t)_{11} - \rho^{(i)}(t)_{00}. \tag{25}$$

The polarization $\boldsymbol{P}^{(i)}(t)$ may be viewed as an indicator of quantum decoherence.

The correlation between the two subsystem spins can be described by the correlation functions defined below,

$$C_{xx} = \langle\sigma_x^{(1)}\sigma_x^{(2)}\rangle - \langle\sigma_x^{(1)}\rangle\langle\sigma_x^{(2)}\rangle, \tag{26}$$

$$C_{yy} = \langle\sigma_y^{(1)}\sigma_y^{(2)}\rangle - \langle\sigma_y^{(1)}\rangle\langle\sigma_y^{(2)}\rangle, \tag{27}$$

$$C_{zz} = \langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle - \langle\sigma_z^{(1)}\rangle\langle\sigma_z^{(2)}\rangle, \tag{28}$$

where $\langle\sigma_\alpha^{(1)}\sigma_\alpha^{(2)}\rangle \equiv {\rm Tr}\left(\rho_S(t)\sigma_\alpha^{(1)}\sigma_\alpha^{(2)}\right)$, $\alpha = x, y, z$. Since $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$, we have $\left(\sigma_\alpha^{(1)} + \sigma_\alpha^{(2)}\right)^2 = 2 + 2\sigma_\alpha^{(1)}\sigma_\alpha^{(2)}$, and the correlation function is also a measurement of the fluctuations of the total spin of the system.

3.2. Concurrence

The concurrence of the two-spin-1/2 system is an indicator of their intra-entanglement, which is defined as[20]

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4,\ 0\}, \tag{29}$$

where λ_i are the square roots of the eigenvalues of the product matrix $\rho_{\rm S}\tilde{\rho}_{\rm S}$ in decreasing order. Equation (29) applies to all kinds of states, either mixed or pure.

The matrix $\tilde{\rho}_{\rm S}$ is constructed as $(\sigma_y \otimes \sigma_y)\rho_{\rm S}^*(\sigma_y \otimes \sigma_y)$. If the bipartite quantum state $\rho_{\rm S}$ is pure,[28] such as the states in Eqs.(7)–(10), they can be written as

$$\begin{aligned}\rho_{\rm S} &= |\psi\rangle\langle\psi|,\\ |\psi\rangle &= a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle,\end{aligned}$$

then Eq.(29) can be simplified to

$$C(|\psi\rangle) = 2|ad - bc|. \tag{30}$$

4. Results and discussions

In this section we give the calculated results of polarization correlations and concurrence with discussions. In all the calculations given here, the parameters are set to be $\lambda_{\rm bb} = \lambda_{\rm ss} = 1.0$ unless otherwise specified. And the temperature is set at $T = 0.1$, much higher than $T = 0.02$ in Ref.[12], in order to stress the generality of the algorithm and conclusion. The results for isolated systems $\lambda_{\rm sb} = 0.0$ are presented as a standard for comparison, in which no decoherence occurs. On the other hand, the case of $\lambda_{\rm bb} = 0.0$ will also be considered, in which a strong decoherence occurs and the subsystem initial state is beyond retrieval.

4.1. Polarization correlation

The polarization correlations of the isolated system are plotted in Figs.1, 2 and 3. We see that the evolutions of the polarization correlations are periodic in time. Then we take into account the case that $\lambda_{\rm bb} = 0.0$, (i.e. no coupling among bath spins), in order to highlight the effect of strong intra-bath coupling. In Fig.4, we show the pure destruction effect on the polarization correlation of subsystem spins imposed by the bath spins ($m = 6$) to the open subsystem, where we neglect the intra-bath coupling strength. The initial state of the subsystem is $|\psi_{\rm S}(0)\rangle^2 = 1/\sqrt{2}(|01\rangle + |10\rangle)$.

Figure 5 through 13 show the polarization correlations when the subsystem coupled to the bath with different $\lambda_{\rm bb}$. The corresponding results for isolated systems are also drawn on each figure as solid lines. In these figures, we have $m = 4$ spins in the bath; though the number of bath spins is small, it still can give some information on the influence of bath coupling to the system. We also perform calculations with more bath spins, $m = 6$, as shown in Fig.14.

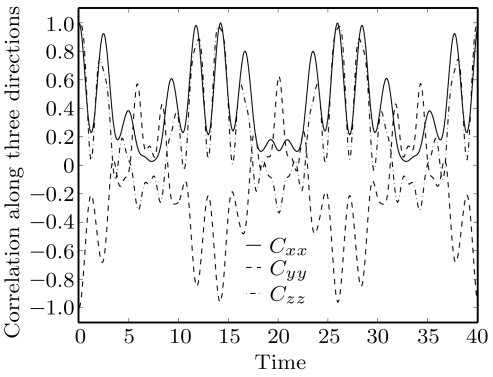


Fig.1. Evolution of polarization correlation in an isolated subsystem. $C_{\alpha\alpha} = \langle\sigma_\alpha^{(1)}\sigma_\alpha^{(2)}\rangle - \langle\sigma_\alpha^{(1)}\rangle\langle\sigma_\alpha^{(2)}\rangle$, where α denotes x, y or z, between the two spins in the subsystem. The initial state of the subsystem is $1/\sqrt{2}(|00\rangle + |11\rangle)$.

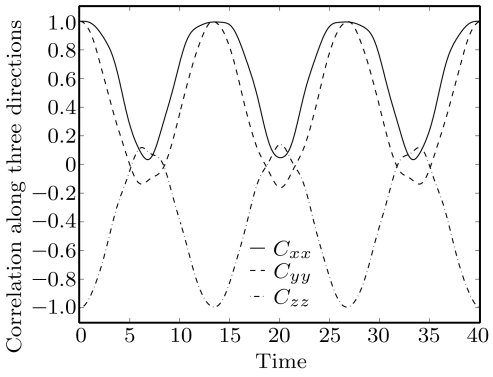


Fig.2. Evolution of polarization correlation in an isolated subsystem. $C_{\alpha\alpha} = \langle\sigma_\alpha^{(1)}\sigma_\alpha^{(2)}\rangle - \langle\sigma_\alpha^{(1)}\rangle\langle\sigma_\alpha^{(2)}\rangle$, where α denotes x, y or z, between the two spins in the subsystem. The initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$.

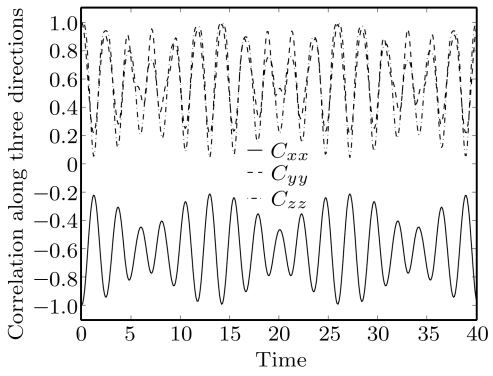
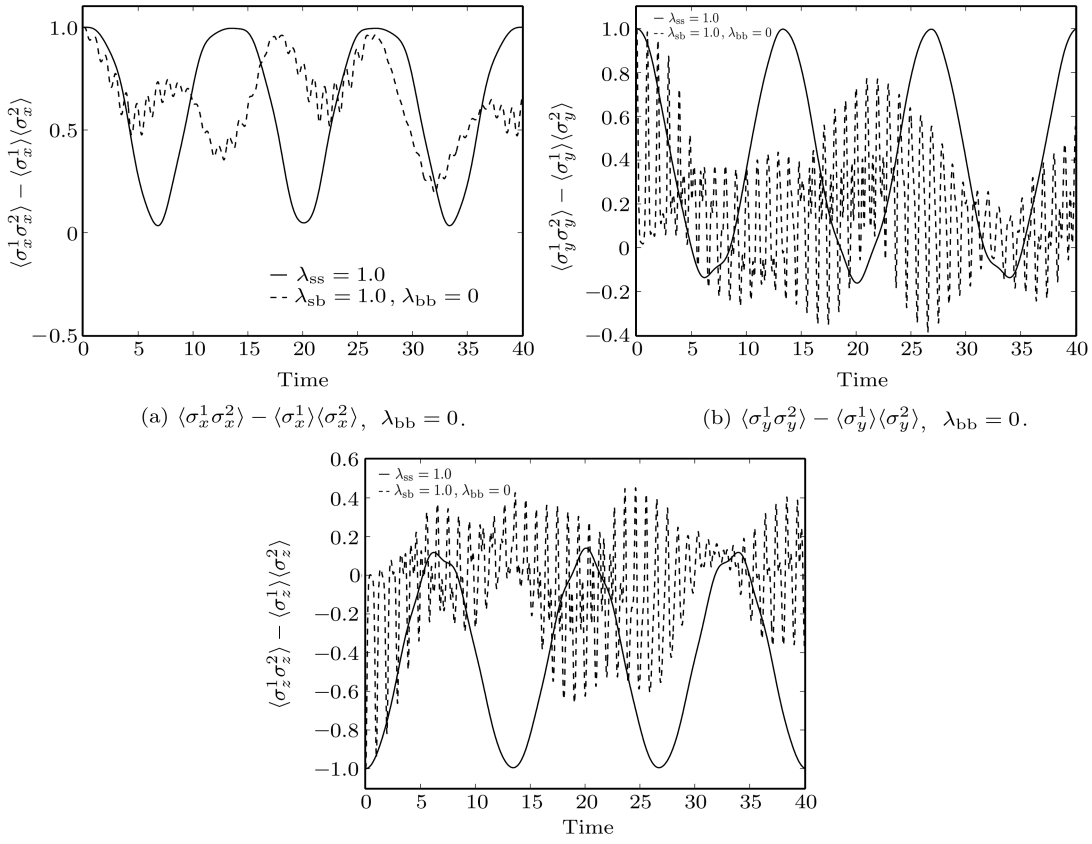


