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Stationary entanglement between two spatially separated atoms driven by a coherent laser field*

Liao Xiang-Ping(廖湘萍)[a)b)c)], Fang Mao-Fa(方卯发)[a)†],
Zheng Xiao-Juan(郑小娟)[a)], and Cai Jian-Wu(蔡建武)[a)b)c)]

[a)] *Department of Physics, Hunan Normal University, Changsha* 410081, *China*
[b)] *Department of Physics and Electronics, Zhuzhou Teacher' College, Zhuzhou* 412007, *China*
[c)] *Department of Physics, Hunan Industrial University, Zhuzhou* 412000, *China*

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This paper studies quantum entanglement between two spatially separated atoms driven by a coherent laser field in the dissipative process of spontaneous emission. It is shown that the entanglement strongly depends on the detuning of the laser frequency from atomic transition frequency, the interatomic separation and the Rabi frequency of the coherent laser field. A considerable amount of steady state entanglement can be obtained near $\Delta = -\alpha$ (i.e., the dipole–dipole interaction and the detuning cancel out mutually) for small atomic separation and large Rabi frequency of the coherent laser field.

Keywords: the coherent laser field, spontaneous emission, concurrence
PACC: 4250

1. Introduction

The subject of generation of entangled states has attracted much interest since it became clear that entanglement lies at the heart of many new applications which come under the general heading of quantum information and quantum computation. Several methods of creating entanglement have been proposed involving trapped and cooled ions or neutral atoms.[1−8] Particular interest is paid to the generation of entangled states in two-atom systems, since they can represent two qubits, the building blocks of the quantum gates that are essential to implement quantum protocols in quantum information processing. It has been shown that entangled states in a two-atom system can be created by a continuous driving of the atoms with a coherent or chaotic thermal field,[5,9,10] or by a pulse excitation followed by a continuous observation of radiative decay.[11−13] Moreover, the effect of spontaneous emission on initially prepared entangled state has also been discussed.[14−17] These studies, however, have been limited to the small sample (Dicke) model[18] or the situation involving noninteracting atoms strongly coupled to a cavity mode. The difficulty of the Dicke model is that it does not include the dipole–dipole interaction among the atoms and does not correspond to realistic experimental situations of atoms located(trapped) at different positions. The present atom trapping and cooling techniques can trap two atoms at distances of order of a resonant wavelength,[19−21] which makes questionable the applicability of the Dicke model to physical systems.

Recently, Ficek and Tanás have shown[22,23] that spontaneous emission from two spatially separated atoms can lead to a transient entanglement of initially unentangled atoms. This result contrasts the result with the Dicke model where spontaneous emission cannot produce entanglement from initially unentangled atoms. However, the entanglement is transient and decays to zero over long times. Also, the results are obtained only under the condition that the coherent laser field is not presented.

In this paper, we study quantum entanglement between two spatially separated atoms driven by a coherent laser field in the dissipative process of spontaneous emission. It is shown that the entanglement strongly depends on the detuning of the laser frequency from atomic transition frequency, the inter-

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†Corresponding author. E-mail: mffang@hunnu.edu.cn

atomic separation and the Rabi frequency of the coherent laser field. A considerable amount of steady state entanglement can be obtained near $\Delta = -\alpha$ (i.e., the dipole–dipole interaction and the detuning cancel out mutually) for small atomic separation and large Rabi frequency of the coherent laser field.

2. Master equation

We consider a system of two identical non-overlapping two-level atoms connected by dipole transition moments $\boldsymbol{\mu}_{\rm i}$. The atoms are located at fixed positions $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ and coupled to all modes of the electromagnetic field, which we assume are in the vacuum state. We restrict ourselves to the interaction of the atoms with the ordinary vacuum, and driven by a coherent laser field of constant amplitude $\xi^{'}$, and of frequency $\omega_{\rm L}$. In a frame rotating at the laser frequency $\omega_{\rm L}$ and in the rotating-wave approximation, the dynamics of such system is given by the master equation[24,25]

$$\begin{aligned}\frac{\partial\rho}{\partial t} = & -{\rm i}\sum_{i=1}^{2}\left\{\delta^{'}[S_i^z,\rho]-\frac{1}{2}\Omega[(S_i^+{\rm e}^{-{\rm i}\varphi_L}+S_i^-{\rm e}^{{\rm i}\varphi_L}),\rho]\right\}\\ & -{\rm i}\sum_{i\neq j}^{2}\Omega_{ij}[S_i^+S_j^-,\rho]-\sum_{i,j=1}^{2}\Gamma_{ij}(\rho S_i^+S_j^-+S_i^+S_j^-\rho-2S_j^-\rho S_i^+).\end{aligned}\tag{1}$$

