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Implementing entanglement teleportation via adiabatic passage*

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This paper proposes a scheme for implementing teleportation of an entangled state of two trapped atoms through adiabatic passage and photonic interference. The scheme is robust against certain noise such as atomic spontaneous emission and the detector inefficiency.

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PACC: 4250

Quantum teleportation, the means of disembodied transfer of an unknown quantum state from one place to another, is probably the most appealing example of many potential applications in quantum information processing. Recently, many schemes have been presented to perform quantum teleportation of a multiqubit state.[1−8] In this paper, we propose a scheme to teleport an entangled state of two trapped atoms via adiabatic passage and photonic interference,[9−12] where we use a three-atom entangled state as quantum channel. Our scheme is robust against certain practical noise such as atomic spontaneous emission, output coupling inefficiency, and the detector inefficiency.

Firstly we briefly introduce the teleportation protocol of an entangled state. Alice has atoms 4, 5 in an unknown entangled state

$$|\Psi\rangle_{45} = \alpha|g_{\rm L}g_{\rm L}\rangle_{45} + \beta|g_{\rm R}g_{\rm R}\rangle_{45}, \tag{1}$$

which is to be transferred to Bob. In order to teleport the entangled state, Alice and Bob must share a Greenberger–Horne–Zeilinger (GHZ) state

$$|\Psi\rangle_{123} = \frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm L}g_{\rm L}\rangle_{123} + |g_{\rm R}g_{\rm R}g_{\rm R}\rangle_{123}), \tag{2}$$

where atom 3 belongs to Alice, atoms 1 and 2 are held by Bob. So the state of the whole system consisted of five atoms can be expanded as follows:

$$\begin{aligned}|\Phi\rangle = \ & \frac{1}{2}\Big[\frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm L}g_{\rm L}\rangle_{345} + |g_{\rm R}g_{\rm R}g_{\rm R}\rangle_{345})(\alpha|g_{\rm L}g_{\rm L}\rangle_{12} + \beta|g_{\rm R}g_{\rm R}\rangle_{12}) \\ & + \frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm L}g_{\rm L}\rangle_{345} - |g_{\rm R}g_{\rm R}g_{\rm R}\rangle_{345})(\alpha|g_{\rm L}g_{\rm L}\rangle_{12} - \beta|g_{\rm R}g_{\rm R}\rangle_{12}) \\ & + \frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm R}g_{\rm R}\rangle_{345} + |g_{\rm R}g_{\rm L}g_{\rm L}\rangle_{345})(\beta|g_{\rm L}g_{\rm L}\rangle_{12} + \alpha|g_{\rm R}g_{\rm R}\rangle_{12}) \\ & + \frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm R}g_{\rm R}\rangle_{345} - |g_{\rm R}g_{\rm L}g_{\rm L}\rangle_{345})(\beta|g_{\rm L}g_{\rm L}\rangle_{12} - \alpha|g_{\rm R}g_{\rm R}\rangle_{12})\Big]. \end{aligned} \tag{3}$$

After making joint measurement on her atoms 3, 4 and 5, Alice sends the result to Bob via classical channel. If the result is

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$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm L}g_{\rm L}\rangle_{345} + |g_{\rm R}g_{\rm R}g_{\rm R}\rangle_{345}), \quad (4)$$

then atoms 1 and 2 collapse to the state

$$|\Psi\rangle_{12} = \alpha|g_{\rm L}g_{\rm L}\rangle_{12} + \beta|g_{\rm R}g_{\rm R}\rangle_{12}. \quad (5)$$

If the result is one of the states described as follows:

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm L}g_{\rm L}\rangle_{345} - |g_{\rm R}g_{\rm R}g_{\rm R}\rangle_{345}), \quad (6)$$

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm R}g_{\rm R}\rangle_{345} + |g_{\rm R}g_{\rm L}g_{\rm L}\rangle_{345}), \quad (7)$$

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm R}g_{\rm R}\rangle_{345} - |g_{\rm R}g_{\rm L}g_{\rm L}\rangle_{345}), \quad (8)$$

Bob may recreate the delivered state after performing local unitary transformations on the entangled atoms 1 and 2.