Fig.3. Evolution of polarization correlation in an isolated subsystem. $C_{\alpha\alpha} = \langle\sigma_\alpha^{(1)}\sigma_\alpha^{(2)}\rangle - \langle\sigma_\alpha^{(1)}\rangle\langle\sigma_\alpha^{(2)}\rangle$, where α denotes x, y or z, between the two spins in the subsystem. The initial state of the subsystem is $1/\sqrt{2}(|00\rangle - |11\rangle)$.

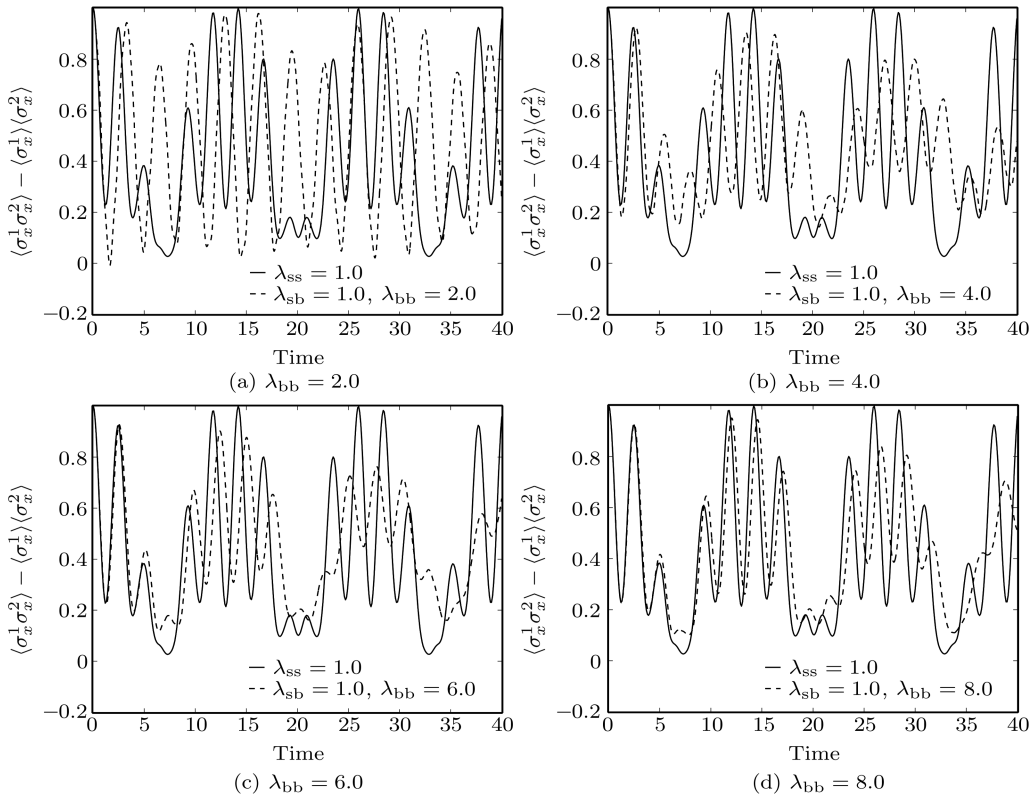


(a) $\langle\sigma_x^1\sigma_x^2\rangle - \langle\sigma_x^1\rangle\langle\sigma_x^2\rangle$, $\lambda_{bb} = 0$.

(b) $\langle\sigma_y^1\sigma_y^2\rangle - \langle\sigma_y^1\rangle\langle\sigma_y^2\rangle$, $\lambda_{bb} = 0$.

(c) $\langle\sigma_z^1\sigma_z^2\rangle - \langle\sigma_z^1\rangle\langle\sigma_z^2\rangle$, $\lambda_{bb} = 0$.

Fig.4. The evolution of polarization correlation along three directions of the open subsystem. There are 6 spins in the bath and the initial state of the subsystem is $1/\sqrt{2}(|10\rangle + |01\rangle)$ and $\lambda_{ss} = \lambda_{sb} = 1.0$, $\lambda_{bb} = 0.0$.



(a) $\lambda_{bb} = 2.0$

(b) $\lambda_{bb} = 4.0$

(c) $\lambda_{bb} = 6.0$

(d) $\lambda_{bb} = 8.0$

Fig.5. Evolution of the polarization correlation along the x direction of the open subsystem, $\langle\sigma_x^1\sigma_x^2\rangle - \langle\sigma_x^1\rangle\langle\sigma_x^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|00\rangle + |11\rangle)$.

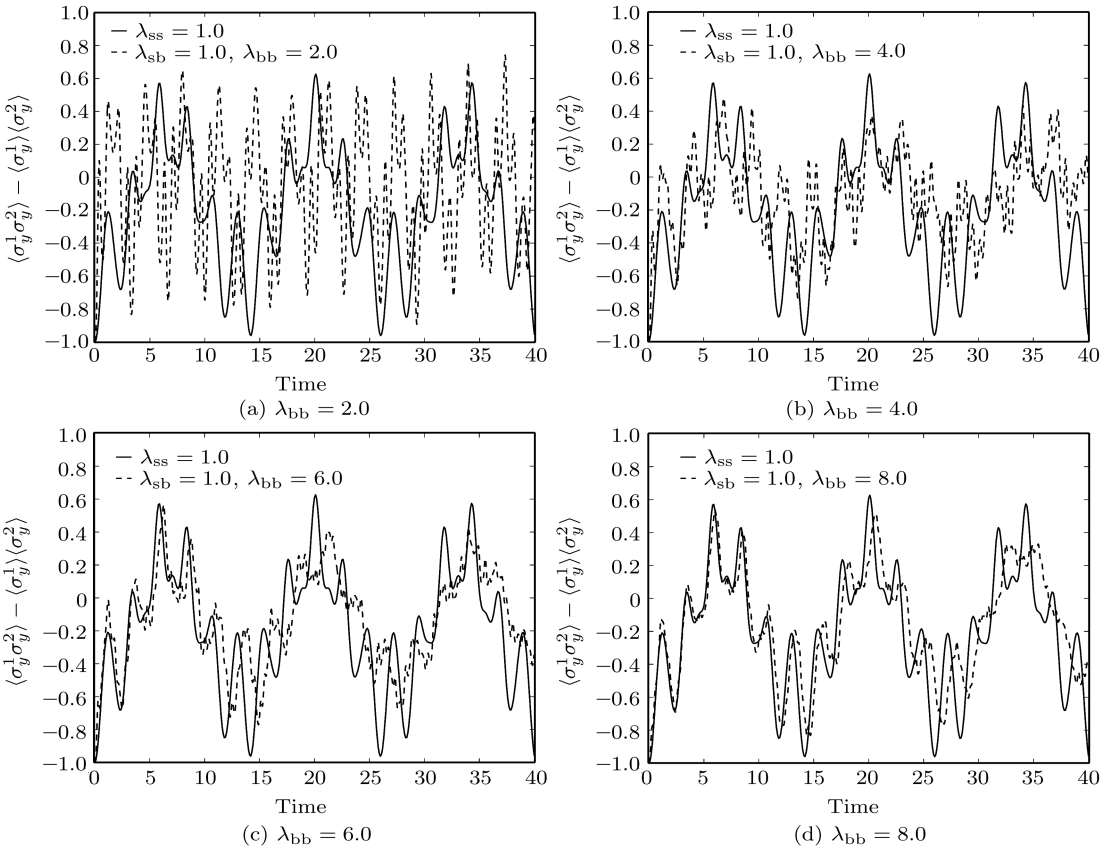


Fig.6. Evolution of the polarization correlation along the y direction of the open subsystem, $\langle\sigma_y^1\sigma_y^2\rangle - \langle\sigma_y^1\rangle\langle\sigma_y^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|00\rangle + |11\rangle)$.