Here $\Omega = |\boldsymbol{\mu}\cdot\boldsymbol{\xi}^{'}|/\hbar$ is the Rabi frequency associated with the driving field of amplitude $\xi^{'}$ and phase $\varphi_{\rm L}$, $\delta^{'} = \omega_0 - \omega_{\rm L}$, $2\Gamma_{ii} = 2\Gamma_{jj} = 2\Gamma$ is the Einstein A coefficient, and $S_i^+(S_i^-)$ are the dipole raising(lowering) operators, S_i^z is the energy operator of the ith atom. The parameters Γ_{ij} and $\Omega_{ij}(i\neq j)$ describe the collective damping and the collective frequency shift respectively, and are produced by the mutual interaction of the atoms through their own electromagnetic field. These parameters depend on the interatomic distance r_{12}, and on the geometrical configuration of the atomic dipoles, and are given by

$$\Gamma_{ij} = \Gamma_{ji} = \frac{3}{2}\Gamma\left\{[1-(\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{ij})^2]\frac{\sin(k_0r_{ij})}{k_0r_{ij}}+[1-3(\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{ij})^2]\left[\frac{\cos(k_0r_{ij})}{(k_0r_{ij})^2}-\frac{\sin(k_0r_{ij})}{(k_0r_{ij})^3}\right]\right\},\tag{2}$$

and

$$\Omega_{ij} = \frac{3}{2}\Gamma\left\{-[1-(\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{ij})^2]\frac{\cos(k_0r_{ij})}{k_0r_{ij}}+[1-3(\hat{\boldsymbol{\mu}}\cdot\hat{\boldsymbol{r}}_{ij})^2]\left[\frac{\sin(k_0r_{ij})}{(k_0r_{ij})^2}+\frac{\cos(k_0r_{ij})}{(k_0r_{ij})^3}\right]\right\},\tag{3}$$

where $k_0 = \omega_0/c$, $r_{ij} = |\boldsymbol{r}_j - \boldsymbol{r}_i|$ is the distance between the atoms, $\hat{\boldsymbol{\mu}}$ is unit vector along the atomic transition dipole moments, assumed to be parallel to each other, and $\hat{\boldsymbol{r}}_{ij}$ is the unit vector along the interatomic axis.

In Eq.(1), we have specified the orientation of the interatomic axis to be perpendicular to the direction of propagation of the laser beam. This is not an essential feature as we are interested in effects which occur at very small interatomic separations where the orientation of the atomic axis is not important. However, these effects are significant for large interatomic separations.

In the following, it will be convenient for us to use the bases of the collective states[18,24] $B = \{|e\rangle, |s\rangle, |a\rangle, |g\rangle\}$, where

$$\begin{aligned}|e\rangle &= |e_1\rangle|e_2\rangle,\\ |s\rangle &= \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle+|g_1\rangle|e_2\rangle),\\ |a\rangle &= \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle-|g_1\rangle|e_2\rangle),\\ |g\rangle &= |g_1\rangle|g_2\rangle.\end{aligned}\tag{4}$$

The most important property of the collective states is

that the symmetric state $|s\rangle$ and antisymmetric state $|a\rangle$ are maximally entangled states.

Using the master equation (1), we can write down the equations of motion for the components of the density matrix of the two-atom system in the presence of the coherent laser field in the collective states bases (for simplicity assuming phase $\varphi_{\rm L} = 0$ in this paper).

