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Quantum entanglement in the system of a moving V-type three-level atom interacting with the SU(1,1)-related coherent fields*

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In a system with a moving V-type three-level atom interacting with the SU(1,1)-related coherent fields, we investigate the entanglement between the moving three-level atom and the SU(1,1)-related coherent fields by using the quantum-reduced entropy, and that between the SU(1,1)-related coherent fields by using the quantum relative entropy of entanglement. It is shown that the two kinds of entanglement are dependent on the atomic motion and exhibit the periodic evolution with a period of $2\pi/p$. The maximal atom–field qutrit entanglement state can be prepared, and the entanglement preservation of the SU(1,1)-related coherent fields can be realized in the interacting process via the appropriate selection of system parameters and interaction time.

Keywords: three-level atoms, SU(1,1)-related coherent fields, entanglement, reduced entropy, relative entropy
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1. Introduction

Quantum entanglement (QE) is one of the most striking features of quantum mechanics. Quantum entangled states, as a fundamental physical resource of quantum information processing, are extensively used in quantum computation and quantum communication,[1,2] quantum crypto-graphy,[3] quantum teleportation,[4] and quantum dense coding,[5] etc. One of the key problems in QE study is the generation and manipulation of quantum entangled states. There has been an important but simple way to generate atom–field entangled states in cavity QED. At present, entanglement of two-level systems has been used to represent information in most quantum information processing. Recently, one has found that using the entanglement of three-level systems has more outstanding advantages than that of two-level systems in the sense of quantum information representation. It is shown in Refs.[6–8] that quantum key distribution schemes in quantum cryptography with three-level systems are more secure against symmetric attacks than protocols using two-level states. In Refs.[9–12] it is found that quantum nonlocality in three-level systems is stronger than that in two-level systems. Thus there has been a great interest in three-level or multilevel systems in quantum information processing.[13−27]

It has been noticed that, in all the studies listed above, the atom is regarded as motionless and the influence of atomic motion on field is neglected. However, atomic motion is in existence. Besides, with the development of the laser-cooling technique, the slow-moving atoms can be obtained easily. When the momentum of the atom is comparable with that of the photon, the effect of quantum mechanical centre-of-mass motion becomes important, therefore the atomic motion must be taken into consideration. On the other hand, one has found that the atomic motion can bring about the nonlinear transient effect of atomic population,[28,29] modulate the fidelities of quantum states of system,[30] shorten the time of field squeezing,[31] change the time to create a Schrödinger cat state,[32] influence quantum fluctuations of a single atom in a weakly driven cavity,[33] and suppress the Rabi oscillations of an atom irradiated by a laser in resonance with the atomic transition.[34] And even atomic spontaneous emission spectra are asymmetric due to the quantized motion of the atom.[35] There-

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fore, some interesting researches on atomic motion have been carried out.[36−38] The SU(1,1)-related coherent state[39] is a continuous variable entangled state and is a generalization of a two-mode squeezed vacuum state. It is generated by applying a two-mode squeeze operator to a state in which there are initially q photons in one mode. It is found that this state exhibits the nonclassical properties: squeezing, sum squeezing, violations of the Cauchy–Schwarz inequality, and enhanced number-phase uncertainty fluctuations. In this paper, we study the QE in a system with a moving V-type three-level atom interacting with the SU(1,1)-related coherent fields. The entanglement between the moving three-level atom and the SU(1,1)-related coherent fields is investigated by using the quantum reduced entropy, and that between the SU(1,1)-related coherent fields is studied by using the quantum relative entropy of entanglement. The influence of the atomic motion on the QE is examined. We show that two kinds of entanglements are dependent on the atomic motion. A maximal atom–field qutrit entanglement state can be prepared, and the entanglement preservation of the SU(1,1)-related coherent fields can be realized in the interacting process via the appropriate selection of system parameters and interaction time.

2. The model

We consider a moving V-type three-level atom in the z-direction with two upper levels $|3\rangle$ and $|2\rangle$ and one lower level $|1\rangle$ as shown in Fig.1.

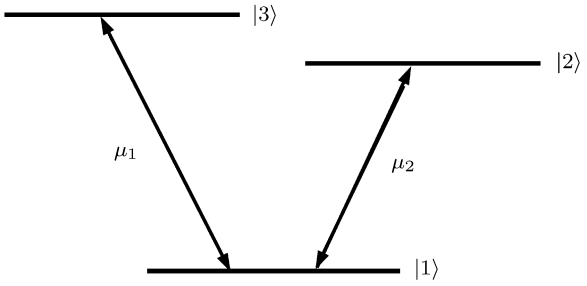


Fig.1. Diagram of a V-type three-level atom interacting with the SU(1,1)-related coherent fields.

