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Amplitude-squared squeezing of the generalized odd–even coherent states of the anharmonic oscillator in a finite-dimensional Hilbert space

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This paper discusses the properties of amplitude-squared squeezing of the generalized odd–even coherent states of anharmonic oscillator in finite-dimensional Hilbert space. It demonstrates that the generalized odd coherent states do exhibit strong amplitude-squared squeezing effects in comparison with the generalized even coherent states.

Keywords: squeezing effect, generalized even and odd coherent states, anharmonic oscillator, finite-dimensional Hilbert space

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1. Introduction

In quantum optics, much attention has paid to the investigation of quantum systems in finite-dimensional Hilbert space. Finite-dimensional quantum systems are also of great interest because of their connection to the quantum communication and quantum computation. In the past few decades, various nonclassical properties of the coherent states of the harmonic oscillator in a finite-dimensional Hilbert space were studied by many authors.[1−3] Buzek *et al*[4] used the Glauber's approach to construct coherent states in finite-dimensional Hilbert space and discussed their squeezing properties.

The anharmonic oscillator is one of the most important models in the study of many problems in physics. New problems in many branches of physics, ranging from molecular dynamics to solid state physics and to optics, have intensified the interest in anharmonic models. Different approaches have been utilized to construct the generalized odd-even coherent states of the anharmonic oscillator.[5,6] Moreover, Q deformation[7−9] and superposition properties[10] of generalized coherent states in the anharmonic oscillator have been discussed. Yu *et al*[11] have discussed the squeezing properties of the generalized odd-even coherent states of anharmonic oscillator in finite-dimensional Hilbert space.

Complex amplitude which describes both the magnitude and the phase of the field can represent the electric field in an electromagnetic wave. If the uncertainties obeying the uncertainty relation in the real and imaginary parts of the complex amplitude are equal, then the minimum uncertainty states are coherent states. Amplitude-squared squeezing is a nonclassical feature which occurs in a number of nonlinear optical processes, states which possess this property can be of use in obtaining noise reduction in the output of certain nonlinear optical devices.[12] The main focus of the present work is to study the amplitude-squared squeezing for generalized odd-even coherent states of anharmonic oscillator in finite-dimensional Hilbert space.

This paper is organized in the following sequence. In Section 2 we briefly summarize some basic definitions which we use in the course of the paper and develop a general formalism concerning anharmonic oscillator in the finite-dimensional Hilbert space. In Section 3 we discuss the amplitude-squared squeezing for generalized odd-even coherent states. The final section gives a short discussion of the results.

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2. Anharmonic oscillator in a finite-dimensional Hilbert space

In this section we shall briefly describe some properties of the generalized coherent states of anharmonic oscillator in finite-dimensional Hilbert space. The Hamiltonian operator of anharmonic oscillator considered by us is defined as[13]

$$H = -\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}x^2 + \frac{1}{2}\frac{\mathrm{A}}{x^2} \quad (A > 0). \tag{1}$$

The position and momentum operator corresponding to Eq.(1) are:

$$Q = x^2 - H, \quad P = \frac{1}{2\mathrm{i}}\left(x\frac{\mathrm{d}}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}x}x\right). \tag{2}$$

They satisfy the commutation relations

$$[H, Q] = -2\mathrm{i}P, \quad [H, P] = 2\mathrm{i}Q, \quad [Q, P] = 2\mathrm{i}H. \tag{3}$$

Let us now consider the ladder operator $b_\pm$ defined by

$$b_\pm = \frac{1}{2}(Q \pm \mathrm{i}P), \tag{4}$$

and it can be shown that H, b_-, and b_+ satisfy the following relations:

$$[H, b_\pm] = \pm 2b_\pm, \quad [b_-, b_+] = H. \tag{5}$$

It is not difficult to identify the algebra in Eq.(5) as the one related to the $SU(1,1)$ Lie algebra. The action of the operators H, b_-, and b_+ on the energy eigenstate $|n\rangle$ is given by

$$\begin{aligned} H|n\rangle &= (2n + 2k)|n\rangle, \\ b_-|n\rangle &= \sqrt{n(n + 2k - 1)}|n - 1\rangle, \\ b_+|n\rangle &= \sqrt{(n + 1)(n + 2k)}|n + 1\rangle, \end{aligned} \tag{6}$$

where $k = \dfrac{1}{2}(1 + \sqrt{2A + 1/4})$.