$$\begin{aligned}
\dot{\rho}_{ee} &= -2\rho_{ee} - \mathrm{i}\beta(\rho_{es} - \rho_{se}),\\
\dot{\rho}_{ss} &= -(1+\gamma)(\rho_{ss} - \rho_{ee}) + \mathrm{i}\beta[(\rho_{gs} - \rho_{sg}) + (\rho_{es} - \rho_{se})],\\
\dot{\rho}_{aa} &= -(1-\gamma)(\rho_{aa} - \rho_{ee}),\\
\dot{\rho}_{se} &= -\frac{1}{2}[(3+\gamma) - \mathrm{i}(\Delta - \alpha)]\rho_{se} + \mathrm{i}\beta[\rho_{ge} + (\rho_{ee} - \rho_{ss})],\\
\dot{\rho}_{gs} &= -\mathrm{i}\beta - \frac{1}{2}[(1+\gamma) - \mathrm{i}(\Delta + \alpha)]\rho_{gs} + (1+\gamma)\rho_{se} + \mathrm{i}\beta[2\rho_{ss} + \rho_{ee} + \rho_{aa} - \rho_{ge}],\\
\dot{\rho}_{ge} &= -(1 - \mathrm{i}\Delta)\rho_{ge} + \mathrm{i}\beta(\rho_{se} - \rho_{gs}),
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
\dot{\rho}_{ga} &= -\frac{1}{2}[(1-\gamma) - \mathrm{i}(\Delta - \alpha)]\rho_{ga} - (1-\gamma)\rho_{ae} + \mathrm{i}\beta\rho_{sa},\\
\dot{\rho}_{ae} &= -\frac{1}{2}[(3-\gamma) - \mathrm{i}(\Delta + \alpha)]\rho_{ae} - \mathrm{i}\beta\rho_{as},\\
\dot{\rho}_{sa} &= -(1 + \mathrm{i}\alpha)\rho_{sa} + \mathrm{i}\beta(\rho_{ga} + \rho_{ea}),
\end{aligned} \tag{6}$$

with the condition $\rho_{gg}+\rho_{ee}+\rho_{ss}+\rho_{aa} = 1$. Where the dot indicates differentiation with respect to $\tau = 2\Gamma t$. The other parameters have also been made dimensionless: $\Delta = \delta'/\Gamma$, $\alpha = \Omega_{12}/\Gamma$, $\gamma = \Gamma_{12}/\Gamma$, and $\beta = \Omega/(2\sqrt{2}\Gamma)$. It is evident from Eqs.(5) and (6) that when the atoms are in their ground states just before the laser is turned on, the matrix elements(6) retain their initial zero values, and only the set of nine equations (5) can have non-zero steady-state solutions.

Setting $\dot{\rho_{ij}} = 0$ in equation (5) we obtain the steady-state values of density matrix elements

$$\begin{aligned}
\rho_{ee} &= \rho_{aa} = 4\beta^4/D,\\
\rho_{ss} &= 4\beta^2(1 + \Delta^2 + \beta^2)/D,\\
\rho_{se} &= -4\mathrm{i}\beta^3(1 + \mathrm{i}\Delta)/D,\\
\rho_{gs} &= -2\mathrm{i}\beta\{2\beta^2(1 + \mathrm{i}\Delta) + (1 + \Delta^2)[(1+\gamma)\\
&\quad +\mathrm{i}(\Delta + \alpha)]\}/D\\
\rho_{ge} &= -2\beta^2(1 + \mathrm{i}\Delta)[(1+\gamma) + \mathrm{i}(\Delta + \alpha)]/D,
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
D = \;& 16\beta^4 + 8(1+\Delta^2)\beta^2 + (1+\Delta^2)[(1+\gamma)^2\\
&+(\Delta+\alpha)^2].
\end{aligned} \tag{8}$$

It is seen from Eqs.(7) and (8) that the populations of the collective atomic levels as well as the coherences exhibit resonant behaviours not only on the one-photon resonance $\Delta = -\alpha$, but also on the two-photon resonance $\Delta = 0$. Next, we transform them into the original bases $|e_1\rangle \otimes |e_2\rangle$, $|e_1\rangle \otimes |g_2\rangle$, $|g_1\rangle \otimes |e_2\rangle$, $|g_1\rangle \otimes |g_2\rangle$ in order to calculate the concurrence. In Section 3, we will use Eqs.(7) and (8) to explore quantum entanglement between two spatially separated atoms driven by a coherent laser field in the dissipative process of spontaneous emission.