The transitions between levels $|3\rangle$ and $|1\rangle$, $|2\rangle$ and $|1\rangle$ are connected by two-mode fields, respectively, to the photons with frequencies μ_1 and μ_2, while that between $|3\rangle$ and $|2\rangle$ is dipole-forbidden. The resonance condition is $\omega_3 - \omega_1 = \mu_1$, and $\omega_2 - \omega_1 = \mu_2$. The Hamiltonian of the considered system in the rotating wave approximation and in the interaction picture is(assuming $\hbar = 1$)[40−42]

$$H_{\rm I} = g_1 f_1(z)(a^+\sigma_{13}+a\sigma_{31})+g_2 f_2(z)(b^+\sigma_{12}+b\sigma_{21}), \quad (1)$$

where $\sigma_{ij} = |i\rangle\langle j|$ $(i,j = 3,2,1)$ are the atomic operators, $a^+(a)$ and $b^+(b)$ are the photon creation (annihilation) operators of two photon fields and $g_i(i = 1,2)$ is the atom–field coupling constant. Our study on the atomic motion is restricted in the z-axis direction so that only the z-dependent field-mode function needs taking into account. $f_i(z)(i = 1,2)$ denotes the mode structure of the ith photon field, which can be specified as[40]

$$f_i(z) \longrightarrow f_i(vt) = \sin\Big(\frac{p_i \pi v t}{L}\Big), \quad (2)$$

where v is the atomic velocity, $p_i(i = 1,2)$ represents the half-wave number of the ith field in the cavity with a length L. For simplicity, we assume that $g_1 = g_2 = g$, $p_1 = p_2 = p$, and the atom enters the cavity in an excited state at $t = 0$, and the two-mode fields are initially in an SU(1,1)-related coherent state[39]

$$|\Psi_{\rm f}(0)\rangle = |\xi, q\rangle = \sum_{n=0}^{\infty} F(n)|n+q, n\rangle, \quad (3)$$

where $F(n) = (1-|\xi|^2)^{(1+q)/2}\sqrt{(n+q)!/n!q!}\,\xi^n$, $(0 < |\xi|^2 < 1, q = 0,1,2,\ldots)$. q is the difference in photon number between the two modes, ξ is related to the two-mode squeezing parameter and indicates that the fields are weak or strong initially. Then the initial-state vector of the system can be written as

$$|\Psi_{\rm I}(0)\rangle = \sum_{n=0}^{\infty} F(n)|3, n+q, n\rangle. \quad (4)$$

At any time t, the state vector is described by

$$\begin{aligned}|\Psi_{\rm I}(t)\rangle = \sum_{n=0}^{\infty} F(n)\Big[&C_3(n,t)|3,n+q,n\rangle \\ &+ C_2(n,t)|2,n+q+1,n-1\rangle \\ &+ C_1(n,t)|1,n+q+1,n\rangle\Big]. \end{aligned} \quad (5)$$

Using the Schrödinger equation in the interaction picture, we can obtain the probability amplitudes

$$\begin{aligned} C_3(n,t) &= \frac{1}{n+q+1+n}[n+(n+q+1) \\ &\quad \times \cos(g\sqrt{n+q+1+n}\,\theta(t))], \\ C_2(n,t) &= \frac{\sqrt{(n+q+1)n}}{n+q+1+n} \\ &\quad \times [-1+\cos(g\sqrt{n+q+1+n}\,\theta(t))], \end{aligned}$$

$$C_1(n,t) = = -\mathrm{i}\sqrt{\frac{n+q+1}{n+q+1+n}} \times \sin(g\sqrt{n+q+1+n}\,\theta(t)), \quad (6)$$

where $\theta(t) = \int_0^t f(vt')\mathrm{d}t' = \frac{1}{\beta}[1-\cos(\beta t)]$, and $\beta = p\pi v/L$. Let the atomic velocity $v = gL/\pi$ (it can be realized in experiment), then $\theta(t)$ will become