According to Ref.[6] the eigenstate $|n\rangle$ can be written as

$$|n\rangle = \frac{{b_+}^n}{\sqrt{n!(2k)_n}}|0\rangle, \tag{7}$$

where $(2k)_n = (2k)(2k + 1)(2k + 2)\cdots(2k + n - 1) = \dfrac{\Gamma(2k + n)}{\Gamma(2k)}$.

Generalized coherent state can be defined as the eigenstate of the annihilation operator:

$$b_-|\beta\rangle = \beta|\beta\rangle. \tag{8}$$

It is easy to get that

$$|\beta\rangle = [F(|\beta|^2)]^{-1/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!(2k)_n}}|n\rangle, \tag{9}$$

where

$$F(R) = \sum_{n=0}^{\infty}\frac{R^n}{n!(2k)_n}. \tag{10}$$

Now consider $(2S + 1)$ dimensional Hilbert space $\sum_{2S+1} = \{|n\rangle, n = 0, 1, 2, \cdots, 2S\}$, where S is any integer. In this space the annihilation and creation operators of the anharmonic oscillator:

$$b_- = \sum_{n=1}^{2S}\sqrt{n(n + 2k + 1)}|n - 1\rangle\langle n|,$$

and

$$b_+ = \sum_{n=0}^{2S-1}\sqrt{(n + 1)(n + 2k)}|n + 1\rangle\langle n|, \tag{11}$$

satisfy the following commutation relation:

$$\begin{aligned} [b_-, b_+] = &\sum_{n=0}^{2S}(2n + 2k)|n\rangle\langle n| \\ &-(2S + 1)(2S + 2k)|2S\rangle\langle 2S|. \end{aligned} \tag{12}$$

The action of the operators b_- and b_+ on the generalized Fock state is given by

$$\begin{aligned} b_-|n\rangle &= \sqrt{n(n + 2k - 1)}|n - 1\rangle, \\ b_-|0\rangle &= 0, \\ b_+|n\rangle &= \sqrt{(n + 1)(n + 2k)}|n + 1\rangle, \\ b_+|2S\rangle &= 0, \quad |n\rangle \in \sum_{2S+1}. \end{aligned} \tag{13}$$

The coherent state of the anharmonic oscillator in a finite-dimensional Hilbert space can be defined as:

$$|\beta, S\rangle = [F(R, 2S)]^{-1/2}\sum_{n=0}^{2S}\frac{\beta^n}{\sqrt{n!(2k)_n}}|n\rangle, \tag{14}$$

where

$$F(R, m) = \sum_{n=0}^{\infty}\frac{R^{2n}}{n!(2k)_n}, \quad \beta = R\mathrm{e}^{\mathrm{i}\phi}. \tag{15}$$

The corresponding generalized odd–even coherent states are

$$|\beta,S\rangle_{\rm e}=[F_{\rm e}(R,S)]^{-1/2}\sum_{n=0}^{S}\frac{\beta^{2n}}{\sqrt{(2n)!(2k)_{2n}}}|2n\rangle,$$
$$|\beta,S\rangle_{\rm o}=[F_{\rm o}(R,S)]^{-1/2}\sum_{n=0}^{S-1}\frac{\beta^{2n+1}}{\sqrt{(2n+1)!(2k)_{2n+1}}}|2n+1\rangle, \tag{16}$$

where

$$F_{\rm e}(R,m)=\sum_{n=0}^{m}\frac{R^{4n}}{(2n)!(2k)_{2n}},$$
$$F_{\rm o}(R,m)=\sum_{n=0}^{m-1}\frac{R^{4n+2}}{(2n+1)!(2k)_{2n+1}}. \tag{17}$$