Now we use atom–cavity system to implement the above protocol. It can be divided into two steps. The first is the preparation of GHZ state, as shown in Fig.1.

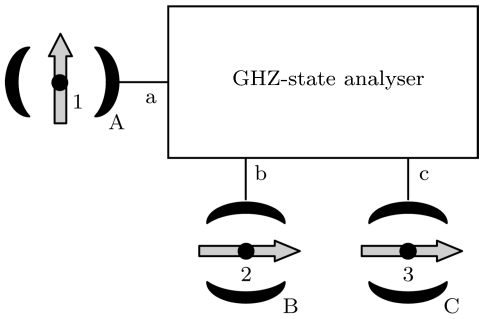


Fig.1. (a) The schematic setup to prepare GHZ state of three atoms. The distance between each cavity and analyser is equal. Each arrow represents the classical driving field.

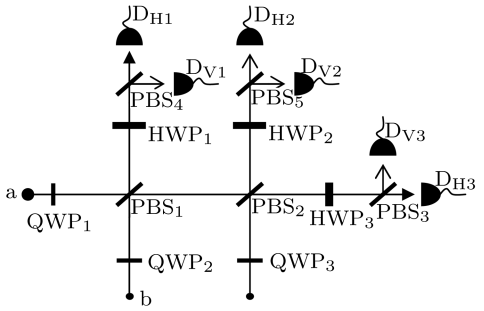


Fig.1. (b) A GHZ-state analyser. PBS denotes polarization beam splitter, QWP signifies quarter wave plate, D represents detector, and HWP is half wave plate.

Three identical ^{87}Rb atoms 1, 2 and 3 are trapped in one-sided leaky cavities A, B and C respectively. The relevant atomic levels and transitions are depicted in Fig.2. $|g_{\rm L}\rangle$ and $|g_{\rm R}\rangle$ correspond to $|F=1,\ m=-1\rangle$ and $|F=1,\ m=1\rangle$ of $5^2S_{1/2}$ respectively, and $|e_0\rangle$ corresponds to $|F=1,\ m=0\rangle$ of $5^2P_{3/2}$, while $|s\rangle$ corresponds to $|F=2,\ m=0\rangle$ of $5^2S_{1/2}$. The transition $|s\rangle \to |e_0\rangle$ is driven adiabatically by a classical field with the Rabi frequency $\Omega(t)$, and the transition $|e_0\rangle \to |g_{\rm L}\rangle(|e_0\rangle \to |g_{\rm R}\rangle)$ is coupled to the right-circularly (left-circularly) polarized mode of the cavity.

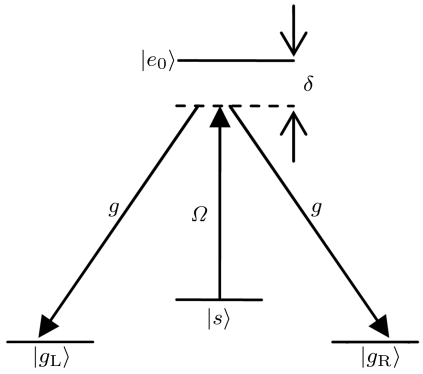


Fig.2. The relevant atomic level structure and transitions.

In the rotating frame, the interaction Hamiltonian for each cavity is (assuming $\eta = 1$)[10]

$$\begin{aligned} H = \ & \Omega(t)|e_0\rangle\langle s|{\rm e}^{{\rm i}\delta t} + g(t)(a_{\rm R}|e_0\rangle\langle g_{\rm L}| \\ & + a_{\rm L}|e_0\rangle\langle g_{\rm R}|){\rm e}^{{\rm i}\delta t} + {\rm H.c.}, \end{aligned} \quad (9)$$