3. Steady state entanglement

Several different measures have been proposed to identify entanglement between two atoms, and we choose the Wootters entanglement measure,[26] the concurrence C, which is defined as

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{9}$$

where $\lambda_1, \cdots, \lambda_4$ are the eigenvalues of the matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, σ_y is the Pauli matrix, ρ is the density matrix representing the quantum state and the matrix elements are taken with respect to the bases $|e_1\rangle\otimes|e_2\rangle$, $|e_1\rangle\otimes|g_2\rangle$, $|g_1\rangle\otimes|e_2\rangle$, $|g_1\rangle\otimes|g_2\rangle$. The range of the concurrence is from 0 to 1. For unentangled atoms $C = 0$ whereas $C = 1$ for the maximally entangled atoms.

We assume that the matrix $\tilde{\rho}$, needed for the calculation of the concurrence, has the form

$$\tilde{\rho} = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{bmatrix}. \tag{10}$$

The eigenvalues of above matrix $\tilde{\rho}$ satisfy the fourth order polynomial

$$\lambda^4 + a\lambda^3 + b\lambda^2 + c\lambda + d = 0, \tag{11}$$

where

$$\begin{aligned}
a = & -\rho_{11} - \rho_{22} - \rho_{33} - \rho_{44}, \\
b = & -\rho_{31}\rho_{13} + \rho_{11}\rho_{33} - \rho_{41}\rho_{14} - \rho_{21}\rho_{12} + \rho_{22}\rho_{33} + \rho_{22}\rho_{44} + \rho_{33}\rho_{44} - \rho_{34}\rho_{43} - \rho_{32}\rho_{23} - \rho_{42}\rho_{24} \\
& +\rho_{11}\rho_{22} + \rho_{11}\rho_{44}, \\
c = & \rho_{21}\rho_{12}\rho_{33} - \rho_{42}\rho_{23}\rho_{34} + \rho_{32}\rho_{23}\rho_{44} - \rho_{21}\rho_{32}\rho_{13} + \rho_{11}\rho_{42}\rho_{24} - \rho_{11}\rho_{33}\rho_{44} + \rho_{22}\rho_{34}\rho_{43} - \rho_{21}\rho_{42}\rho_{14} \\
& -\rho_{32}\rho_{24}\rho_{43} - \rho_{22}\rho_{33}\rho_{44} + \rho_{42}\rho_{24}\rho_{33} + \rho_{11}\rho_{34}\rho_{43} + \rho_{31}\rho_{22}\rho_{13} - \rho_{31}\rho_{12}\rho_{23} + \rho_{31}\rho_{13}\rho_{44} - \rho_{31}\rho_{14}\rho_{43} \\
& -\rho_{41}\rho_{12}\rho_{24} - \rho_{11}\rho_{22}\rho_{33} - \rho_{11}\rho_{22}\rho_{44} + \rho_{41}\rho_{22}\rho_{14} - \rho_{41}\rho_{13}\rho_{34} + \rho_{41}\rho_{14}\rho_{33} + \rho_{21}\rho_{12}\rho_{44} + \rho_{11}\rho_{32}\rho_{23}, \\
d = & \rho_{11}\rho_{22}\rho_{33}\rho_{44} - \rho_{11}\rho_{22}\rho_{34}\rho_{43} - \rho_{11}\rho_{32}\rho_{23}\rho_{44} + \rho_{11}\rho_{32}\rho_{24}\rho_{43} + \rho_{11}\rho_{42}\rho_{23}\rho_{34} \\
& -\rho_{11}\rho_{42}\rho_{24}\rho_{33} - \rho_{21}\rho_{12}\rho_{33}\rho_{44} + \rho_{21}\rho_{12}\rho_{34}\rho_{43} + \rho_{21}\rho_{32}\rho_{13}\rho_{44} - \rho_{21}\rho_{32}\rho_{14}\rho_{43} \\
& -\rho_{21}\rho_{42}\rho_{13}\rho_{34} + \rho_{21}\rho_{42}\rho_{14}\rho_{33} + \rho_{31}\rho_{12}\rho_{23}\rho_{44} - \rho_{31}\rho_{12}\rho_{24}\rho_{43} - \rho_{31}\rho_{22}\rho_{13}\rho_{44} \\
& +\rho_{31}\rho_{22}\rho_{14}\rho_{43} + \rho_{31}\rho_{42}\rho_{13}\rho_{24} - \rho_{31}\rho_{42}\rho_{14}\rho_{23} - \rho_{41}\rho_{12}\rho_{23}\rho_{34} + \rho_{41}\rho_{12}\rho_{24}\rho_{33} \\
& +\rho_{41}\rho_{22}\rho_{13}\rho_{34} - \rho_{41}\rho_{22}\rho_{14}\rho_{33} - \rho_{41}\rho_{32}\rho_{13}\rho_{24} + \rho_{41}\rho_{32}\rho_{14}\rho_{23}.
\end{aligned} \tag{12}$$