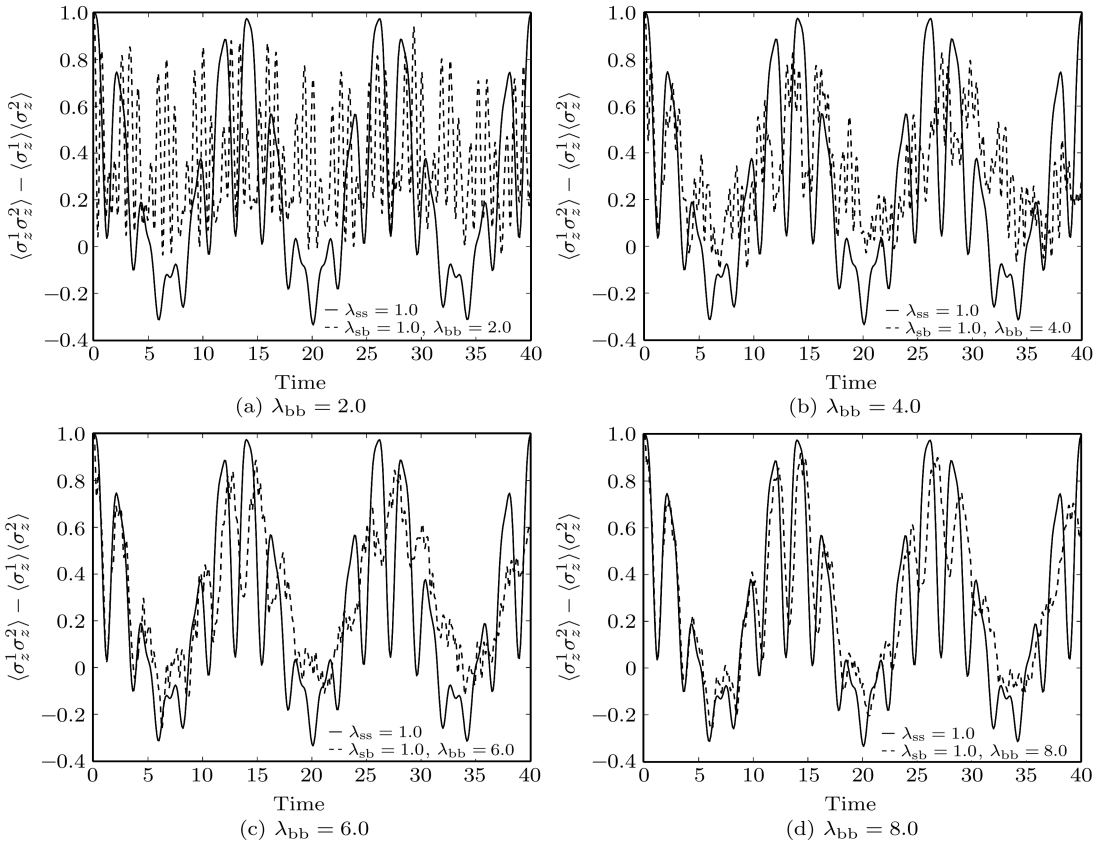
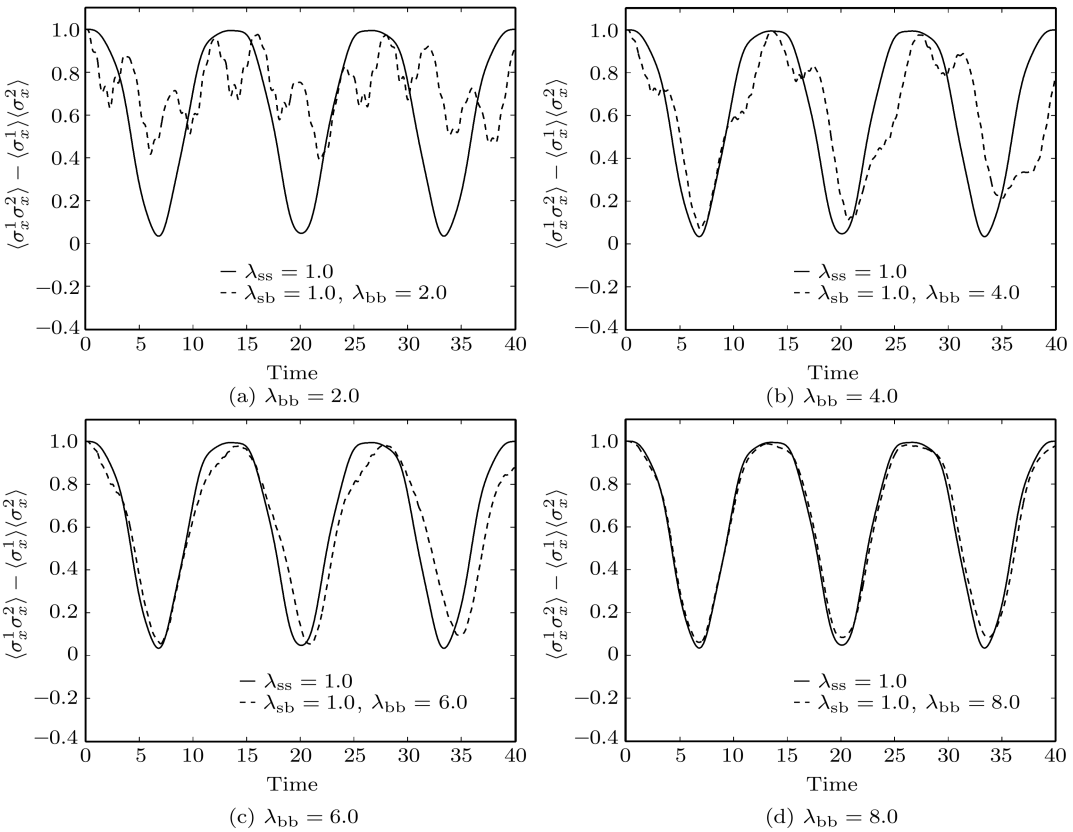


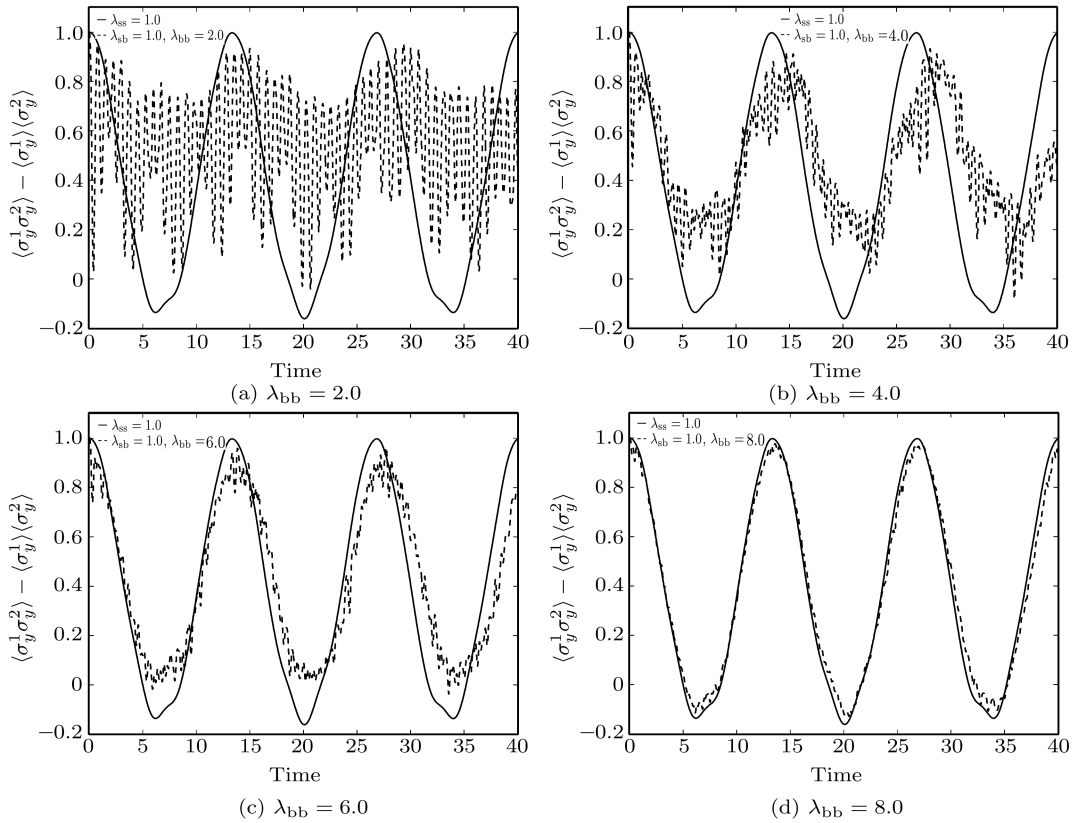
Fig.7. Evolution of the polarization correlation along the z direction of the open subsystem, $\langle\sigma_z^1\sigma_z^2\rangle - \langle\sigma_z^1\rangle\langle\sigma_z^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|00\rangle + |11\rangle)$.