The inner product of the generalized odd–even coherent states of the anharmonic oscillator is

$${}_{\rm e}\langle\beta,S|\beta,S\rangle_{\rm e}={}_{\rm o}\langle\beta,S|\beta,S\rangle_{\rm o}=1,$$
$${}_{\rm o}\langle\beta,S|\beta,S\rangle_{\rm e}={}_{\rm e}\langle\beta,S|\beta,S\rangle_{\rm o}=0. \tag{18}$$

3. Amplitude-squared squeezing

Let us proceed to examine amplitude-squared squeezing. We consider the following two Hermitian operators, which represent the real and imaginary parts of the square of the amplitude.

$$X=\frac{1}{2}(b_-^2+b_+^2),\quad Y=\frac{1}{2{\rm i}}(b_-^2-b_+^2). \tag{19}$$

These operators obey the commutation relation

$$[X,Y]\;=\;\frac{\rm i}{2}\;[b_-^2,b_+^2]. \tag{20}$$

The generalized Heisenberg uncertainty relation takes the form

$$\left\langle(\Delta X)^2\right\rangle\left\langle(\Delta Y)^2\right\rangle\;\geq\frac{1}{4}\;|\;\langle\;[X,\;Y]\;\rangle\;|^2$$
$$=\frac{1}{16}\;\left|\;\langle[\;b_-^2,\;b_+^2]\;\rangle\;\right|^2. \tag{21}$$

In general for any two operators X or Y the squeezing condition is given by

$$S_x=\left\langle\Delta X^2\right\rangle\;-\frac{1}{4}\;\left|\;\left\langle[\;b_-^2,\;b_+^2]\;\right\rangle\;\right|<0, \tag{22a}$$
$$S_y=\left\langle\Delta Y^2\right\rangle\;-\frac{1}{4}\;\left|\;\left\langle[\;b_-^2,\;b_+^2]\;\right\rangle\;\right|<0. \tag{22b}$$

With the help of Eqs.(11) and (16), we obtain the following expectation values

$$\begin{cases}
{}_{\rm e}\langle\beta,S|\,b_-^4\;+\;b_+^4|\beta,S\rangle_{\rm e}=[F_{\rm e}(R,S)]^{-1}\displaystyle\sum_{n=0}^{S-2}\frac{R^{4n}}{(2n)!(2k)_{2n}}\times 2R^4\cos(4\phi),\\
{}_{\rm e}\langle\beta,S|\,b_-^2\;+\;b_+^2|\beta,S\rangle_{\rm e}=[F_{\rm e}(R,S)]^{-1}\displaystyle\sum_{n=0}^{S-1}\frac{R^{4n}}{(2n)!(2k)_{2n}}\times 2R^2\cos(2\phi),\\
{}_{\rm e}\langle\beta,S|\,b_-^2b_+^2\;\pm\;b_+^2b_-^2|\beta,S\rangle_{\rm e}=[F_{\rm e}(R,S)]^{-1}\\
\qquad\times\left\{\displaystyle\sum_{n=0}^{S-1}\left[\frac{(2n+1)(2n+2k)(2n+2)(2n+1+2k)\;R^{4n}}{(2n)!(2k)_{2n}}\right.\right.\\
\qquad\left.\left.\pm\displaystyle\sum_{n=0}^{S-1}\frac{R^{4n+4}}{(2n)!(2k)_{2n}}\right]\right\}.
\end{cases} \tag{23}$$