$$\theta(t) = \frac{1}{pg}[1-\cos(pgt)]. \quad (7)$$

Thus the state vector of the system at any time t can be given by

$$|\Psi_{\mathrm{I}}(t)\rangle = |Q\rangle|3\rangle + |P\rangle|2\rangle + |S\rangle|1\rangle, \quad (8)$$

where

$$\begin{aligned} |Q\rangle &= \sum_{n=0}^{\infty} F(n)C_3(n,t)|n+q,n\rangle, \\ |P\rangle &= \sum_{n=0}^{\infty} F(n)C_2(n,t)|n+q+1,n-1\rangle, \\ |S\rangle &= \sum_{n=0}^{\infty} F(n)C_1(n,t)|n+q+1,n\rangle. \end{aligned} \quad (9)$$

The density operator for the considered system is written as

$$\rho_{\mathrm{af}}^{\mathrm{I}}(t) = |\Psi_{\mathrm{I}}(t)\rangle\langle\Psi_{\mathrm{I}}(t)| = \begin{pmatrix} |Q\rangle\langle Q| & |Q\rangle\langle P| & |Q\rangle\langle S| \\ |P\rangle\langle Q| & |P\rangle\langle P| & |P\rangle\langle S| \\ |S\rangle\langle Q| & |S\rangle\langle P| & |S\rangle\langle S| \end{pmatrix}. \quad (10)$$

The reduced density operator for the V-type three-level atom can be obtained as

$$\rho_{\mathrm{a}}^{\mathrm{I}}(t) = \mathrm{Tr}_{\mathrm{f}}|\Psi_{\mathrm{I}}(t)\rangle\langle\Psi_{\mathrm{I}}(t)| = \begin{pmatrix} \rho_{\mathrm{a}11} & \rho_{\mathrm{a}12} & \rho_{\mathrm{a}13} \\ \rho_{\mathrm{a}21} & \rho_{\mathrm{a}22} & \rho_{\mathrm{a}23} \\ \rho_{\mathrm{a}31} & \rho_{\mathrm{a}32} & \rho_{\mathrm{a}33} \end{pmatrix}, \quad (11)$$

where

$$\begin{aligned} &\rho_{\mathrm{a}11} = \langle Q|Q\rangle, \quad \rho_{\mathrm{a}21} = \langle Q|P\rangle = \rho_{\mathrm{a}12}^*, \\ &\rho_{\mathrm{a}31} = \langle Q|S\rangle = \rho_{\mathrm{a}13}^*, \quad \rho_{\mathrm{a}22} = \langle P|P\rangle, \\ &\rho_{\mathrm{a}32} = \langle P|S\rangle = \rho_{\mathrm{a}23}^*, \quad \rho_{\mathrm{a}33} = \langle S|S\rangle. \end{aligned} \quad (12)$$

Then the eigenvalues of the reduced atomic density operator are given by the cubic equation

$$\lambda_{\mathrm{a}}^3(t) + A\lambda_{\mathrm{a}}^2(t) + B\lambda_{\mathrm{a}}(t) + C = 0, \quad (13)$$

where the coefficients A, B and C are given by the matrix elements of Eq.(11)

$$\begin{aligned} A &= -(\rho_{\mathrm{a}11}+\rho_{\mathrm{a}22}+\rho_{\mathrm{a}33}) = -1, \\ B &= \rho_{\mathrm{a}11}\rho_{\mathrm{a}22} + \rho_{\mathrm{a}22}\rho_{\mathrm{a}33} + \rho_{\mathrm{a}11}\rho_{\mathrm{a}33} \\ &\quad - \rho_{\mathrm{a}13}\rho_{\mathrm{a}31} - \rho_{\mathrm{a}12}\rho_{\mathrm{a}21} - \rho_{\mathrm{a}23}\rho_{\mathrm{a}32}, \\ C &= \rho_{\mathrm{a}13}\rho_{\mathrm{a}31}\rho_{\mathrm{a}22} + \rho_{\mathrm{a}12}\rho_{\mathrm{a}21}\rho_{\mathrm{a}33} + \rho_{\mathrm{a}23}\rho_{\mathrm{a}32}\rho_{\mathrm{a}11} \\ &\quad - \rho_{\mathrm{a}11}\rho_{\mathrm{a}22}\rho_{\mathrm{a}33} - \rho_{\mathrm{a}23}\rho_{\mathrm{a}12}\rho_{\mathrm{a}31} - \rho_{\mathrm{a}21}\rho_{\mathrm{a}32}\rho_{\mathrm{a}13}. \end{aligned} \quad (14)$$