(a) $\lambda_{bb} = 2.0$ (b) $\lambda_{bb} = 4.0$

(c) $\lambda_{bb} = 6.0$ (d) $\lambda_{bb} = 8.0$

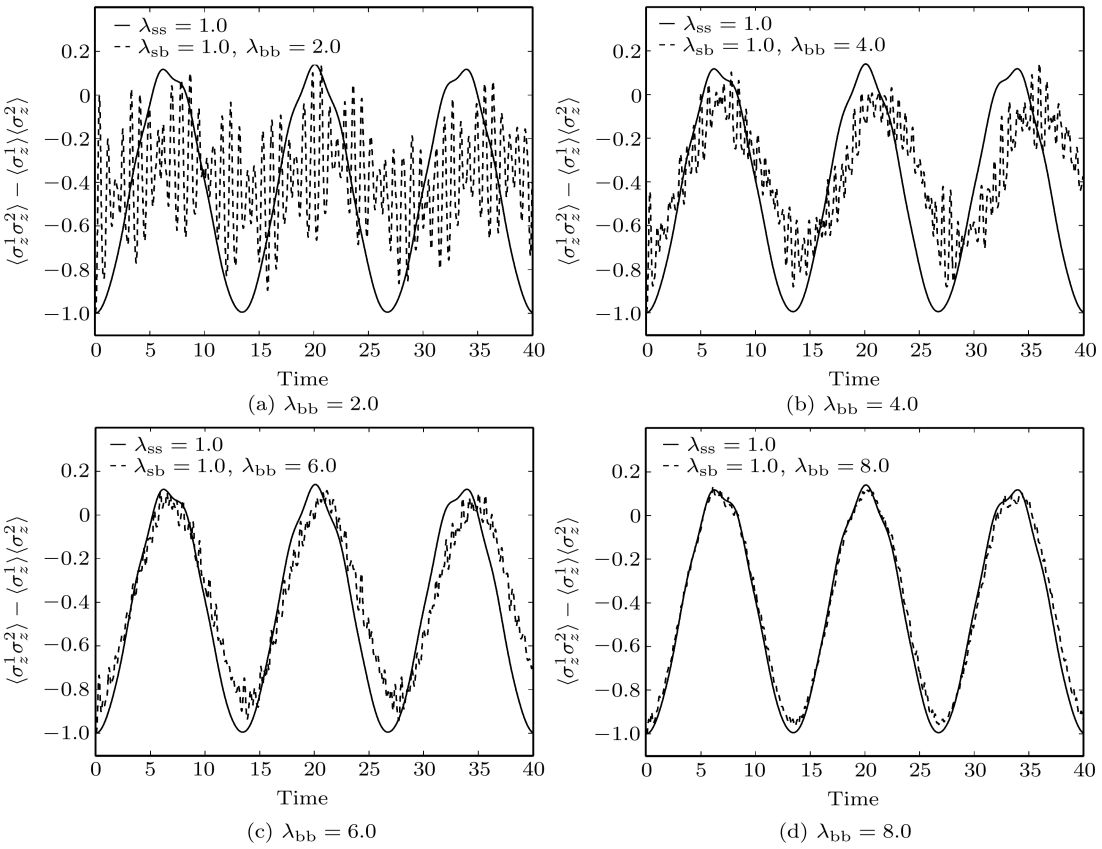
Fig.8. Evolution of the polarization correlation along the x direction of the open subsystem, $\langle\sigma_x^1\sigma_x^2\rangle - \langle\sigma_x^1\rangle\langle\sigma_x^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$.



(a) $\lambda_{bb} = 2.0$ (b) $\lambda_{bb} = 4.0$

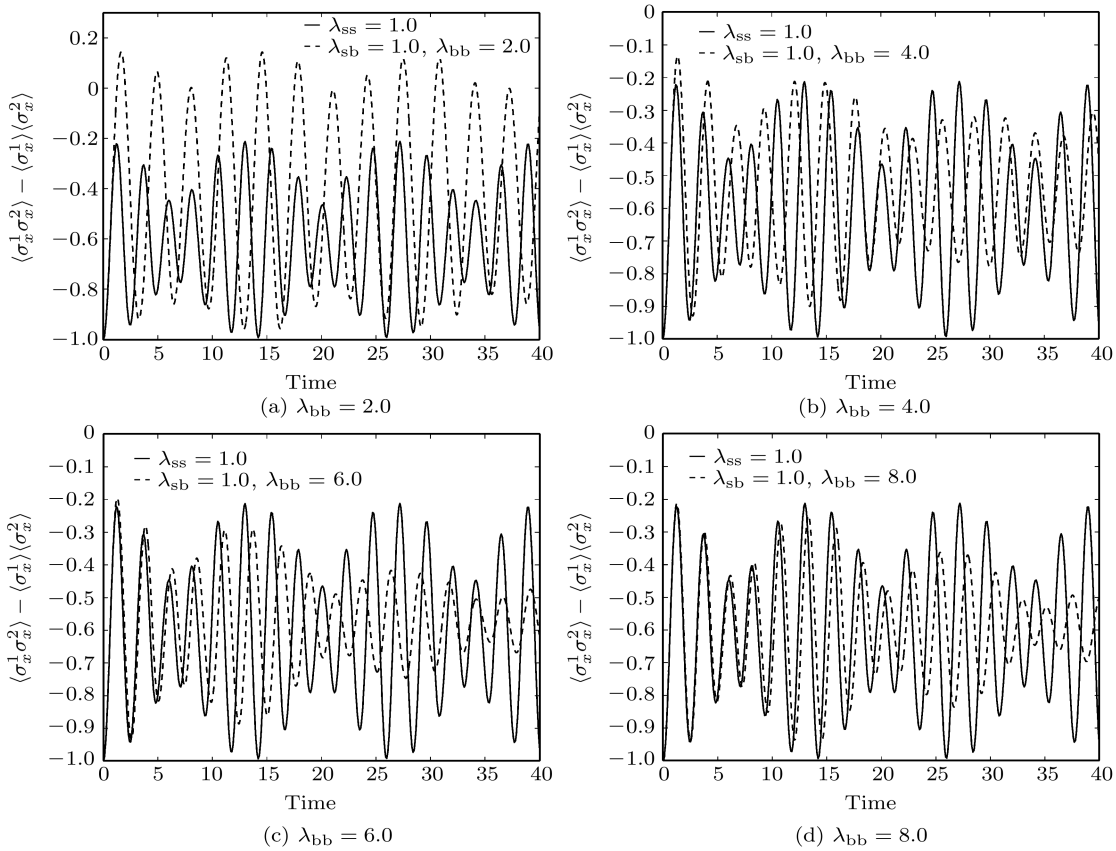
(c) $\lambda_{bb} = 6.0$ (d) $\lambda_{bb} = 8.0$

Fig.9. Evolution of the polarization correlation along the y direction of the open subsystem, $\langle\sigma_y^1\sigma_y^2\rangle - \langle\sigma_y^1\rangle\langle\sigma_y^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$.