$$\begin{cases}
{}_{\rm o}\langle\beta,S|\,b_-^4\;+\;b_+^4|\beta,S\rangle_{\rm o}=[F_{\rm o}(R,S)]^{-1}\displaystyle\sum_{n=0}^{S-3}\frac{R^{4n+2}}{(2n+1)!(2k)_{2n+1}}\times 2R^4\cos(4\phi),\\
{}_{\rm o}\langle\beta,S|\,b_-^2\;+\;b_+^2|\beta,S\rangle_{\rm o}=[F_{\rm o}(R,S)]^{-1}\displaystyle\sum_{n=0}^{S-2}\frac{R^{4n+2}}{(2n+1)!(2k)_{2n+1}}\times 2R^2\cos(2\phi),\\
{}_{\rm o}\langle\beta,S|\,b_-^2b_+^2\;\pm\;b_+^2b_-^2|\beta,S\rangle_{\rm o}=[F_{\rm o}(R,S)]^{-1}\times\left\{\displaystyle\sum_{n=0}^{S-2}\left[\frac{(2n+1)(2n+2k)(2n+2)(2n+1+2k)\;R^{4n+2}}{(2n+1)!(2k)_{2n+1}}\right.\right.\\
\qquad\left.\left.\pm\displaystyle\sum_{n=0}^{S-2}\frac{R^{4n+6}}{(2n+1)!(2k)_{2n+1}}\right]\right\}.
\end{cases} \tag{24}$$

Finally the squeezing condition $S_x^{\rm e}$ for the generalized even coherent states of anharmonic oscillator in finite-dimensional Hilbert space is given by

$$S_x^{\rm e} = \frac{1}{4}[F_{\rm e}(R,S)]^{-1} \times \left\{ \sum_{n=0}^{S-2} \frac{R^{4n}}{(2n)!(2k)_{2n}} 2R^4 \cos(4\phi) + \sum_{n=0}^{S-1} \left[\frac{(2n+1)(2n+2k)(2n+2)(2n+1+2k)\ R^{4n}}{(2n)!(2k)_{2n}} + \frac{R^{4n+4}}{(2n)!(2k)_{2n}} \right] - [F_{\rm e}(R,S)]^{-1} \left[\sum_{n=0}^{S-1} \frac{R^{4n}}{(2n)!(2k)_{2n}} 2R^2 \cos(2\phi) \right]^2 - \left| \sum_{n=0}^{S-1} \left[\frac{(2n+1)(2n+2k)(2n+2)(2n+1+2k)\ R^{4n}}{(2n)!(2k)_{2n}} - \frac{R^{4n+4}}{(2n)!(2k)_{2n}} \right] \right| \right\}. \quad (25)$$

It is difficult to analyse the squeezing condition $S_x^{\rm e}$ for generalized even coherent states analytically. However, from this equation it is clear that if $\cos(4\phi) > 0$ then there exists amplitude-squared squeezing for large values of R. In Figs.1–4 we analyse the amplitude-squared squeezing effects in detail by depicting the squeezing condition $S_x^{\rm e}$ of Eq.(25) as a function of S, k, and ϕ. The curves are plotted for several values of S, k, and ϕ. Figures 1 and 2 show that generalized even coherent states amplitude-squared squeezing effects only for large values of $\cos(4\phi)$ and R, and no squeezing is observed for small values of $\cos(4\phi)$ even when R increases. It is also evident that the maximum amplitude-squared squeezing in the case of generalized even coherent states is obtained for large values of $\cos(4\phi)$. That is, $S_x^{\rm e} \to -1$ for $\cos(4\phi) \to 1$ or $\phi \to 0$. Along the increase of k and S the squeezing regions also increase and shift towards the larger values of R. It is also interesting to note that generalized even coherent states do exhibit maximum degree of amplitude-squared squeezing for large values of S and k, in Figs.3 and 4.

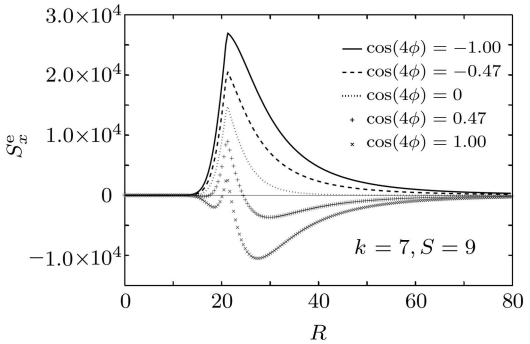


Fig.1. The squeezing condition $S_x^{\rm e}$ of Eq.(25) vs R for several values of $\cos(4\phi)$. All curves here are plotted for definiteness for the same value of $k = 7$ and $S = 9$.

