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Light beams with orbital angular momentum for free space optics

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The light's orbital angular momentum (OAM) is a consequence of the spiral flow of the electromagnetic energy. In this paper, an analysis of light beams with OAM used for free space optics (FSO) is conducted. The basic description and conception of light's OAM are reviewed. Both encoding information into OAM states of single light beam and encoding information into spatial structure of the mixed optical vortex with OAM are discussed, and feasibility to improve the FSO's performance of security and obstruction of line of sight is examined.

Keywords: orbital angular momentum, free space optics, pseudo-nondiffracting beams
PACC: 4230, 4260

1. Introduction

Free space optics (FSO) is increasingly considered to play a significant role in many situations because of its low fabrication costs compared with optical fibre and short installation times.[1,2] Technical problems encountered are those happening to the early wireless optical systems. For example, reliability problems were encountered under adverse weather conditions such as heavy rain or dense fog.[2] The relevant technology has developed in recent years and most problems can be readily solved by carefully designing networks.

During the last decade the increasing attention has been paid to the optical vortices.[3−6] Optical encoding is now being applied to free-space communication links. Such links have been demonstrated at both the classical[7] and the single photon levels.[8] Most of these links rely on modulations of intensity, frequency, or polarization of the light. Because light's orbital angular momentum (OAM) has an infinite number of eigenstates, the OAM is a promising parameter onto which classical or quantum information may be encoded.[5,9]

The rest of this paper is organized as follows. In Section 2, the basic description and conception of light's OAM as well as several methods of generating light beams with OAM are reviewed. In Section 3 discussed is the issue of using light beams with OAM for FSO based on the available literature. Concluding remarks are presented in Section 4.

2. Light's orbital angular momentum

Light beams can carry both spin and orbital angular momenta associated, respectively, with circular polarization and the helicity of their phasefronts. The angular momentum associated with circular polarization arises from the spin of individual photon and is termed spin angular momentum. As identified by Allen *et al*, each photon in a beam comprising l helical phasefronts, described by an azimuthal phase term $\exp(il\phi)$, carries an OAM in its propagation direction, where l can take any integer value.[6] Such a beam is usually called optical vortices (OVs).

Some types of nondiffracting beams can also belong in the optical vortices. The physical concept of the nondiffracting propagation has been presented and the basic properties of the nondiffracting beams have been reviewed in Ref.[10]. Nondiffracting beams possess unique properties useful for the free space communications. The ideal nondiffracting beams carry an infinite energy, and their transverse intensity profile remains unchanged from $-\infty$ to $+\infty$. In experiments only their approximations known as the pseudo-nondiffracting (P-N) beams can be obtained. During the free propagation, their transverse intensity profile remains nearly invariant in a range which is dependent

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on the geometrical parameters of the experiment. Another important advantage of the P-N beams is their self-regeneration ability. Owing to this property, the P-N beam perturbed by amplitude or phase obstacles can revive and during free propagation obtain its initial amplitude and phase profile.

Light's OAM has been studied theoretically and experimentally;[11,12] in this section we briefly present its mathematical description and generation method for completeness and also for providing a basis for using light with OAM for FSO.

2.1. Description of single light beam with OAM

Light with orbital angular momentum can be described in terms of the Laguerre–Gaussian (LG) mode which contains an $\exp(\mathrm{i}l\phi)$ term describing an on-axis phase singularity of strength l. In addition to the index l, which can take any integer number and determines the azimuthal phase dependence of the mode, the LG mode is also characterized by an index p, which can take any non-negative value and determines the radial shape, or node number, of the light distribution. The normalized $\mathrm{LG}_{l,p}$ mode at its beam waist is given in cylindrical coordinates by[13]

$$\mathrm{LG}_{l,p}(\rho,\phi) = \left(\frac{2p!}{\pi(|m|+p)!}\right)^{1/2}\frac{1}{\eta}\left(\frac{\sqrt{2}\rho}{\eta}\right)^{|m|} \times L_p^{|m|}\left(\frac{2\rho^2}{\eta^2}\right)\exp\left(-\frac{\rho^2}{\eta^2}\right)\exp(\mathrm{i}l\phi), \quad (1)$$

where L_p^m are the associated Laguerre polynomial, ρ is the radial cylindrical coordinate, ϕ is the azimuthal angle, and η is the beam waist.

Consider the slowly varying electric field envelope $u(x,y;z)$ of a continues wave paraxial beam propagating in the z-direction, where x and y are the transverse coordinates. The time averaged energy per unit length, carried by the beam, is

$$U = 2\varepsilon_0 \iint |u|^2 \mathrm{d}x\mathrm{d}y, \quad (2)$$

where ε_0 is the permittivity of vacuum. The time-averaged z-component of the orbital angular momentum per unit of length, carried by the light beam, is given by

$$L_z = \iint \lfloor \boldsymbol{r}_\perp \times \boldsymbol{p} \rfloor \mathrm{d}x\mathrm{d}y, \quad (3)$$

where $\boldsymbol{r}_\perp$ is the position vector in the X–Y plane, $\boldsymbol{p} = (\mathrm{i}\varepsilon_0/\omega)[u\nabla_\perp u^* - u^*\nabla_\perp u]$, and ω is the angular frequency.