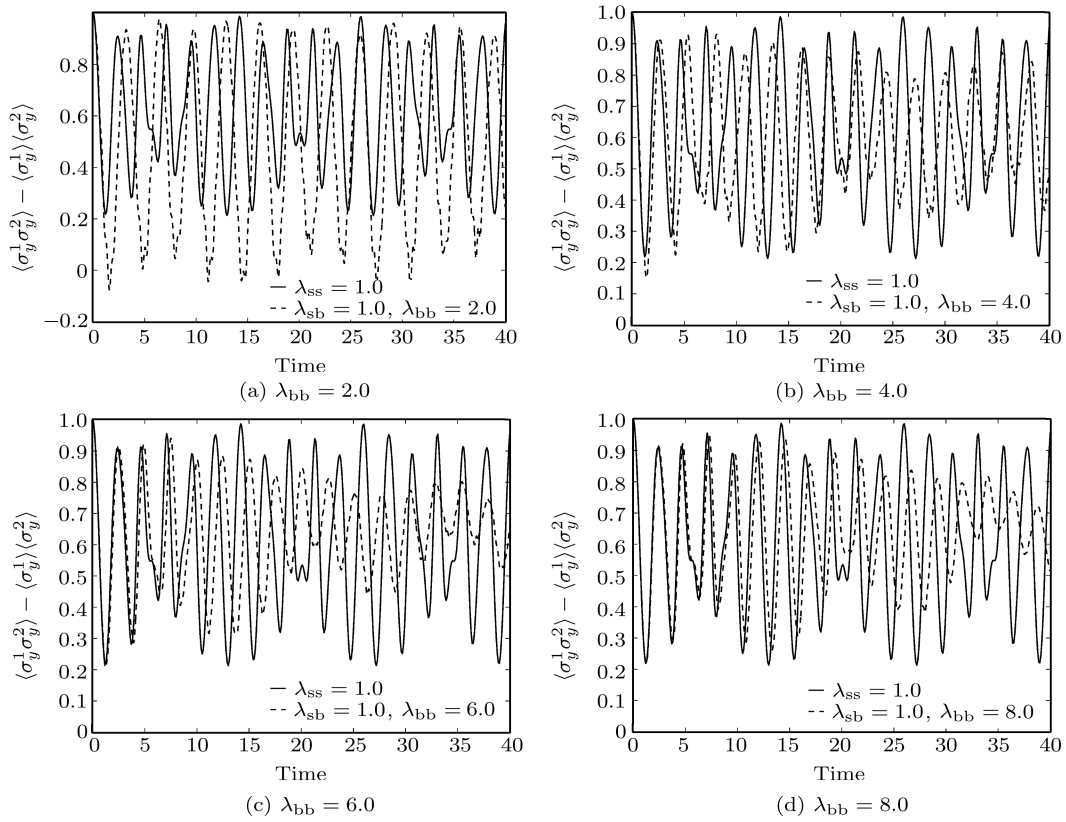
(a) $\lambda_{bb} = 2.0$ (b) $\lambda_{bb} = 4.0$ (c) $\lambda_{bb} = 6.0$ (d) $\lambda_{bb} = 8.0$

Fig.10. Evolution of the polarization correlation along the z direction of the open subsystem, $\langle\sigma_z^1\sigma_z^2\rangle - \langle\sigma_z^1\rangle\langle\sigma_z^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$.



(a) $\lambda_{bb} = 2.0$ (b) $\lambda_{bb} = 4.0$ (c) $\lambda_{bb} = 6.0$ (d) $\lambda_{bb} = 8.0$

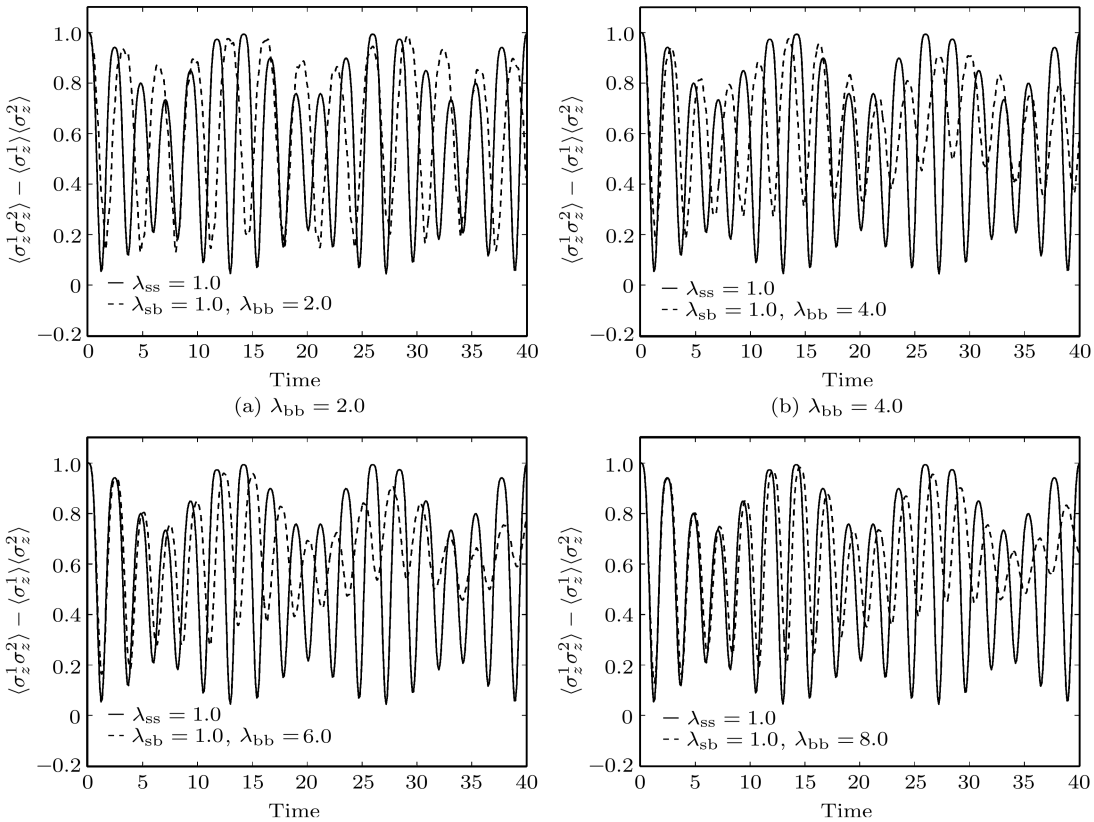
Fig.11. Evolution of the polarization correlation along the x direction of the open subsystem, $\langle\sigma_x^1\sigma_x^2\rangle - \langle\sigma_x^1\rangle\langle\sigma_x^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|11\rangle - |00\rangle)$.



(a) $\lambda_{bb} = 2.0$ (b) $\lambda_{bb} = 4.0$

(c) $\lambda_{bb} = 6.0$ (d) $\lambda_{bb} = 8.0$

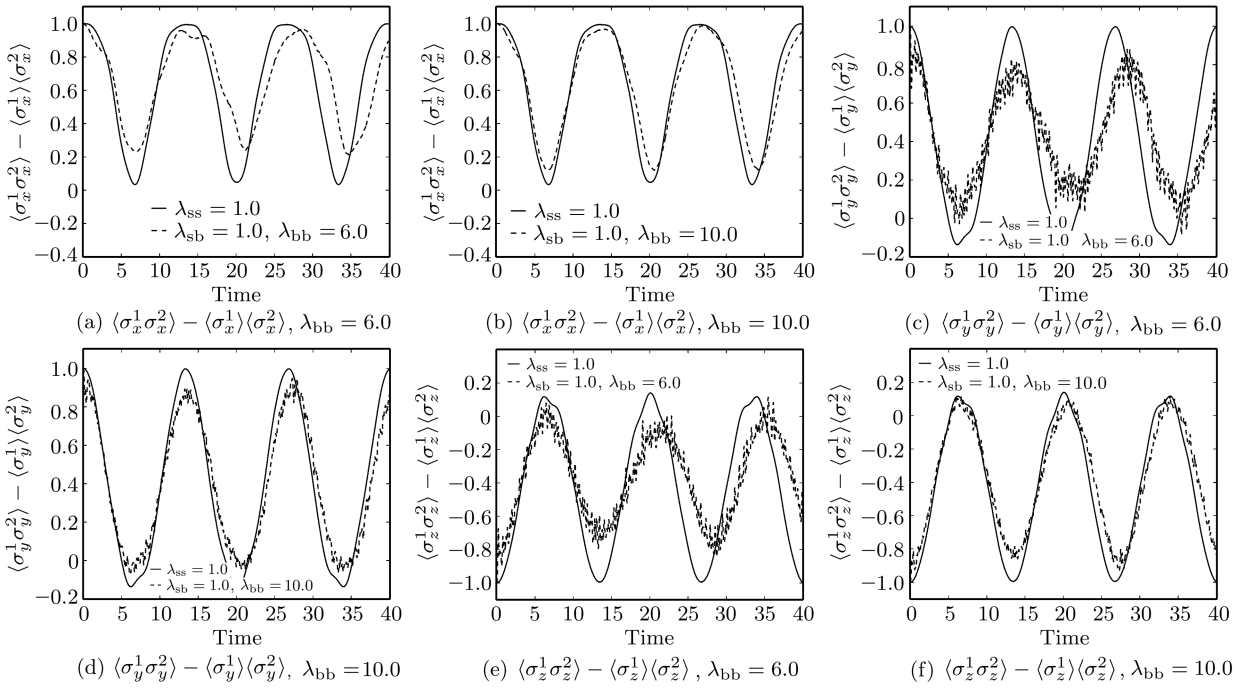
Fig.12. Evolution of the polarization correlation along the y direction of the open subsystem, $\langle\sigma_y^1\sigma_y^2\rangle - \langle\sigma_y^1\rangle\langle\sigma_y^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|11\rangle - |00\rangle)$.