From Eq.(11), we can obtain the eigenvalues of matrix $\tilde{\rho}$

$$\begin{aligned}
\lambda_1 = & -\frac{1}{4}a + \frac{1}{12}3^{\frac{1}{2}}\xi^{\frac{1}{2}} + \frac{1}{12}[18a^2 - 48b - 6\delta^{\frac{1}{3}} + 72/\delta^{\frac{1}{3}}ca - 288/\delta^{\frac{1}{3}}d - 24/\delta^{\frac{1}{3}}b^2 \\
& -18/\xi^{\frac{1}{2}}3^{\frac{1}{2}}a^3 + 72/\xi^{\frac{1}{2}}3^{\frac{1}{2}}ab - 144/\xi^{\frac{1}{2}}3^{\frac{1}{2}}c]^{\frac{1}{2}}, \\
\lambda_2 = & -\frac{1}{4}a + \frac{1}{12}3^{\frac{1}{2}}\xi^{\frac{1}{2}} - \frac{1}{12}[18a^2 - 48b - 6\delta^{\frac{1}{3}} + 72/\delta^{\frac{1}{3}}ca - 288/\delta^{\frac{1}{3}}d - 24/\delta^{\frac{1}{3}}b^2 \\
& -18/\xi^{\frac{1}{2}}3^{\frac{1}{2}}a^3 + 72/\xi^{\frac{1}{2}}3^{\frac{1}{2}}ab - 144/\xi^{\frac{1}{2}}3^{\frac{1}{2}}c]^{\frac{1}{2}}, \\
\lambda_3 = & -\frac{1}{4}a - \frac{1}{12}3^{\frac{1}{2}}\xi^{\frac{1}{2}} + \frac{1}{12}6^{\frac{1}{2}}[3a^2 - 8b - \delta^{\frac{1}{3}} + 12/\delta^{\frac{1}{3}}ca - 48/\delta^{\frac{1}{3}}d - 4/\delta^{\frac{1}{3}}b^2 \\
& +3/\xi^{\frac{1}{2}}3^{\frac{1}{2}}a^3 - 12/\xi^{\frac{1}{2}}3^{\frac{1}{2}}ab + 24/\xi^{\frac{1}{2}}3^{\frac{1}{2}}c]^{\frac{1}{2}}, \\
\lambda_4 = & -\frac{1}{4}a - \frac{1}{12}3^{\frac{1}{2}}\xi^{\frac{1}{2}} - \frac{1}{12}6^{\frac{1}{2}}[3a^2 - 8b - \delta^{\frac{1}{3}} + 12/\delta^{\frac{1}{3}}ca - 48/\delta^{\frac{1}{3}}d - 4/\delta^{\frac{1}{3}}b^2 \\
& +3/\xi^{\frac{1}{2}}3^{\frac{1}{2}}a^3 - 12/\xi^{\frac{1}{2}}3^{\frac{1}{2}}ab + 24/\xi^{\frac{1}{2}}3^{\frac{1}{2}}c]^{\frac{1}{2}},
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
\delta = & -36bca - 288db + 108c^2 + 108da^2 + 8b^3 + 12[18c^2a^2d - 3c^2a^2b^2 + 240cadb^2 \\
& -54bca^3d + 12c^3a^3 + 384d^2b^2 - 48db^4 + 12c^2b^3 + 81d^2a^4 - 768d^3 + 576cad^2 \\
& -54bc^3a - 432dbc^2 - 432d^2ba^2 + 12da^2b^3 + 81c^4]^{\frac{1}{2}}, \\
\xi = & [3a^2\delta^{\frac{1}{3}} - 8b\delta^{\frac{1}{3}} + 2\delta^{\frac{2}{3}} - 24ca + 96d + 8b^2]/\delta^{\frac{1}{3}}.
\end{aligned} \tag{14}$$

In the following, we use the Wootters entanglement measure to quantify the degree of entanglement for two identical atoms.