Therefore, the eigenvalues $\lambda_{\mathrm{a}j}(t)$, $(j=1,2,3)$ is given as

$$\begin{aligned} \lambda_{\mathrm{a}1}(t) &= -2R\cos\Big(\frac{\phi}{3}\Big) + \frac{1}{3}, \\ \lambda_{\mathrm{a}2}(t) &= 2R\cos\Big(\frac{\pi}{3}+\frac{\phi}{3}\Big) + \frac{1}{3}, \\ \lambda_{\mathrm{a}3}(t) &= 2R\cos\Big(\frac{\pi}{3}-\frac{\phi}{3}\Big) + \frac{1}{3}, \end{aligned} \quad (15)$$

where

$$\phi = \arccos\Big(\frac{q}{R^3}\Big), \quad (16)$$

$$q = -\frac{1}{27} + \frac{1}{6}B + \frac{1}{2}C, \quad (17)$$

$$R = \pm\Big(\frac{1}{3}B - \frac{1}{9}\Big)^{1/2}, \quad (18)$$

while the reduced density operator of the coherent fields is given by

$$\rho_{\mathrm{f}}^{\mathrm{I}}(t) = |Q\rangle\langle Q| + |P\rangle\langle P| + |S\rangle\langle S|. \quad (19)$$

Similarly, the eigenvalues of $\rho_{\mathrm{f}}^{\mathrm{I}}(t)$ can be obtained, and proved to be equal to those of $\rho_{\mathrm{a}}^{\mathrm{I}}(t)$. That is, $\lambda_{\mathrm{a}i}(t) = \lambda_{\mathrm{f}i}(t)(i=1,2,3)$.

In the following sections, we investigate the entanglement properties of the considered system by using the results mentioned above.

3. The entanglement between a moving V-type three-level atom and the SU(1,1)-related coherent fields

Since we have assumed that the moving V-type three-level atom and the SU(1,1)-related coherent fields are initially in a disentangled pure state, the considered system is a bicomponent quantum system in a pure state. For such a system, Phoenix and Knight [43,44] have shown that the reduced entropy is an accurate measure of entanglement between two subsystems. The higher the reduced entropy, the greater the degree of entanglement. Here, we investigate the entanglement between the moving

V-type three-level atom and the SU(1,1)-related coherent fields by using the reduced entropy.

The reduced entropies of the moving V-type three-level atom and the SU(1,1)-related coherent fields can be determined through their corresponding reduced density operators by

$$S_j(t) = -\mathrm{Tr}_j\{\rho_j^{\mathrm{I}}(t)\log_2\rho_j^{\mathrm{I}}(t)\}, \quad (j = \mathrm{a\ \ or\ \ f}). \quad (20)$$

The entropies of a bicomponent quantum system obey the triangle inequality of the entropy[45]

$$|S_{\mathrm{a}}(t) - S_{\mathrm{f}}(t)| \leq S_{\mathrm{af}}(t) \leq S_{\mathrm{a}}(t) + S_{\mathrm{f}}(t), \quad (21)$$

where $S_{\mathrm{af}}(t) = -\mathrm{Tr}\{\rho_{\mathrm{af}}^{\mathrm{I}}(t)\log_2\rho_{\mathrm{af}}^{\mathrm{I}}(t)\}$ is the total entropy of the considered system. Since the considered system is a bicomponent quantum system in a pure state, the total entropy $S_{\mathrm{af}}(t)$ is zero. Using Eq.(21), we can obtain that the reduced entropies of the two subsystems are equal, namely $S_{\mathrm{a}}(t) = S_{\mathrm{f}}(t)$. Consequently we can obtain the degree of entanglement between the moving V-type three-level atom and the SU(1,1)-related coherent fields(DEAF) only through calculating the atomic reduced entropy $S_{\mathrm{a}}(t)$. Following the work by Phoenix and Knight,[43,44] we can express the reduced entropy $S_{\mathrm{a}}(t)$ by using the eigenvalues of $\rho_{\mathrm{a}}(t)$ given by Eq.(15)

$$S_{\mathrm{a}}(t) = -\sum_{i=1}^{3}\{\lambda_{\mathrm{a}i}(t)\log\lambda_{\mathrm{a}i}(t)\}. \quad (22)$$

We numerically calculate the entanglement between the moving three-level atom and the SU(1,1)-related coherent fields, and illustrate it according to Eq.(22) in Fig.2 for different values of p and given $\xi = 0.6$, $q = 10$, and in Fig.3 for different values of q and given $p = 2$ and $\xi = 0.6$ by the solid lines.