(a) $\lambda_{bb} = 2.0$ (b) $\lambda_{bb} = 4.0$

(c) $\lambda_{bb} = 6.0$ (d) $\lambda_{bb} = 8.0$

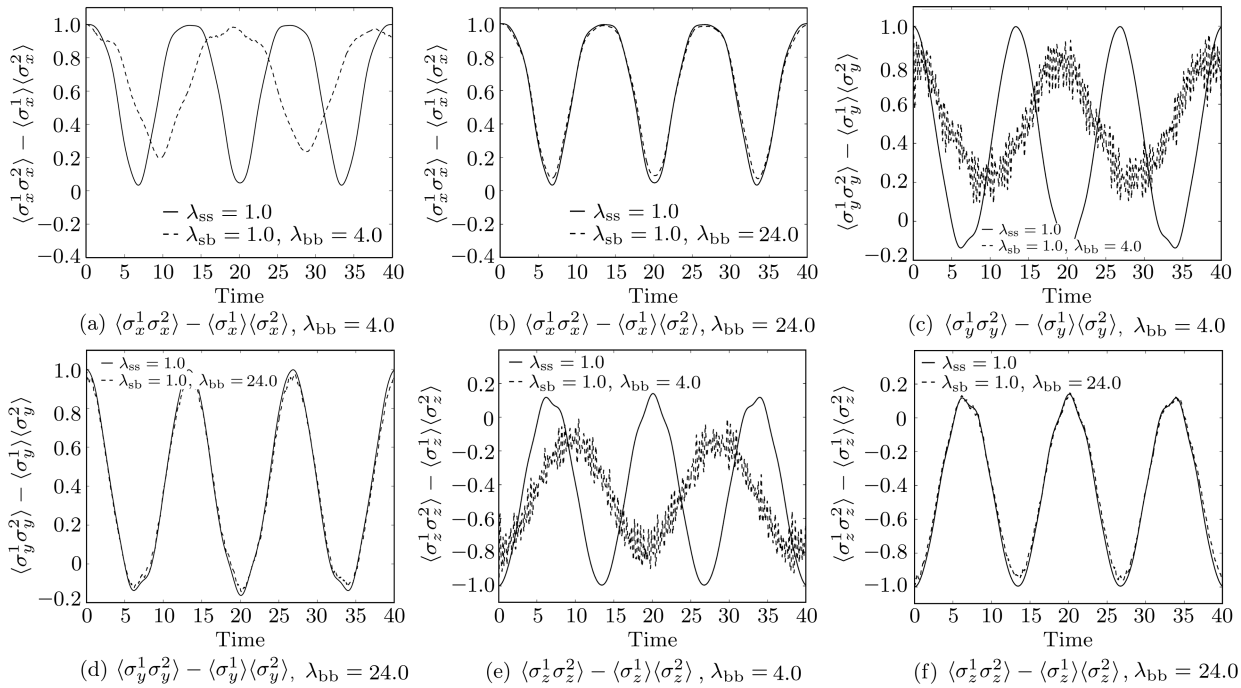
Fig.13. Evolution of the polarization correlation along the z direction of the open subsystem, $\langle\sigma_z^1\sigma_z^2\rangle - \langle\sigma_z^1\rangle\langle\sigma_z^2\rangle$. The initial state of the subsystem is $1/\sqrt{2}(|11\rangle - |00\rangle)$.



(a) $\langle\sigma_x^1\sigma_x^2\rangle - \langle\sigma_x^1\rangle\langle\sigma_x^2\rangle$, $\lambda_{bb} = 6.0$ (b) $\langle\sigma_x^1\sigma_x^2\rangle - \langle\sigma_x^1\rangle\langle\sigma_x^2\rangle$, $\lambda_{bb} = 10.0$ (c) $\langle\sigma_y^1\sigma_y^2\rangle - \langle\sigma_y^1\rangle\langle\sigma_y^2\rangle$, $\lambda_{bb} = 6.0$

(d) $\langle\sigma_y^1\sigma_y^2\rangle - \langle\sigma_y^1\rangle\langle\sigma_y^2\rangle$, $\lambda_{bb} = 10.0$ (e) $\langle\sigma_z^1\sigma_z^2\rangle - \langle\sigma_z^1\rangle\langle\sigma_z^2\rangle$, $\lambda_{bb} = 6.0$ (f) $\langle\sigma_z^1\sigma_z^2\rangle - \langle\sigma_z^1\rangle\langle\sigma_z^2\rangle$, $\lambda_{bb} = 10.0$

Fig.14. Evolution of the polarization correlation along the three directions of the open subsystem, where there are 6 spins in the bath and the initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$.



(a) $\langle\sigma_x^1\sigma_x^2\rangle - \langle\sigma_x^1\rangle\langle\sigma_x^2\rangle$, $\lambda_{bb} = 4.0$ (b) $\langle\sigma_x^1\sigma_x^2\rangle - \langle\sigma_x^1\rangle\langle\sigma_x^2\rangle$, $\lambda_{bb} = 24.0$ (c) $\langle\sigma_y^1\sigma_y^2\rangle - \langle\sigma_y^1\rangle\langle\sigma_y^2\rangle$, $\lambda_{bb} = 4.0$

(d) $\langle\sigma_y^1\sigma_y^2\rangle - \langle\sigma_y^1\rangle\langle\sigma_y^2\rangle$, $\lambda_{bb} = 24.0$ (e) $\langle\sigma_z^1\sigma_z^2\rangle - \langle\sigma_z^1\rangle\langle\sigma_z^2\rangle$, $\lambda_{bb} = 4.0$ (f) $\langle\sigma_z^1\sigma_z^2\rangle - \langle\sigma_z^1\rangle\langle\sigma_z^2\rangle$, $\lambda_{bb} = 24.0$

Fig.15. Evolution of polarization correlations along the three directions of the open subsystem, where there are 8 spins in the bath and the initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$.

It is clear from the figures that when the subsystem is coupled to the bath, all the polarization correlations are different from those in the isolated systems. The difference reflects the influences of the bath and decoherence of the subsystem. However, as pointed out in Refs.[12,14] for the case of a single spin sys-

tem, the coupling strength among environment spins can suppress the decoherence and partially restore the subsystem coherence. Figures 5 through 13 are the plots of the polarization correlations as functions of the time with different initial states and increasing intra-bath coupling strength. At a weak intra-bath coupling, the results are remarkably different from the isolated systems and similar to Fig.4; as the intra-bath coupling is increased, the difference between the open subsystem and the isolated system becomes smaller; and at very strong coupling strengths, the difference becomes very small especially in the case of $|\psi_S(0)\rangle^2$. The other two initial Bell states will be dissipated when the time is prolonged.

These results suggest that by changing the bath coupling strength, one can effectively control the subsystem coherence properties. The suppression of decoherence effect also depends on the number of bath spins; it is found that the more the bath spins, the less effective the suppression of decoherence with the same bath coupling strength. To show this, we provide the results of $\lambda_{\rm bb} = 6.0$ and $\lambda_{\rm bb} = 10.0$ on the Bell state of $|\psi_{\rm S}(0)\rangle^2 = 1/\sqrt{2}(|01\rangle + |10\rangle)$ in Fig.14, where there are $m = 6$ bath spins. The values of other parameters are the same as before. Through the comparison of Fig.14(a) with Fig.8(c), we see that when $m = 4$, $\lambda_{\rm bb} = 6.0$ is almost sufficient to recover the isolated case except a slightly prolonged period (see Fig.8(c)), but when m is increased to 6, the coupling strength need be increased at least to $\lambda_{\rm bb} = 10.0$ to obtain the same effect (see Fig.14(b)). The same feature also appears in the evolution of C_{yy} and C_{zz} which can be seen by comparing Fig.14(c) with Fig.9(c) and Fig.14(e) with Fig.10(c). If further increasing the coupling strength $\lambda_{\rm bb}$, we can suppress the decoherence and disentanglement effect due to the bath with more spins. In Fig.15, we compare the results of the cases of $\lambda_{\rm bb} = 4.0$ and $\lambda_{\rm bb} = 24.0$, in which the subsystem seems to be decoupled from the bath; the initial state is also $1/\sqrt{2}(|01\rangle + |10\rangle)$.

4.2. Concurrence

The effect of coupling strengths of bath spins on the concurrence are also studied. In Figs.16, 17 and 18 we plot the time evolution of the concurrence of three Bell states for the isolated subsystem. An example that the subsystem-bath coupling is considered but the intra-bath coupling strength is zero is shown in Fig.19, which can be compared with Fig.20 because of the same compact condition except for $\lambda_{\rm bb}$.