To elucidate the details of orbital angular momentum of a field distribution $u(x,y;z)$ one has to compute its projection onto the spiral harmonics $\exp(\mathrm{i}l\phi)$, because for a given winding number n, we have to add the weights of all LG modes with the same index n, independent of the index p. We thus let

$$u(\rho,\phi;z) = \frac{1}{\sqrt{2\pi}}\sum_{n=-\infty}^{\infty} a_n(\rho,z)\exp(\mathrm{i}n\phi), \quad (4)$$

where $a_n = 1/(2\pi)^{1/2}\int_0^{2\pi} u(\rho,\phi;z)\exp(-\mathrm{i}n\phi)\mathrm{d}\phi$.

The energy carried by the corresponding light beam can be written as

$$U = 2\varepsilon_0 \sum_{n=-\infty}^{\infty} C_n, \quad (5)$$

where $C_n = \int_0^\infty |a_n(\rho,z)|^2 \rho \mathrm{d}\rho$, which can be shown to be a constant independent of z. The angular momentum of the light beam is thus given by

$$L_z = (2\varepsilon_0/\omega)\sum_{n=-\infty}^{\infty} nC_n. \quad (6)$$

2.2. Description of the mixed vortex field with OAM

The creation of mixed vortex fields is based on a variability of nondiffracting P-N fields. The mixed vortex fields with a multi-helix phase propagating along the z-axis can be expressed in the form[12] as follows:

$$U(r,t) = u_0(z)G(r,t)\sum_{l=1}^{M}\sum_{n=1}^{M} a_l u_{l,n}(r_n)\exp(\mathrm{i}l\phi_n), \quad (7)$$

which describes an array of the Bessel beams carried by a background Gaussian beam G modified by a propagation function u_0. And a_l denotes the weight coefficient. The Gaussian beam can be conveniently described by the confocal parameter q_0 or by the complex parameter $q = z + \mathrm{i}q_0$. The function u_0 changes under a free propagation. It can be expressed as

$$u_0(z) = 2\pi\exp\left[\mathrm{i}\frac{\alpha^2 z}{2k}\left(1-\frac{z}{q}\right)\right],$$

where α is the parameter influencing the spot size of the superposed vortex modes. The complex amplitude G has a known form

$$G(r,t) = \frac{\omega_0}{\omega}\exp\left(-\frac{r^2}{\omega^2} - \mathrm{i}\frac{kr^2}{2R} + \mathrm{i}\Phi\right)\exp(\mathrm{i}\omega t - \mathrm{i}kz),$$

where ω, k, ω_0, R and Φ denote the angular frequency, the wavenumber and the parameters of the common Gaussian beam, respectively.

2.3. Methods of generating light with OAM

Several methods of generating an LG beam have been proposed including conversion from the Hermite–Gaussian (HG) mode using a cylindrical lens,[14] a spiral phase plate (SPP),[15] and a phase hologram element.[16] The output beam mode converted by a cylindrical lens for transforming HG into LG modes gives a pure LG mode. The computer hologram method can generate a beam with high-order phase singular points and obtain a high conversion efficiency. However, these methods are not suitable for the short-pulse high-intensity laser because of laser damage, difficulties in fabricating a large-diameter device, and the energy loss by passing through many optical components. In Ref.[17] employed is a large-diameter optical device which has a multilevel spiral phase distribution in consideration of its application to ultrashort high-intensity laser pulses. The methods of generating PN beams are reviewed in detail in Ref.[10].

3. Light beams with OAM for FSO

3.1. Encoding information into OAM states of single light beam

In Ref.[5] are realized the generation and measurement of weighted LG beams via phase holograms, corresponding to the information code. The experimental configuration is shown in Fig.1. The transmitter unit is based on a HeNe laser, a computer-controlled phase hologram implemented using a reflective spatial light modulator (SLM) and a telescope. The receiver unit comprises a similar telescope, SLM and CCD array detector.

Using the OAM to encode the data offers an inherent security enhancement as it is difficult to read the data without positioning the detector directly in the path of the intended receiver.[5,18] The OAM data are extremely difficult to recover from the light that is scattered by the atmosphere since the time-dependent scattering process randomizes the phase structure of the beam. The light scattered from optical components, which, although, are not subjected to temporal variation, is still highly complex, and the recovery of the original OAM data is non-trivial. Even if the detector is positioned within the beam, an eavesdropper still has some difficulty in establishing a system to measure the OAM.