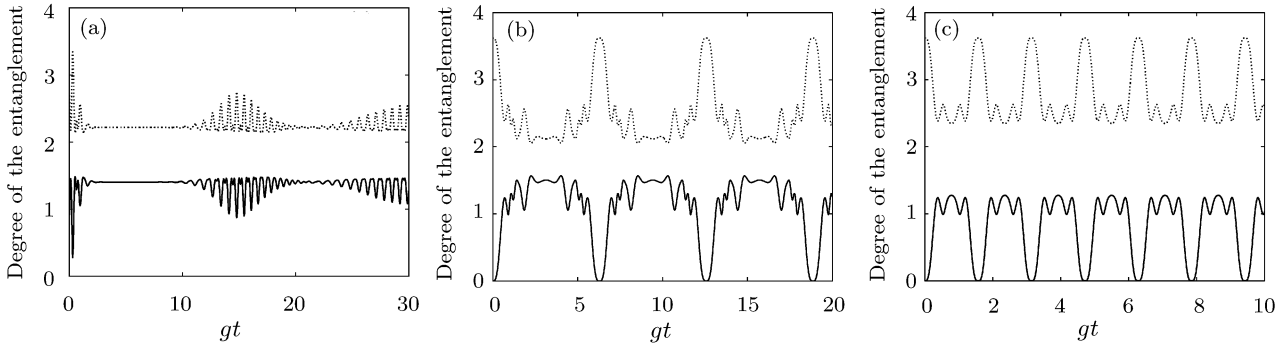


Fig.2. The time evolution of the degree of the entanglement. $S_{\mathrm{a}}(t)$ is shown by the solid line and $E_{\mathrm{R}}(\rho_{\mathrm{f}}^{\mathrm{I}}(t))$ by the dotted line for $\xi = 0.6$, $q = 10$: (a) is for the atomic motion being neglected; (b) for the atom being in motion at a velocity $v = gL/\pi$ and $p = 1$, and (c) for the atom being in motion at a velocity $v = gL/\pi$ and $p = 4$.

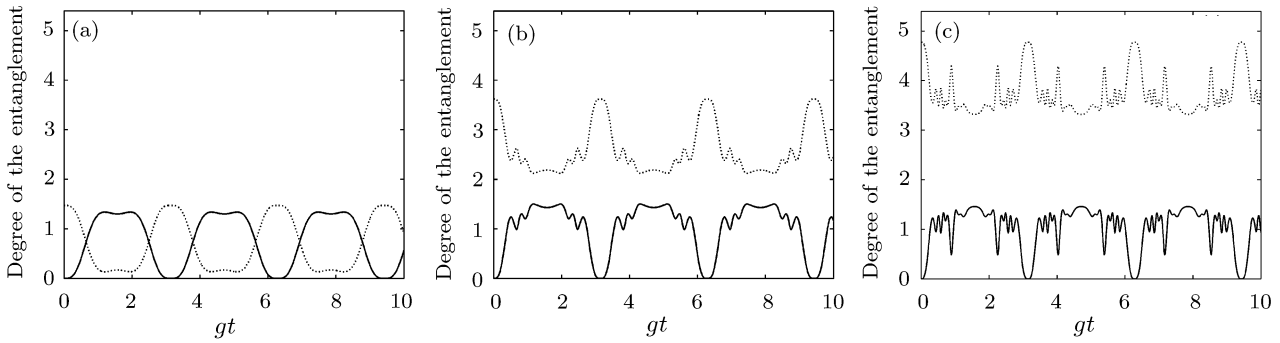


Fig.3. The time evolution of the degree of the entanglement. $S_{\mathrm{a}}(t)$ is shown by the solid line and $E_{\mathrm{R}}(\rho_{\mathrm{f}}^{\mathrm{I}}(t))$ by the dotted line for $p = 2$ and $\xi = 0.6$: (a) is for $q = 0$, (b) for $q = 10$ and (c) for $q = 50$.