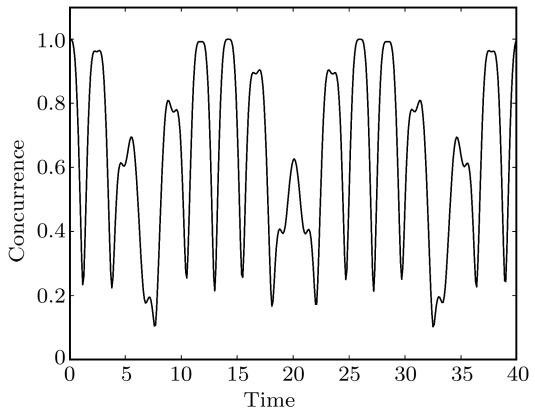


Fig.16. Evolution of the concurrence of the two subsystem spins for the isolated subsystem. The initial state of the subsystem is $1/\sqrt{2}(|00\rangle + |11\rangle)$.

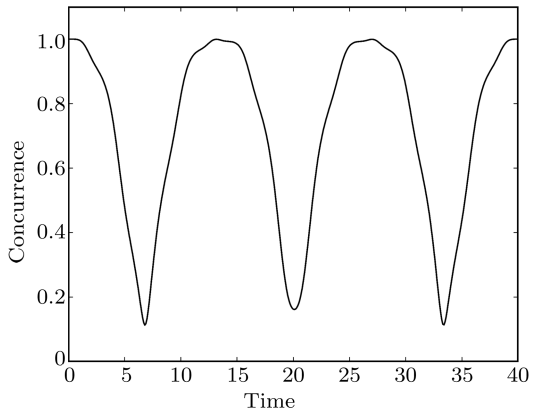


Fig.17. Evolution of the concurrence of the two subsystem spins for the isolated subsystem. The initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$.

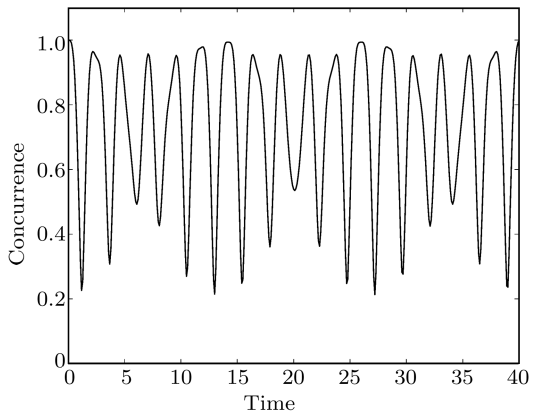


Fig.18. Evolution of the concurrence of the two subsystem spins for the isolated subsystem. The initial state of the subsystem is $1/\sqrt{2}(|00\rangle - |11\rangle)$.

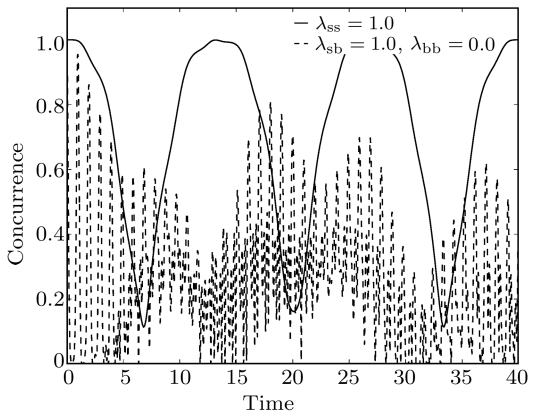
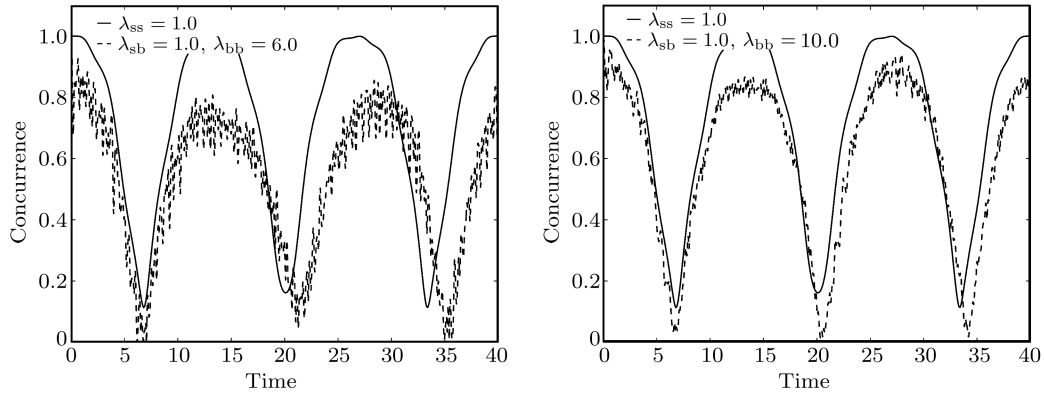


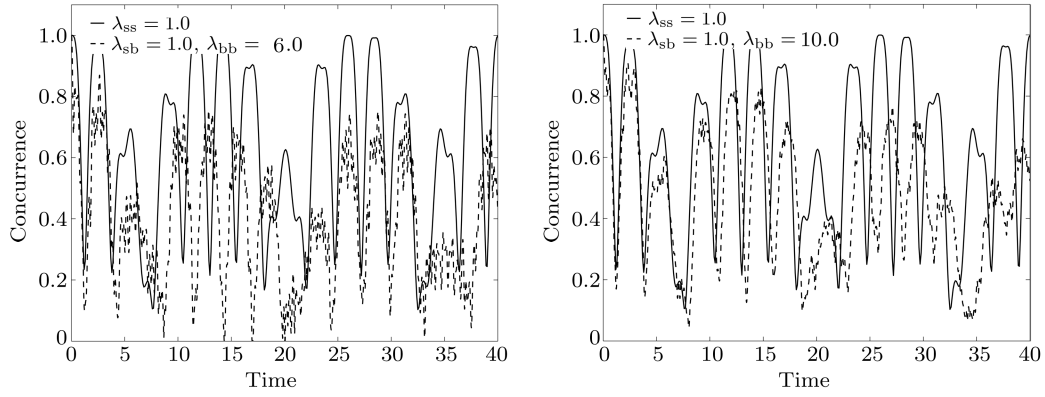
Fig.19. The evolution of concurrence of the open subsystem, where there are 6 spins in the bath and the initial state of the subsystem is $1/\sqrt{2}(|10\rangle + |01\rangle)$ and $\lambda_{ss} = \lambda_{sb} = 1.0$, $\lambda_{bb} = 0.0$.

Figures 20, 21 and 22 are plots of the evolution of concurrence of the three Bell states of the open subsystem. The number of bath spins is $m = 6$ and two coupling strengths of the bath spins ($\lambda_{bb} = 6.0$ and $\lambda_{bb} = 10.0$) are used in the calculation. By comparing the results for $\lambda_{bb} = 6.0$ with that for $\lambda_{bb} = 10.0$, there are visible improvements of the concurrence by increasing the coupling strength of the bath spins. However, it is found that the influence of the same coupling strengths of bath spins has less effect on the concurrence than on the polarization correlations. From Fig.22 we see that even for the strength $\lambda_{bb} = 10.0$, the concurrences are still somewhat different from the isolated case. It is also noted that the influence is different for different initial stats, and it can be seen from Fig.20 that for the case of Bell state $|\psi_S(0)\rangle^2 = 1/\sqrt{2}(|01\rangle + |10\rangle)$, much better suppression of decoherence is observed than for the other two states. Thus we obtain the results for a larger bath with the initial state $|\psi_S(0)\rangle^2$ in Fig.23, where there is $m = 8$ of bath spins. We find that for $\lambda_{bb} = 24$ the evolution of concurrence approaches the dynamics of subsystem evolving in the isolated case.



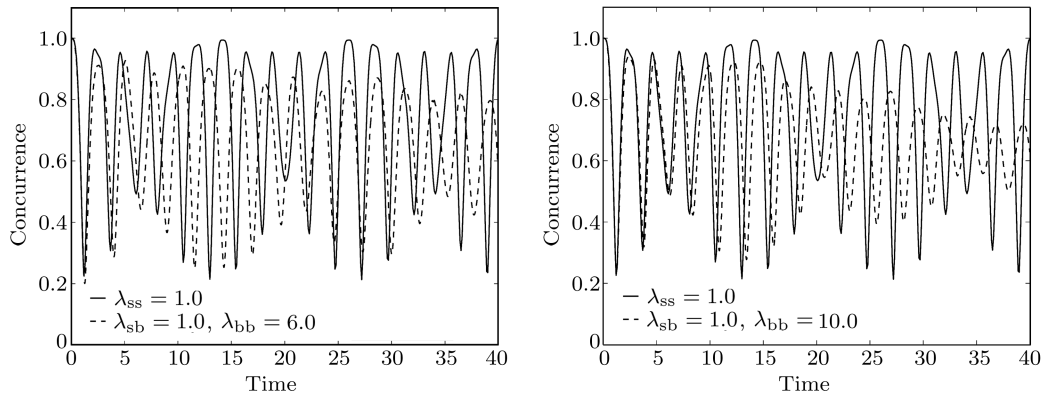
(a) $\lambda_{bb} = 6.0$ (b) $\lambda_{bb} = 10.0$

Fig.20. The evolution of concurrence of the open subsystem, where there are 6 spins in the bath and the initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$, with $\lambda_{ss} = \lambda_{sb} = 1.0$.