In Fig.1, we plot the concurrence as a function of the detuning Δ for different values of the interatomic distance $r_{12} = \frac{\lambda}{12}, \frac{\lambda}{15}, \frac{\lambda}{20}$, which gives the

values $\gamma = 0.95, \alpha = 9.30$, $\gamma = 0.965, \alpha = 18.85$, $\gamma = 0.98, \alpha = 46.17$ respectively, and at fixed $\beta = 1.6$. The figure shows that the concurrence strongly depends on the detuning Δ. And, the concurrence exhibits that peaks are always near $\Delta = 0$ and $\Delta = -\alpha$ though the interatomic distance is different. While, for a given interatomic distance r_{12}, the maximum of the concurrence near $\Delta = 0$ is less than that near $\Delta = -\alpha$ (i.e., when the dipole–dipole interaction α and the detuning Δ cancel out mutually, which means that the laser frequency is tuned to resonate with a particular pair of energy levels of the two-atom system that are shifted by the dipole–dipole interaction).

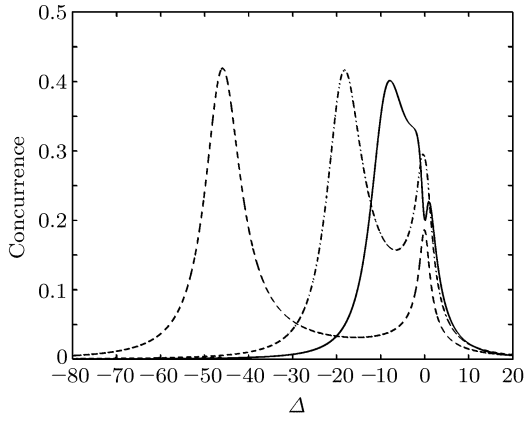


Fig.1. The concurrence as a function of the detuning Δ for $\beta = 1.6$ and different values of the interatomic distance: $r_{12} = \dfrac{\lambda}{12}$ (solid line), $r_{12} = \dfrac{\lambda}{15}$ (dashed-dotted line), $r_{12} = \dfrac{\lambda}{20}$ (dashed line).

In Fig.2, we plot the concurrence as a function of the detuning Δ for different values of the coherent laser field strength $\beta = 0.5, 1, 3, 4$ respectively and at fixed interatomic distance $r_{12} = \dfrac{\lambda}{20}$ ($\gamma = 0.98, \alpha = 46.17$). It is shown that the maximum of the concurrence for a given interatomic distance r_{12} (i.e., given dipole–dipole interaction value α) appears near the same value of $\Delta = -\alpha = -46.17$, even at different values of the field strength β. Moreover, as the Rabi frequency of the coherent laser field increases, the maximum of the concurrence increases. However, when the Rabi frequency of the coherent laser field increases to certain value, an increase in the maximum of the concurrence is very small with the increase of the Rabi frequency (see the dotted line for $\beta = 4$). Moreover, we find that when interatomic distance $r_{12} = \dfrac{\lambda}{20}$, the maximum of the concurrence cannot attain 0.5 no matter how high value of the coherent laser field strength is.

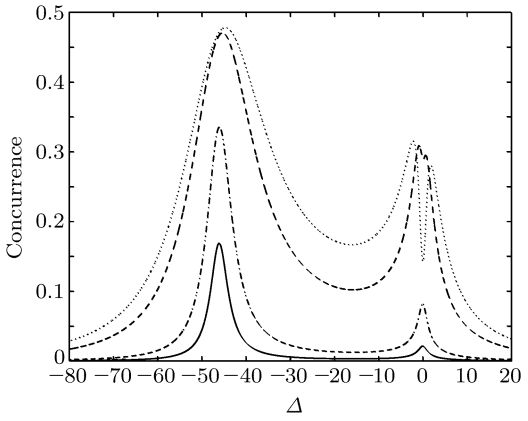


Fig.2. The concurrence as a function of the detuning Δ for interatomic distance $r_{12} = \dfrac{\lambda}{20}$ and different values of the field strength $\beta = 0.5$ (solid line), $\beta = 1$ (dashed-dotted line), $\beta = 3$ (dashed line), $\beta = 4$ (dotted line).