where H.c. denotes the Hermitian conjugate, $a_{\rm L}$ and $a_{\rm R}$ denote the annihilation operator for the corresponding polarized mode of the cavity respectively, $g(t)$ represents the atom–cavity coupling rate, and δ is the detuning of atomic transition from cavity mode or classical field (see Fig.2). Without loss of generality, we have supposed that $\Omega(t)$ and $g(t)$ are real. The state vector of the atom–cavity system can be expanded in terms of the possible basis states in the following way:[13]

$$\begin{aligned} |\Psi(t)\rangle = \ & c_1|s\rangle|n,\mu\rangle + c_2|e_0\rangle|n,\mu\rangle + c_3|g_{\rm L}\rangle|n,\mu+1\rangle \\ & + c_4|g_{\rm R}\rangle|n+1,\ \mu\rangle, \end{aligned} \quad (10)$$

where n and μ are the initial number of left-circularly polarized and right-circularly polarized photons respectively, $c_i (i = 1, 2, 3, 4)$ are the probability amplitudes for the corresponding states. The amplitude equation can be obtained from the Schrödinger equation as

$$\dot{d}_1 = -\mathrm{i}\Omega d_2, \tag{11}$$

$$\dot{d}_2 = -\mathrm{i}\Omega d_1 - \mathrm{i}g\sqrt{\mu+1}d_3 - \mathrm{i}g\sqrt{n+1}d_4 - \mathrm{i}\delta d_2, \tag{12}$$

$$\dot{d}_3 = -\mathrm{i}g\sqrt{\mu+1}d_2, \tag{13}$$

$$\dot{d}_4 = -\mathrm{i}g\sqrt{n+1}d_2, \tag{14}$$

where the following transformations have been used:

$$c_1 \to d_1,\ c_2\mathrm{e}^{-\mathrm{i}\delta t} \to d_2,\ c_3 \to d_3,\ c_4 \to d_4, \tag{15}$$

In the large detuned case, i.e. $\delta \gg g, \delta \gg \Omega$ we put $\dot{d}_2 = 0$, then the amplitude equation reduces to

$$\dot{d}_1 = \frac{\mathrm{i}\Omega}{\delta}(\Omega d_1 + g\sqrt{\mu+1}d_3 + g\sqrt{n+1}d_4), \tag{16}$$

$$\dot{d}_3 = \frac{\mathrm{i}g}{\delta}\sqrt{\mu+1}(\Omega d_1 + g\sqrt{\mu+1}d_3 + g\sqrt{n+1}d_4), \tag{17}$$

$$\dot{d}_4 = \frac{\mathrm{i}g}{\delta}\sqrt{n+1}(\Omega d_1 + g\sqrt{\mu+1}d_3 + g\sqrt{n+1}d_4). \tag{18}$$

We note that Eqs.(16–18) can be obtained from an effective Hamiltonian:

$$H_{\mathrm{eff}} = -\Big[\frac{\Omega^2}{\delta}|s\rangle\langle s| + \frac{\Omega g}{\delta}\alpha_{\mathrm{R}}^{+}|g_{\mathrm{L}}\rangle\langle s| + \frac{\Omega g}{\delta}\alpha_{\mathrm{L}}^{+}|g_{\mathrm{R}}\rangle\langle s| + \frac{\Omega g}{\delta}\alpha_{\mathrm{R}}|s\rangle\langle g_{\mathrm{L}}| + \frac{g^2}{\delta}\alpha_{\mathrm{R}}^{+}\alpha_{\mathrm{R}}|g_{\mathrm{L}}\rangle\langle g_{\mathrm{L}}| + \frac{g^2}{\delta}\alpha_{\mathrm{L}}^{+}\alpha_{\mathrm{R}}|g_{\mathrm{R}}\rangle\langle g_{\mathrm{L}}| + \frac{\Omega g}{\delta}\alpha_{\mathrm{L}}|s\rangle\langle g_{\mathrm{R}}| + \frac{g^2}{\delta}\alpha_{\mathrm{L}}^{+}\alpha_{\mathrm{L}}|g_{\mathrm{R}}\rangle\langle g_{\mathrm{R}}| + \frac{g^2}{\delta}\alpha_{\mathrm{R}}^{+}\alpha_{\mathrm{L}}|g_{\mathrm{L}}\rangle\langle g_{\mathrm{R}}|\Big]. \tag{19}$$