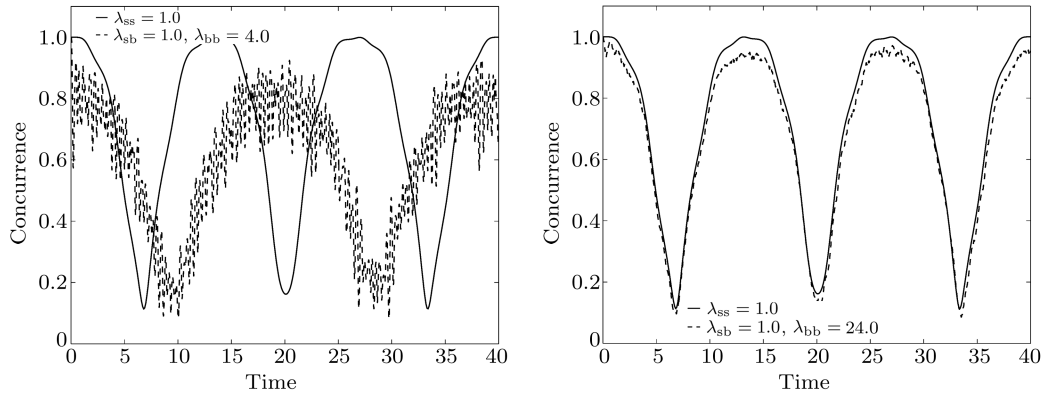
(a) $\lambda_{bb} = 6.0$ (b) $\lambda_{bb} = 10.0$

Fig.21. The evolution of concurrence of the open subsystem, where there are 6 spins in the bath and the initial state of the subsystem is $1/\sqrt{2}(|00\rangle + |11\rangle)$, with $\lambda_{ss} = \lambda_{sb} = 1.0$.



(a) $\lambda_{bb} = 6.0$ (b) $\lambda_{bb} = 10.0$

Fig.22. The evolution of concurrence of the open subsystem, where there are 6 spins in the bath and the initial state of the subsystem is $1/\sqrt{2}(|00\rangle - |11\rangle)$, with $\lambda_{ss} = \lambda_{sb} = 1.0$.



(a) $\lambda_{bb} = 4.0$ (b) $\lambda_{bb} = 24.0$

Fig.23. The evolution of concurrence of the open subsystem, where there are 8 spins in the bath and the initial state of the subsystem is $1/\sqrt{2}(|01\rangle + |10\rangle)$, with $\lambda_{ss} = \lambda_{sb} = 1.0$.

5. Discussion

All the results of comparison in the above suggest that if we did not consider the intra-bath coupling, or we cannot control the intra-bath coupling, the decoherence occurring in our subsystem due to the bath is very serious. Although the number of bath spins in our simulation is not very big, these bath spins can be regarded as the nearest-neighbours to our open subsystem, and the interaction λ_{sb} between them is much larger than the coupling existing between the subsystem spin and other spins in the environment. So the bath we consider in this study can form a safeguard device for the open subsystem, in which the intra-coupling could be adjusted to a high level to counteract the dissipation by itself and the environment.

To understand the physics behind the decoherence suppression effects due to the strong coupling among bath spins, we investigate the states of the bath when the coupling strength changes. The bath spins couple antiferromagnetically with each other in the x direction, causing the bath spins to be in a kind of frustrated state. The other terms in the Hamiltonian favours an aligned ordered state. The final state is produced by the competition between different terms and the thermal fluctuations, and turns out to be very complicated. However, we believe that the decoherence suppression effects are somewhat related to the state ordering of the bath spins.

At temperatures not high enough, the properties of the bath are determined by a few of lowest energy levels, so we will concentrate on these levels and try to figure out the characteristics of the states. In our representation where the z component of the spin is diagonal, it is hard to observe the ordering properties of the states. Since the coupling is in the x direction, we thus transform our states to the x component diag-

onal representation for clarity. This is simply achieved by the following recipe for each spin state:

$$|\varphi^i\rangle_x = U^{-1}|\varphi^i\rangle_z, \tag{31}$$

where U is a 2×2 matrix defined as

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{32}$$

Now we consider a system with 6 bath spins and 2 spins in the subsystem, thus the bath states can be expanded with the product state of the form $|i_1 i_2 i_3 i_4 i_5 i_6\rangle_x = |i_1\rangle_x|i_2\rangle_x|i_3\rangle_x|i_4\rangle_x|i_5\rangle_x|i_6\rangle_x$, where i_α equals 0 or 1; the subscript x means the x-component diagonal representation, and $\alpha = 1, \cdots, 6$ are the index of the 6 bath spins. There are $2^6 = 64$ product states, and for each product state, the subsystem can be in four states $|00\rangle_x$, $|10\rangle_x$,$|01\rangle_x$ and $|11\rangle_x$. Since the Hamiltonian $H_{\rm B}$ is invariant under permutations of the bath spins, the states with the same number of 'up' spins in the x direction have the same expectation values of energy, and thus we can collect the states with the same number of 'up' spins together. Using the number of 'up' spins n to represent such states, we denote it as $||n\rangle$.

The probability that each kind of the product states may appear in the bath state can be determined from the full density matrix $\rho(t)$, i.e.

$$\begin{aligned} P(n) = & \ {\rm Tr}_{\rm S}\langle n||\rho(t)||n\rangle \\ = & \ {\rm Tr}_{\rm s} \sum \omega_m \left|\langle n||\, \Psi_m(t)\rangle\right|^2 . \end{aligned} \tag{33}$$

The ${\rm Tr}_{\rm s}$ means to trace out degrees of freedom of the subsystem. In the case of 6 bath spins, we have calculated the probability for each state $||n\rangle$ and found that the state $||3\rangle$, having 3 spins 'up' and 3 spins 'down' in the x direction, has the major probability, which increases with the coupling strength. Figure 24 is a plot of the probability $P(3)$. We see that for small coupling strength, the probability is about 0.1, and as the coupling strength increases, the probability increases monotonically and reaches 0.8 at $\lambda_{\rm bb} = 10$, where the big suppression effect is obtained. Based on this observation we can conclude that as the coupling strength increases, the bath spins are self organized to a resonant antiferromagnetically ordered state in the x direction, one half of the spins are in the 'up' state and the other half are in the 'down' state. It is this ordering that brings the subsystem to the more coherent state. This mechanism may play an important role in the controlling of the subsystem coherence in quantum devices, despite the inevitable influences of thermal noise upon the quantum device; we may also couple the quantum device to a system as described in this paper. By changing the coupling strength of the added bath spins, one may reduce the decoherence to a level for practical applications.

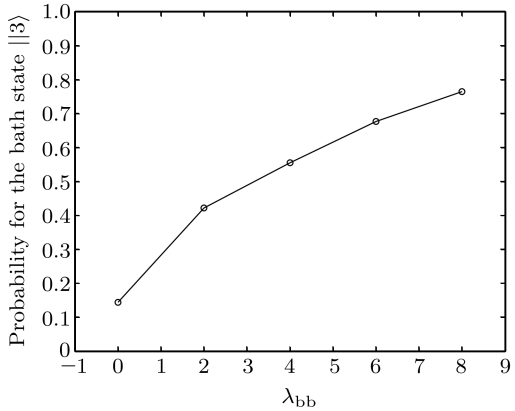


Fig.24. The probability that the bath state has half of the spins in the 'up' state and half in the 'down' state as a function of the coupling strength $\lambda_{\rm bb}$. The total number of bath states is $m = 6$. The initial subsystem state is $1/\sqrt{2}(|01\rangle + |10\rangle)$ with temperature $T = 0.1$.

6. Conclusion

In this paper, we have extended the one-centre-spin spin-bath model[12,14] to a two-centre-spin spin-bath model, which could be thought as an analogue to two correlated qubits in quantum computer. By calculating the polarization correlation and concurrence of the subsystem, we find that the spin-bath can play a revival role in the evolution of polarization correlation and entanglement between two subsystem spins. In the process of calculation, we combine the techniques reported in Refs.[12,26] to reduce the computer resources greatly. The physics of this suppression is found to be the effect of the antiferromagnetically ordering of the bath spins in the x direction. We expect that the results may be of use in the controlling of decoherence of quantum devices.

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