In Fig.3, we plot the concurrence as a function of the detuning Δ for different values of the coherent laser field strength $\beta = 0.5, 1, 3, 13$ respectively and at fixed interatomic distance $r_{12} = \dfrac{\lambda}{40}$ ($\gamma = 0.995, \alpha = 382.33$). As in the case of interatomic distance $r_{12} = \dfrac{\lambda}{20}$, shown in Fig.2, the maximum of the concurrence for a given interatomic distance $r_{12} = \dfrac{\lambda}{40}$ appears near the same value $\Delta = -\alpha = -382.33$, even at different values of the laser field strength β. Moreover, as the Rabi frequency of the coherent laser field increases, the maximum of the concurrence increases and attains 0.5 for $\beta = 13$. Comparing Fig.3 with Fig.2, we can see that the smaller the interatomic distance is, the bigger the maximum of concurrence we can obtain. In the experiment, this result may provide us with a potential way to obtain the high amount of stationary entanglement between the two atoms by choosing certain system parameters.

According to the Eqs.(9) and (7), we know that the entanglement between the two identical atoms not only relates to the one-photon atomic coherences but also the two-photon atomic coherence. To show this, in Fig.4, we plot the one-photon coherences $|\rho_{gs}|$, $|\rho_{se}|$ and the two-photon coherence $|\rho_{ge}|$ as a function of the detuning Δ for interatomic distance $r_{12} = \dfrac{\lambda}{20}$ ($\gamma = 0.98, \alpha = 46.17$) and $\beta = 3$. It is seen that near one-photon resonance $\Delta = -\alpha$ only the one-photon coherence $|\rho_{gs}|$ is significant, but near two-photon resonance $\Delta = 0$ only the two-photon coherence $|\rho_{ge}|$ is large.

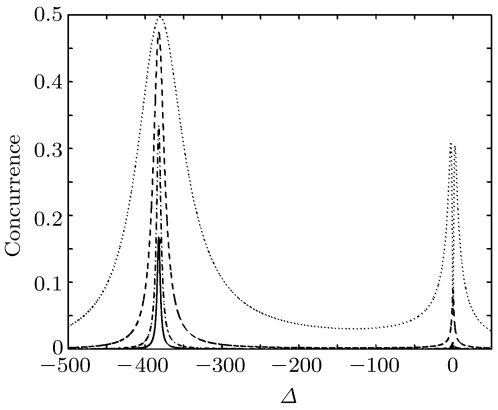


Fig.3. The concurrence as a function of the detuning Δ for interatomic distance $r_{12} = \dfrac{\lambda}{40}$ and different values of the field strength $\beta = 0.5$ (solid line), $\beta = 1$ (dashed-dotted line), $\beta = 3$ (dashed line), $\beta = 13$ (dotted line).

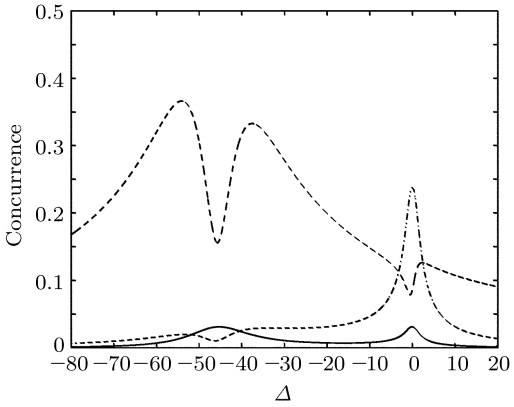


Fig.4. The one-photon coherences $|\rho_{gs}|$(dashed line), $|\rho_{se}|$ (solid line) and the two-photon coherence $|\rho_{ge}|$ (dashed-dotted line) as a function of the detuning Δ for interatomic distance $r_{12} = \dfrac{\lambda}{20}$ and $\beta = 3$.