In the adiabatic limit, the state of the atom–cavity in the initial state $|s\rangle|0\rangle$ evolves into the following dark state:

$$|D_0\rangle = -\frac{\sqrt{2}g}{\sqrt{\Omega^2+2g^2}}|s\rangle|0\rangle + \frac{\Omega}{\sqrt{\Omega^2+2g^2}}\frac{1}{\sqrt{2}}(|g_{\mathrm{L}}\rangle|R\rangle + |g_{\mathrm{R}}\rangle|L\rangle), \tag{20}$$

where $|0\rangle, |R\rangle(|L\rangle)$ denote none and one right- (left-) circularly polarized photon in the cavity respectively. When we adiabatically increase the Rabi frequency $\Omega(t)$ to satisfy the condition $|\Omega(t)| \gg |g(t)|$, the state of the atom–cavity evolves into the following state

$$|\Psi\rangle_{ij} = \frac{1}{\sqrt{2}}(|g_{\mathrm{L}}R\rangle + |g_{\mathrm{R}}L\rangle)_{ij}, \tag{21}$$

where ij = 1A, 2B, 3C. Because of the imperfection of the cavities three photons leak out from three cavities and are used for GHZ-state measurement by virtue of the photonic interference effect. A GHZ-state analyzer[14] consists of three quarter-wave plates (QWP), three half-wave plates (HWP), five polarization beam splitters (PBS) and six photon detectors (D), where QWP transfers the left-circularly (right-circularly) polarized photons into vertical (horizontal) polarized photons, PBS transmits only the horizontal polarization component and reflects the vertical component, and the angle between the HWP axis and the horizontal direction is 22.5° such that it corresponds to a 45° rotation of the polarization. Through simple calculations, it is found that atoms 1, 2 and 3 will be prepared into the GHZ state described by Eq.(2) when $\mathrm{D_{H1}}$, $\mathrm{D_{H2}}$ and $\mathrm{D_{H3}}$(or any one of the following three cases: $\mathrm{D_{H1}}$, $\mathrm{D_{V2}}$ and $\mathrm{D_{V3}}$; $\mathrm{D_{V1}}$, $\mathrm{D_{H2}}$ and $\mathrm{D_{V3}}$; $\mathrm{D_{V1}}$, $\mathrm{D_{V2}}$ and $\mathrm{D_{H3}}$) click at the same time; If $\mathrm{D_{H1}}$, $\mathrm{D_{H2}}$ and $\mathrm{D_{V3}}$(or any one of the following three cases: $\mathrm{D_{H1}}$, $\mathrm{D_{V2}}$ and $\mathrm{D_{H3}}$; $\mathrm{D_{V1}}$, $\mathrm{D_{H2}}$ and $\mathrm{D_{H3}}$; $\mathrm{D_{V1}}$, $\mathrm{D_{V2}}$ and $\mathrm{D_{V3}}$) coincide, atoms 1, 2 and 3 collapse to the state described by Eq.(6). The latter state can be transformed into the former one by making an appropriate local unitary operation, such as performing operation σ_{z} on atom 3. Thus we have prepared atoms 1, 2, 3 in the GHZ state described by Eq.(2).

The next is that a measurement is performed on atoms 3, 4 and 5. The schematic setup is shown in Fig.3, where three atoms 3, 4 and 5 driven by a classical field are trapped in vacuum cavities C, D and E respectively. The involved atomic levels and transitions are depicted in Fig.4. The states $|g_{\mathrm{L}}\rangle, |g_{\mathrm{R}}\rangle$ and $|e_0\rangle$ are the same as those described above, $|e_{\mathrm{L}}\rangle$ and $|e_{\mathrm{R}}\rangle$ are also the Zeeman sublevels of electronic excited states $5^2P_{3/2}$ of ^{87}Rb atom, corresponding to $|F=1, m=-1\rangle$ and $|F=1, m=1\rangle$ respectively, $|g_0\rangle$ is another Zeeman sublevel of electronic ground state $5^2S_{1/2}$, corresponding to $|F=1, m=0\rangle$. The transitions $|g_{\mathrm{L}}\rangle \to |e_{\mathrm{L}}\rangle$ and $|g_{\mathrm{R}}\rangle \to |e_{\mathrm{R}}\rangle$ are driven adiabatically through the π-polarized classical field with