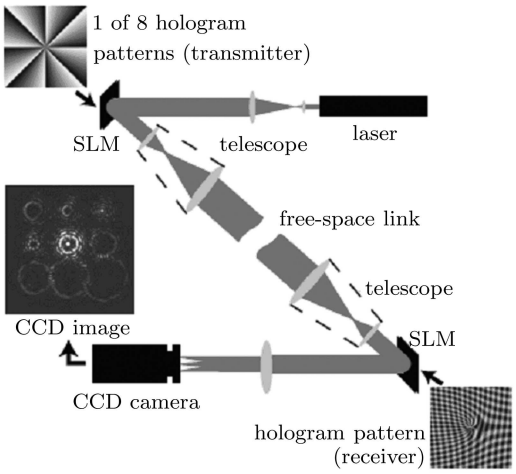


Fig.1. Optical scheme configuration of encoding information into OAM states of single light beam.[5]

3.2. Encoding information into the spatial structure of the mixed vortex with OAM

3.2.1. Mixed vortex generated by single light source

The mixed vortex field representing the superposition of OVs with different topological charges is examined as a convenient carrier of information in Ref.[12]. Applying a spatial light modulator, the mixed vortex field of the form $U = \sum_{l=1}^{M} a_l u_l \exp(\mathrm{i}l\phi)$ can be generated directly by a single Gaussian beam. The number of superposed vortices M, their topological charges l, amplitudes u_l and weight coefficients a_l are encoded into the structure of the computer-generated hologram sent to the spatial light modulator. If the modulator is driven dynamically, the weight coefficients can represent bits of information encoded into the spatial structure of the mixed vortex field. After Fourier filtration performed behind the spatial light modulator, the vortex field propagates as the P-N field. At the receiver, the weight coefficients must be measured to decode information. In the proposed decoding scheme, it is performed by the spatial separation of the OVs. The method is realizable by means of amplitude and phase modulations and works in such a way that the weight coefficients of the OVs are identified as bright intensity spots localized at positions uniquely specified by their topological charges.

The application of the mixed vortex field, expression (7), to the information encoding is feasible due to the fact that it can be obtained by the direct transformation of the single Gaussian beam. The generated mixed vortex field possesses degrees of freedom which are unavailable by the common light field. As additional parameters, the quantities and signs of the topological charges and the weight coefficients of the OVs can be applied to the information encoding. While the weight coefficients are used as carriers of bits of information, the topological charges serve as 'markers' enabling the information decoding. The encoding of information into the spatial structure of the mixed vortex field is based on an application of field (7). It represents the required superposition of the OVs with different topological charges and can be generated by the transmission of the collimating laser beam through the spatial light modulator (SLM). It can be realized by means of the setup illustrated in Fig.2.

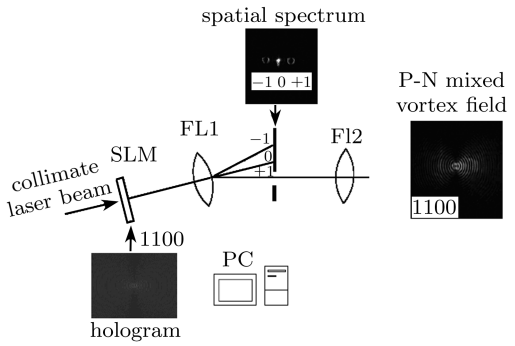


Fig.2. Optical scheme configuration of encoding information into the spatial structure of the mixed vortex generated by single light source.[12]

The calculated intensity pattern representing the computer-generated hologram is sent to the SLM placed at the front focal plane of the first Fourier lens. If the SLM is irradiated by the laser beam, the spatial spectrum of the transmitted field appears at the back focal plane of the Fourier lens. After a convenient spatial filtering a very good approximation of the field (7) is obtained behind the second Fourier lens.

3.2.2. Mixed vortex generated by multi-light-source

In Ref.[19] is realized the generation of weighted superposition of the Bessel beams, corresponding to the information code. The information code is represented by an area of N point sources in such a way that each of them can be bright or dark. The mixed vortex field can be generated with frequencies that are usually in a range of optical communication by a dynamical modulation of the intensity of the point sources.

As shown in Fig.3, if the array of point sources is placed in front of focal plane of a lens, a weighted superposition of plane waves is generated behind the lens. The superposition of plane waves goes through a complex mask placed behind the lens. The complex mask splits each impinging plane wave into N waves with different helicities and directions of propagation. When a superposition of N plane waves impinges on the mask, N^2 helical waves appear behind the mask. Of them, N helical waves propagate along the z-axis, $N(N-1)$ ones have a nonzero transversal component of the wave vector and propagate in a direction inclined with respect to the z-axis. The next component of the optical setup is an axicon placed just behind the complex mask. By means of the axicon, the helical wave with the topological charge m_{j} propagating along the optical axis is transformed to the m_{j}-th order Bessel beam. After filtration and an inverse Fourier transformation realized by the second Fourier lens, the required distribution of an optical field is obtained. The complex amplitude at the back focal plane of the second Fourier lens is of the form

$$U = \sum_{j=1}^{N} a_j u_{0,j} \exp(\mathrm{i} m_j \varphi), \tag{8}$$

where a_j represents one bit of the information code and its value is 0 or 1. u_{0j} denotes a complex amplitude of the m_j-th order Bessel beam at the output of the encoder.