Moreover, from Eqs.(7) and (8), we can obtain the same result as shown in Fig.4. Near the one-photon resonance $|s\rangle \longrightarrow |g\rangle$, the detuning $\Delta = -\alpha$, and when $\alpha \gg \beta$ in the case of the interatomic distance $r_{12} = \dfrac{\lambda}{20}$, the populations and the coherences reduce to

$$\begin{aligned}
&\rho_{ee} = \rho_{aa} = 4\beta^4/[\alpha^2(8\beta^2+1+\gamma^2)],\\
&\rho_{ss} = 4\beta^2/(8\beta^2+1+\gamma^2),\\
&\rho_{se} = -4\beta^3/[\alpha(8\beta^2+1+\gamma^2)],\\
&\rho_{gs} = -2\mathrm{i}\beta(1+\gamma)/[(8\beta^2+1+\gamma^2)],\\
&\rho_{ge} = 2\mathrm{i}\beta^2(1+\gamma)/[\alpha(8\beta^2+1+\gamma^2)]. \qquad (15)
\end{aligned}$$

It is clear from the above equations that near the one-photon resonance $|s\rangle \longrightarrow |g\rangle$ the one-photon coherence ρ_{gs} and the population ρ_{ss} are large, whereas the two-photon coherence ρ_{ge} is of order of α^{-1} and thus is negligible for large α.

Near two-photon resonance, $\Delta \approx 0$, and it follows from Eqs.(7) and (8) that in the limit of $\alpha \gg \beta$ the populations and the coherences reduce to

$$\begin{aligned}
&\rho_{ee} = \rho_{aa} = 4\beta^4/\alpha^2, \quad \rho_{ss} = 4\beta^4/\alpha^2,\\
&\rho_{se} = -4\mathrm{i}\beta^3/\alpha^2, \quad \rho_{gs} = 2\beta/\alpha,\\
&\rho_{ge} = -2\mathrm{i}\beta^2/\alpha. \qquad (16)
\end{aligned}$$

In this region, the coherences ρ_{gs} and ρ_{ge} are of order of magnitude α^{-1}, but the two-photon coherence ρ_{ge} dominates over the one-photon coherence ρ_{gs} when the driving field is strong. The above result means that in this case, the two-photon coherence ρ_{ge} plays the key role in the appearance of the maximum of the concurrence near two-photon resonance, whereas near one-photon resonance, the one-photon coherence ρ_{gs} plays an important role in the appearance of the maximum of the concurrence.

In Refs.[22,23], the entanglement induced by spontaneous emission is transient and decays to zero over long times due to the dissipation of energy in the system. However, in this paper, we can obtain the steady state entanglement because of the influence of the coherent driving field. The coherent driving field plays a double role. It compensates the losses of energy and stabilizes the entanglement carried by the system. A certain balance between the spontaneous decay and the influence of the coherent driving field leads to creation of the steady state entanglement in the system. Moreover, our result is really valuable. First, the non-maximally entangled state is used[27] to demonstrate the intrinsic difference between quantum and classical information transfers. The difference arises from the different ways in which the probabilities occur and is particularly clear in terms of entangled states. Second, for such a two-qubit system, any amount of entanglement, even if very small, is distillable,[28] and therefore the entangled atoms are useful for quantum information processing. Third, it is possible to use this kind of entanglement, irrespective of how small it is, to create maximally entangled Gaussian state,[29,30] which is a resource to build a quantum network. In addition, in this paper, we propose a simple scheme to realize an easily engineered two-atom entangled state. The entanglement depends

on the detuning of the laser frequency from the atomic transition frequency, the interatomic separation and the Rabi frequency of the coherent laser field. For a fixed interatomic separation, the maximum entanglement takes place at $\Delta = (\omega_0 - \omega_L)/\Gamma = 0$ and $\Delta = -\alpha$ whatever the Rabi frequency of the coherent laser field is. From this point of view, the laser frequency ω_L acts as a maximum entanglement generation tuning switch. The advantage of this scheme is that the relative distance between two atoms can be controlled. Current technology can produce an ion crystal with atom–atom separations comparable to the wavelength of the driving field.[31] So, this system device may be further developed into an engineered atom–atom entangler.[32]

4. Conclusion

In this paper, we study quantum entanglement between two spatially separated atoms driven by a coherent laser field in the dissipative process of spontaneous emission. It is shown that the entanglement strongly depends on the detuning of the laser frequency from atomic transition frequency, the interatomic separation and the Rabi frequency of the coherent laser field. A considerable amount of steady state entanglement can be obtained near $\Delta = -\alpha$ (i.e., the dipole–dipole interaction and the detuning cancel out mutually) for small atomic separation and large Rabi frequency of the coherent laser field.

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