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Off-axial elliptical cosine-Gaussian beams and their propagation properties

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In this paper, a new kind of light beam called off-axial elliptical cosine-Gaussian beam (ECosGBs) is defined by using the tensor method. An analytical propagation expression for the ECosGBs passing through axially nonsymmetrical optical systems is derived by using vector integration. The intensity distributions of ECosGBs on the input plane, on the output plane with the equivalent Fresnel number being equal to 0.1 and on the focal plane are respectively illustrated for the propagation properties. The results indicate that an ECosGB is eventually transformed into an elliptical cosh-Gaussian beam. In other words, ECosGBs and cosh-Gaussian beams act in a reciprocal manner after propagation.

Keywords: elliptical cosine-Gaussian beams, nonsymmetrical optical systems, diffraction integral
PACC: 4225B, 4260H

1. Introduction

It is well known that Hermite-sinusoidal-Gaussian beams are obtained as the paraxial solutions of the Helmoltz equation in the rectangular coordinate system, as introduced by Casperson and Tovar.[1,2] Their propagation properties through axially symmetrical optical systems have been extensively studied.[3−10] Actually, the Hermite-sinusoidal-Gaussian beams represent more general beams, and the Gaussian, Hermite–Gaussian, cosh (sinh)-Gaussian, and cosine (sine)-Gaussian beams can be regarded as their special cases. The propagation characteristics of Hermite-cosh-Gaussian beams in apertured fractional Fourier transforming systems have been shown.[11] The generation and propagation of elliptical cosh-Gaussian beams[12] and off-axial elliptical Hermite-cosh-Gaussian beams[13] have also been studied. The propagation properties of Hermite-sinusoidal-Gaussian beams in turbulent atmosphere have been presented by Baykal[14] and Eyyuboglu.[15]

Up to now, the propagation of elliptical cosine-Gaussian beams (ECosGBs) passing through axially nonsymmetrical $ABCD$ optical systems has not been shown in the existent literatures. The aim of the present paper is to study the propagation and transformation of ECosGBs by use of the tensor method.

This paper is organized as follows. The definition of the off-axial ECosGBs by use of tensor method is given in Section 2. The analytical propagation formula of an off-axial ECosGB passing through an axially nonsymmetrical optical system is derived through vector integral in Section 3. Some numerical simulations are given in Section 4 to illustrate for the propagation in a free space and a focusing optical system. Finally, a simple conclusion is outlined in Section 5.

2. Definition of off-axial elliptical cosine-Gaussian beams

The optical field distribution of the one dimensional cosine-Gaussian beam on the input plane ($z = 0$) is given by[16]

$$E_1(x_1) = \exp(-x_1^2/w_{0x}^2)\cos(u_x x_1), \qquad (1)$$

where w_{0x} is the waist size of a Gaussian beam in the x direction, and u_x is the displacement parameter associated with the cosine part.

For a two-dimensional case, the off-axial elliptical Gaussian beam could be expressed in tensor form as[17]

$$E_1(r_1) = \exp\Big[-\frac{\mathrm{i}k}{2}(r_1 - r_0)^{\mathrm{T}} Q_{\mathrm{e}}^{-1}(r_1 - r_0)\Big], \qquad (2)$$

where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, r_1 denotes a position vector on a transverse plane given by $r_1^{\mathrm{T}} = (x_1, y_1)$, $r_0^{\mathrm{T}} = (x_0, y_0)$ is the

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off-axial parameter, and Q_{e}^{-1} is the 2×2 complex curvature tensor for the generalized astigmatic elliptical Gaussian beam. If the waist of the beam is located on the initial plane, then[18,19]

$$Q_{\mathrm{e}}^{-1} = \begin{bmatrix} q_{xx}^{-1} & q_{xy}^{-1} \\ q_{xy}^{-1} & q_{yy}^{-1} \end{bmatrix} = -\frac{\mathrm{i}\lambda}{\pi} \begin{bmatrix} w_{0x}^{-2} & w_{0xy}^{-2} \\ w_{0xy}^{-2} & w_{0y}^{-2} \end{bmatrix}. \tag{3}$$