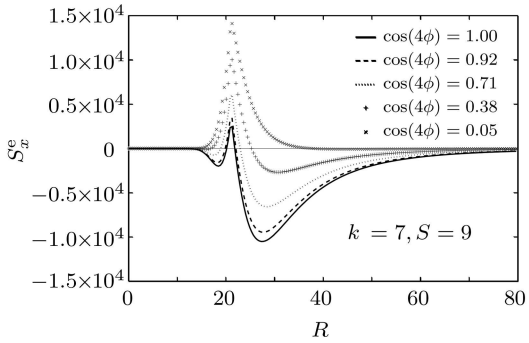


Fig.2. The squeezing condition $S_x^{\rm e}$ of Eq.(25) vs R for several values of $\cos(4\phi) > 0$. All curves here are plotted for definiteness for the same value of $k = 7$ and $S = 9$.

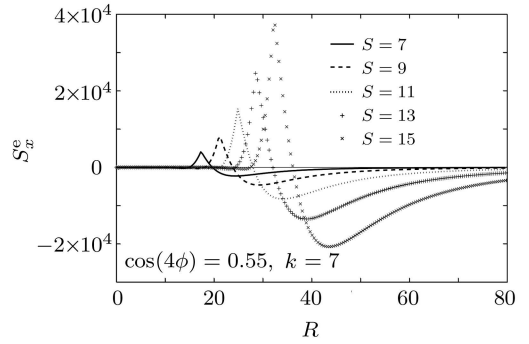


Fig.3. The squeezing condition $S_x^{\rm e}$ of Eq.(25) vs R for several values of S. All curves here are plotted for definiteness for the same value of $k = 7$ and $\cos(4\phi) = 0.55$.

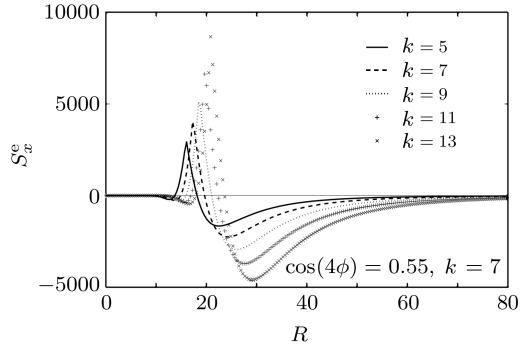


Fig.4. The squeezing condition $S_x^{\rm e}$ of Eq.(25) vs R for several values of k. All curves here are plotted for definiteness for the same value of $S = 7$ and $\cos(4\phi) = 0.55$.

The squeezing condition for the generalized odd coherent states is given by

$$S_x^{\rm o} = \frac{1}{4}[F_{\rm o}(R,S)]^{-1} \times \left\{ \sum_{n=0}^{S-3} \frac{R^{4n+2}}{(2n+1)!(2k)_{2n+1}} 2R^4 \cos(4\phi) \right.$$
$$+ \sum_{n=0}^{S-2} \left[\frac{(2n+1)(2n+2k)(2n+2)(2n+1+2k)\ R^{4n+2}}{(2n+1)!(2k)_{2n+1}} + \frac{R^{4n+6}}{(2n+1)!(2k)_{2n+1}} \right]$$
$$-[F_{\rm o}(R,S)]^{-1} \left[\sum_{n=0}^{S-2} \frac{R^{4n+2}}{(2n+1)!(2k)_{2n+1}} 2R^2 \cos(2\phi) \right]^2$$
$$\left. - \left| \sum_{n=0}^{S-2} \left[\frac{(2n+2)(2n+1+2k)\ R^{4n+2}}{(2n+1)!(2k)_{2n+1}} - \frac{R^{4n+6}}{(2n+1)!(2k)_{2n+1}} \right] \right| \right\}. \quad (26)$$