Firstly, we discuss the influence of the atomic motion and the field-mode structure parameter p on the DEAF $S_{\mathrm{a}}(t)$ as shown in Fig.2 by the solid line. In Fig.2(a), the atomic motion is neglected. We find that the DEAF oscillates at the beginning, and then arrives at an approximately steady value of 1.5, which

is close to the upper limit $\log_2 3 = 1.58$ of $S_{\rm a}(t)$. This means that the moving V-type three-level atom and the SU(1,1)-related coherent fields are in a maximally atom–field qutrit entanglement state. When the atom is in motion at a velocity $v = gL/\pi$ and for the different values of field-mode structure parameter p, the results for the DEAF $S_{\rm a}(t)$ are plotted in Figs.2(b) and 2(c). From these figures one can see the influence of the atomic motion and the field-mode structure parameter p on the DEAF. We can draw the following conclusions. (1) The DEAF is periodic with a period of $2\pi/p$ when we take the atomic motion into account. It can be understood by using Eq.(7). From Eq.(7), $g\theta(t) = \frac{1}{p}[1-\cos(pgt)]$, we observe that $g\theta(t)$ is a periodical function on the scaled time gt with a period of $2\pi/p$. When $p = 1$, the period is 2π (as shown in Fig.2(b)); when $p = 4$, the period is $\pi/2$ (as shown in Fig.2(c)). It is shown that the atomic motion leads the entanglement between the moving V-type three-level atom and the SU(1,1)-related coherent fields to have a periodic dynamic behaviour. (2) The DEAF is zero at the times $gt = 2n\pi/p$ $(n = 0, 1, 2, \ldots)$, which means that the moving three-level atom and the SU(1,1)-related coherent fields are in a disentangled state. This point can be qualitatively interpreted as follows. At $gt = 2n\pi/p$ $(n = 0, 1, 2, \ldots)$, $\theta(t) = 0$, and the probability amplitudes $C_3(n,t) = 1$, $C_2(n,t) = 0$, $C_1(n,t) = 0$. The system state is the same as that at the initial time, whose vector may be expressed as Eq.(4), $|\Psi_{\rm I}(t)\rangle = \sum_{n=0}^{\infty} F(n)|3, n+q, n\rangle = |3\rangle \otimes \sum_{n=0}^{\infty} F(n)|n+q, n\rangle$, which is a direct product state of the moving three-level atom and the SU(1,1)-related coherent fields. Hence, the moving three-level atom is disentangled from the SU(1,1)-related coherent fields. (3) The increase in the value of the field-mode structure parameter p results in a reduction of maximum of the DEAF (comparing Fig.2(b) with Fig.2(c)). (4) For the rest of the time, the moving three-level atom is always entangled with the SU(1,1)-related coherent fields. In particular, from Figs.2(b) and 2(c), the DEAF evolves to a maximum at $gt = (2n+1)\pi/p$, $(n = 0, 1, 2, \ldots)$. This means at these time points, the maximal atom–field qutrit entanglement state can be prepared.

The influence of the difference in photon number between the two modes of q is displayed in Figs.3(a), 3(b) and 3(c) by the solid line. From these figures, we find that the change of q does not basically influence the periodicity of entanglement dynamics and the maximum of the DEAF.

4. The entanglement between the SU(1,1)-related coherent fields

The above discussion shows that Hamiltonian given by Eq.(1) results in the entanglement between the moving V-type three-level atom and the SU(1,1)-related coherent fields, and the state of the SU(1,1)-related coherent fields may evolve into a mixed state. In this case, the degree of entanglement between the SU(1,1)-related coherent fields cannot be measured by the reduced entropy of the entanglement. However, the relative entropy of entanglement is a good measure for the degree of entanglement of the mixed states, which is defined as[46,47]

$$E_{\rm R}(\rho) = \min_{\sigma \in D} S(\rho||\sigma), \tag{23}$$

where $S(\rho||\sigma) = {\rm Tr}[\rho(\ln\rho - \ln\sigma)]$ is the quantum relative entropy, its minimum is taken over D, the set of all disentangled states. If a bipartite quantum state can be expressed as[48,49]

$$\rho = \sum_{n_1,n_2} a_{n_1 n_2} |\Phi_{n_1}, \phi_{n_1}\rangle\langle\Phi_{n_2}, \phi_{n_2}|, \tag{24}$$

where $|\Phi_{n_1}\rangle$ and $|\phi_{n_1}\rangle$ are the orthogonal states of each subsystem, then the relative entropy of the entanglement is given by

$$E_{\rm R}(\rho) = -\sum_n a_{n,n} \log_2 a_{n,n} + {\rm Tr}(\rho \log_2(\rho)), \tag{25}$$

and the disentangled state σ that minimizes the quantum relative entropy $S(\rho||\sigma)$ is

$$\sigma = \sum_n a_{n,n} |\Phi_n, \phi_n\rangle\langle\Phi_n, \phi_n|. \tag{26}$$