the corresponding Rabi frequency $\Omega'(t)$. The transitions $|e_{\rm L}\rangle \rightarrow |g_0\rangle$ and $|e_{\rm R}\rangle \rightarrow |g_0\rangle$ are coupled to the left-circularly and right-circularly polarized mode of the cavity with the coupling rate $g'(t), |g_0\rangle \rightarrow |e_0\rangle$ is forbidden for electric dipole, and δ is the detuning of atomic transition from cavity mode or classical field.

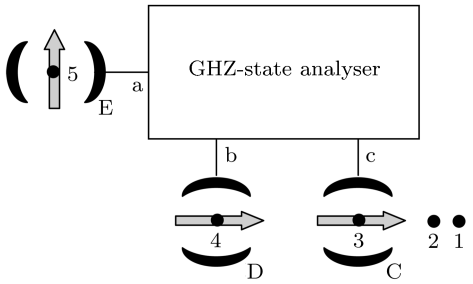


Fig.3. The schematic setup for teleportation of an entangled state. After making GHZ-state measurements on atoms 3, 4 and 5, the state of the atoms 1 and 2 collapses to the state of atoms 4 and 5 with appropriate unitary operations.

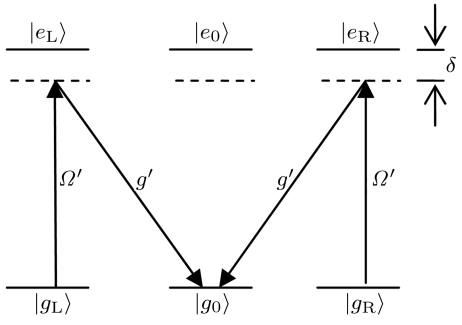


Fig.4. The relevant atomic level structure and transitions.

In the rotating frame, the interaction Hamiltonian for each of the cavities has the form [10]

$$\begin{aligned} H = \ & \Omega'(t)(|e_L\rangle\langle g_{\rm L}| + |e_{\rm R}\rangle\langle g_{\rm R}|){\rm e}^{{\rm i}\delta t} \\ & + g'(t)\Big(a_{\rm L}|e_{\rm L}\rangle\langle g_0| + a_{\rm R}|e_{\rm R}\rangle\langle g_0|\Big){\rm e}^{{\rm i}\delta t} + {\rm H.c.}. \end{aligned} \tag{22}$$

In the large detuned case, i.e., $\delta \gg g'$, $\delta \gg \Omega'$, based on similar analysis above, the effective Hamiltonian is described as follows:

$$\begin{aligned} H_{\rm eff} = \ & -\frac{\Omega'^2}{\delta}|g_{\rm L}\rangle\langle g_{\rm L}| - \frac{\Omega' g'}{\delta}a_{\rm L}^{+}|g_0\rangle\langle g_{\rm L}| \\ & -\frac{\Omega'^2}{\delta}|g_{\rm R}\rangle\langle g_{\rm R}| - \frac{\Omega' g'}{\delta}a_{\rm R}^{+}|g_0\rangle\langle g_{\rm R}| \\ & -\frac{g'^2}{\delta}(a_{\rm L}^{+}a_{\rm L} + a_{\rm R}^{+}a_{\rm R})|g_0\rangle\langle g_0| \\ & -\frac{\Omega' g'}{\delta}(a_{\rm L}|g_{\rm L}\rangle\langle g_0| + a_{\rm R}|g_{\rm R}\rangle\langle g_0|). \end{aligned} \tag{23}$$