The squeezing condition for generalized odd coherent states is plotted in Figs.5–7. From Fig.5 it is clear that generalized odd coherent states exhibit amplitude-squared squeezing for any value of $\cos(4\phi)$ in contrast to the generalized even coherent states. Note that the maximum degree of amplitude-squared squeezing in the generalized odd coherent states is observed for large values of $\cos(4\phi)$ (i.e., $\phi \to 0$). It is also observed that the maximum degree of amplitude-squared squeezing in the generalized odd coherent states is greater than that in the case of the generalized even coherent states. That is, generalized odd coherent states have comparatively strong amplitude-squared squeezing effects. This can be seen by comparing the Figs.5–7 with Figs.1–4. The squeezing effects of the square of the amplitude in the generalized odd coherent states are directly proportional to the values of k and S, see Figs.6 and 7. Moreover, it is interesting to note that both generalized even coherent states and generalized odd coherent states have maximum degree of amplitude-squared squeezing for large values of $\cos(4\phi)$ (i.e., $\phi \to 0$), k, and S. In a similar way the squeezing effects of Y can be calculated.

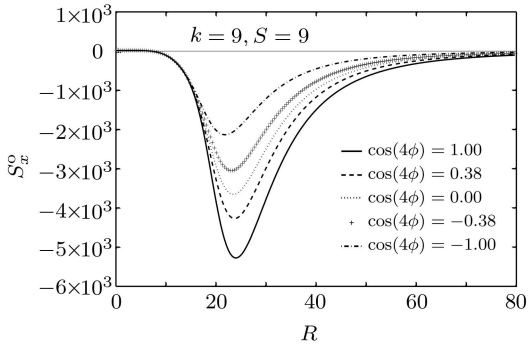


Fig.5. The squeezing condition $S_x^{\rm o}$ of Eq.(26) vs R for several values of $\cos(4\phi)$. All curves here are plotted for definiteness for the same value of $k = 9$ and $S = 9$.

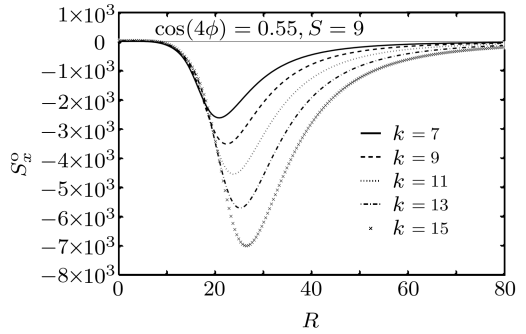


Fig.6. The squeezing condition $S_x^{\rm o}$ of Eq.(26) vs R for several values of k. All curves here are plotted for definiteness for the same value of $\cos(4\phi) = 0.55$ and $S = 9$.

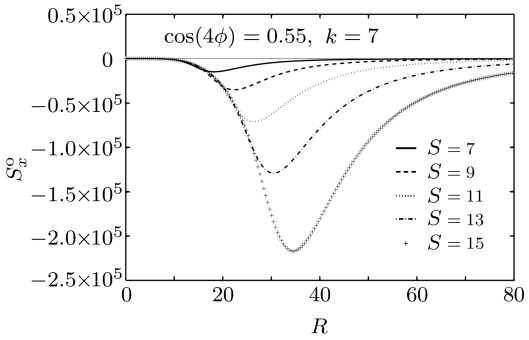


Fig.7. The squeezing condition $S_x^{\rm o}$ of Eq.(26) vs R for several values of S. All curves here are plotted for definiteness for the same value of $\cos(4\phi) = 0.55$ and $k = 7$.

4. Conclusion

In this paper we have considered anharmonic oscillator in a finite-dimensional Hilbert space and studied the properties of amplitude-squared squeezing of generalized odd–even coherent states. It has been shown that generalized even coherent states do exhibit amplitude-squared squeezing when $\cos(4\phi) > 0$ and do not exhibit when $\cos(4\phi) < 0$. On the other hand, generalized odd coherent states exhibit amplitude-squared squeezing regardless the values of $\cos(4\phi)$.

Thus the generalized odd coherent states are found to exhibit even stronger amplitude-squared squeezing and can be used for obtaining noise reduction in the output of certain nonlinear optical devices.

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