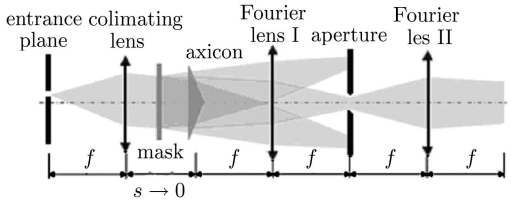


Fig.3. Optical scheme configuration of encoding information into the spatial structure of the mixed vortex generated by multi-light-source.[19]

The obtained field (8) exhibits non-diffracting properties. It can propagate without changing of its intensity profile in an infinite region, which cannot be realized experimentally in this ideal form. A good experimental approximation of the ideal Bessel beam can be achieved using the Gaussian envelope to modulate the transversal intensity profile of a Bessel beam.

On the receiver side, the optical field goes through a hologram. Behind the hologram, each helical wave is split into N waves. Applying the Fourier transformation realized by a Fourier lens, a separation of individual deflected waves can be achieved. And the information code can be decoded by measuring the intensity.

4. Conclusion

The basic principle of using light beams with orbital angular momentum for FSO is analysed in this paper. Both encoding information into OAM states of single light beam and encoding information into spatial structure of the mixed optical vortex are discussed. The mixed optical vortex can be generated by single light source and multi-light-source. Light's OAM has an infinite number of eigenstates, thus the number of bits that the OAM of a single photon can represent is unlimited in principle. Using the OAM of single light beam to encode the data offers an inherent security as it is difficult to read the data without positioning the detector directly in the path of the intended receiver. Mixed vortex fields represent P-N fields possessing a self-reconstruction property. These properties are very significant to the FSO. Using P-N optical fields with OAM to encode information is potential to improve the performance of FSO.

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References

[1] Liu X M, Sun L R, Lang J F, Hai T, Pan W Q and Zhao D 2005 *Acta Phys. Sin.* **54** 5149 (in Chinese)
[2] Wu J Z and Li Y J 2005 *Proc. SPIE ICMIT* **2005** 6041
[3] Liu C X, Guo H L, Jiang Y Q, Li Z L, Cheng B Y and Zhang D Z 2005 *Acta Phys. Sin.* **54** 1162 (in Chinese)
[4] Zhao Z G and Lü B D 2006 *Acta Phys. Sin.* **55** 1798 (in Chinese)
[5] Graham Gibson, Johannes Courtial and Miles J Padgett 2004 *Optics Express* **12** 22
[6] Allen L, Beijersbergen M W, Spreeuw R J C and Woerdman J P 1992 *Phys. Rev.* A **45** 8185
[7] Amico M D, Leva A and Micheli B 2003 *IEEE Microwave and Wireless Components Lett.* **13** 305
[8] Hughes R J, Nordholt J E, Derkacs D and Peterson C G 2002 *New J. Phys.* **4** 43
[9] Molina-Terriza G, Torres J P and Torner L 2002 *Phys. Rev. Lett.* **88** 013601
[10] Zdeněk Bouchal 2003 *Czech. J. Phys. Sect.* A **53** 537
[11] O'Neil A T, MacVicar I, Allen L and Padgett M J 2002 *Phys. Rev. Lett.* **88** 5
[12] Zdeněk Bouchal and Radek Celechovsky 2004 *New J. Phys.* **6** 131
[13] Lluis Torner, Juan P Torres and Silvia Carrasco 2005 *Optics Express* **13** 3
[14] Padgett M, Arlt J, Simpson N and Allen L 1996 *Am. J. Phys.* **64** 77
[15] Beijersbergen M W, Coerwinkel R P C, Kristensen M and Woerdman J P 1994 *Opt. Commun.* **112** 321
[16] Heckenberg N R, McDuff R, Smith C P and White A G 1992 *Opt. Lett.* **17** 221
[17] Sueda K, Miyaji G, Miyanaga N and Nakatsuka M 2004 *Optics Express* **12** 15
[18] Franke-Arnold S, Barnett S, Yao E, Leach J, Courtial J and Padgett M 2004 *New J. Phys.* **6** 103
[19] Radek Celechovsky and Zdeněk Bouchal 2006 *J. Mod. Opt.* **53** 473