The Hamiltonian H has two dark states

$$|D_1\rangle = \frac{g'}{\sqrt{\Omega'^2 + g'^2}}|g_{\rm L}\rangle|0\rangle - \frac{\Omega'}{\sqrt{\Omega'^2 + g'^2}}|g_0\rangle|L\rangle, \tag{24}$$

$$|D_2\rangle = \frac{g'}{\sqrt{\Omega'^2 + g'^2}}|g_{\rm R}\rangle|0\rangle - \frac{\Omega'}{\sqrt{\Omega'^2 + g'^2}}|g_0\rangle|R\rangle. \tag{25}$$

When the adiabatically increasing $\Omega'(t)$ satisfies the condition $|\Omega'(t)| \gg |g'(t)|$, the state of atoms 4, 5 and cavities D, E in initial state $(\alpha|g_{\rm L}|g_{\rm L}\rangle_{45} + \beta|g_{\rm R}g_{\rm R}\rangle_{45})|0\rangle_{\rm D}|0\rangle_{\rm E}$ evolves into the form

$$|\Psi\rangle = |g_0g_0\rangle_{45}(\alpha|LL\rangle + \beta|RR\rangle)_{\rm DE}. \tag{26}$$

The state of atoms 1, 2, 3 and cavities C in initial state $\frac{1}{\sqrt{2}}(|g_{\rm L}g_{\rm L}g_{\rm L}\rangle_{123} + |g_{\rm R}g_{\rm R}g_{\rm R}\rangle_{123})|0\rangle_{\rm C}$ evolves into

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}|g_0\rangle_3(|g_{\rm L}g_{\rm L}L\rangle + |g_{\rm R}g_{\rm R}R\rangle)_{12\rm C}. \tag{27}$$

So the state of the system, which consists of atoms 1, 2, 3, 4, 5 and cavities C, D, E, is

$$\begin{aligned} |\Phi\rangle = \ & \frac{1}{\sqrt{2}}|g_0g_0g_0\rangle_{345}(|g_{\rm L}g_{\rm L}L\rangle + |g_{\rm R}g_{\rm R}R\rangle)_{12\rm C}(\alpha|LL\rangle \\ & +\beta|RR\rangle)_{\rm DE}. \end{aligned} \tag{28}$$

Three photons leaking out from the cavities simultaneously reach the GHZ-analyser, then through an analysis similar to the above, one can find that when a coincidence in detectors $\rm D_{H1}$, $\rm D_{H2}$ and $\rm D_{H3}$ (or $\rm D_{H1}$, $\rm D_{V2}$ and $\rm D_{V3}$, or $\rm D_{V1}$, $\rm D_{H2}$ and $\rm D_{V3}$, or $\rm D_{V1}$, $\rm D_{V2}$ and $\rm D_{H3}$) is observed, the state of atoms 1 and 2 collapses into the state described by Eq.(5). Whereas when a coincidence in $\rm D_{H1}$, $\rm D_{H2}$ and $\rm D_{V3}$ (or any one of the following three cases $\rm D_{H1}$, $\rm D_{V2}$ and $\rm D_{H3}$; $\rm D_{V1}$, $\rm D_{H2}$ and $\rm D_{H3}$; $\rm D_{V1}$, $\rm D_{V2}$ and $\rm D_{V3}$) is observed, it collapses to

$$|\Psi\rangle_{12} = -\alpha|g_{\rm L}g_{\rm L}\rangle_{12} + \beta|g_{\rm R}g_{\rm R}\rangle_{12}, \tag{29}$$

which can be transformed into the state of Eq.(5) after an appropriate local unitary operation.

Thus, we complete the teleportation of an entangled state.

In summary, we have presented a scheme to prepare GHZ state and teleport an entangled state of two atoms based on adiabatic passage and photonic interference. The qubit is represented by degenerate ground states and the atomic state evolves only

in the ground dark space through adiabatic passage, so the scheme is robust against the effects of spontaneous emission. Additionally, due to adopting coincidence technology, the detection inefficiency and output coupling inefficiency can only decrease the successful probability of our protocol but have no influence on the fidelity of the final state.

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