It should be noted that the density operator given by Eq.(19) can take the form of Eq.(24). Hence, the relative entropy of entanglement of the state given by Eq.(19) is expressed as

$$\begin{aligned} E_{\rm R}(\rho_{\rm f}^{\rm I}(t)) = & -\sum_n a_{n,n} \log_2 a_{n,n} \\ & + \sum_{i=1}^{4} \lambda_{{\rm f}i}(t) \log_2 \lambda_{{\rm f}i}(t), \end{aligned} \tag{27}$$

where $a_{n,n} = \langle n+q, n|\rho_{\rm f}^{\rm I}(t)|n+q, n\rangle$.

The time evolution of $E_{\rm R}(\rho_{\rm f}^{\rm I}(t))$ given by Eq.(26) is plotted in Fig.2 for different values of p and given $\xi = 0.6$, and $q = 10$, and in Fig.3 for different values of q and given $\xi = 0.6$ and $p = 2$ by the dotted lines, and reflects the dynamics of entanglement of between the SU(1,1)-related coherent fields. Like

the DEAF, when the atomic motion is neglected, the dynamic behaviour of the entanglement between the SU(1,1)-related coherent fields displays oscillating at the beginning, and then arriving at an approximately steady value. When the atom moves at a velocity $v = gL/\pi$ and for the different values of field-mode structure parameter p, the entanglement between the SU(1,1)-related coherent fields exhibits the periodic dynamics with a period of $2\pi/p$. And when $p = 1$, the period is 2π (as shown in Fig.2(b)), and when $p = 2$, the period is π (as shown in Fig.3), etc. Especially, from the above discussion we know that at $gt = 2n\pi/p(n = 0, 1, 2, \ldots)$, the moving three-level atom is disentangled from the SU(1,1)-related coherent fields. However, at the same time, the relative entropy of entanglement achieves its maximum, which is equal to the value of the relative entropy of entanglement at the initial time. This indicates that at these time points, the entanglement between the SU(1,1)-related coherent fields can be preserved in the interacting process. Comparing the dotted line with the solid line in Fig.2 and in Fig.3, we find that the entanglement between the moving three-level atom and the SU(1,1)-related coherent fields and that between the SU(1,1)-related coherent fields go up and down in a contrary way, and they can impair each other due to the moving three-level atom interacting with the SU(1,1)-related coherent fields. It can be clearly seen from Figs.2(b) and 2(c) that, when the value of field-mode structure parameter p increases, the maximum of the degree of the entanglement between the SU(1,1)-related coherent fields does not change, but the minimum rises. From Figs.3(a), 3(b) and 3(c), we find that, when the values of field-mode structure parameter p is given and the difference in photon number between the two modes q increases, both the maximum and the minimum of the degree of entanglement between the SU(1,1)-related coherent fields increase.

5. Conclusions

In this paper, we have studied the entanglement in a system of a moving V-type three-level atom interacting with the SU(1,1)-related coherent fields. The entanglement between the moving three-level atom and the SU(1,1)-related coherent fields is investigated by using the quantum reduced entropy, and that between the SU(1,1)-related coherent fields by using the quantum relative entropy of entanglement. We can draw some conclusions as follows. Firstly, the atomic motion leads to the periodic dynamics of the entanglement of the system with a period of $2\pi/p$. Secondly, a maximal atom–field qutrit entanglement state can be prepared via the appropriate selection of system parameters and interaction time. Thirdly, the DEAF and the entanglement between the SU(1,1)-related coherent fields go up and down in a contrary way, and they can impair each other due to the moving three-level atom interacting with the two-mode coherent fields. At $gt = 2n\pi/p(n = 0, 1, 2, \ldots)$, the entanglement preservation of the SU(1,1)-related coherent fields can be realized in the interacting process. Finally, the maximum and the minimum of the DEAF and the degree of entanglement between the SU(1,1)-related coherent fields depend on the value of field-mode structure parameter p and the difference in photon between the two modes of q.

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