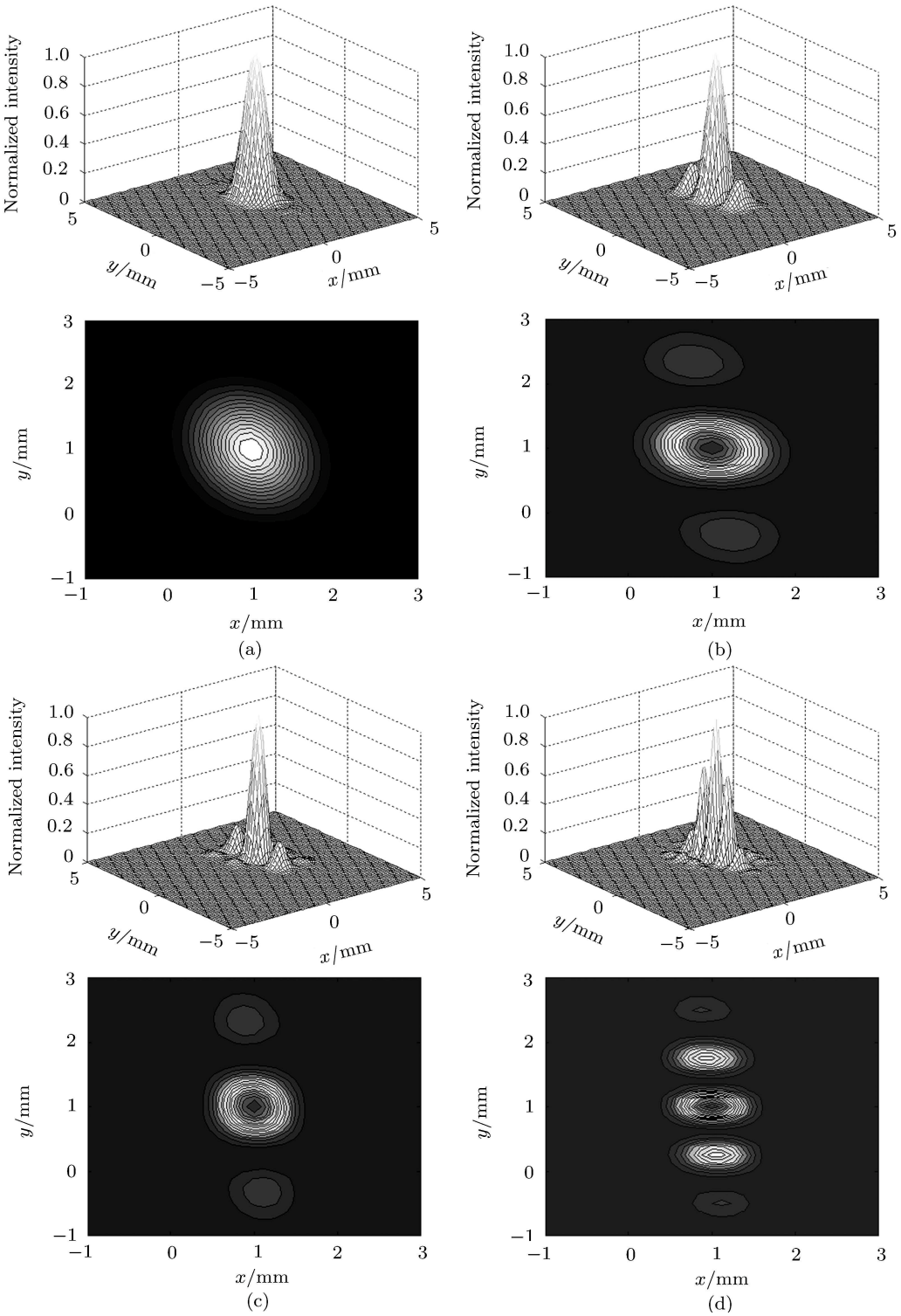


Fig.1. Three-dimensional normalized intensity distributions and their contour graphs of the off-axial ECosGBs on the input plane of $z = 0$, with $r_0^{\mathrm{T}} = (1, 1)$. The displacement parameters of cosine parts: (a) $u_x = 1$ mm^{-1}, $u_y = 1$ mm^{-1}; (b) $u_x = 1$ mm^{-1}, $u_y = 2$ mm^{-1}; (c) $u_x = 2$ mm^{-1}, $u_y = 2$ mm^{-1}; (d) $u_x = 2$ mm^{-1}, $u_y = 4$ mm^{-1}.

Up to now, one-dimensional or two-dimensional (x and y directions are independent) cosine Gaussian beams and off-axial cosine Gaussian beams passing through a nonsymmetrical optical system have been discussed intensively.[18,19] In practice, laser beams generated from a resonator of neither completely circular symmetry nor completely rectangular symmetry are ECosGBs or off-axial ECosGBs. In the following, we will define the mode of off-axial ECosGBs by using tensor method.

Comparing Eq.(2) with Eq.(1) and after some manipulations by using of the tensor method, we can define the off-axial ECosGBs as follows:

$$E_1(r_1) = \frac{1}{2}\exp\Big[-\frac{\mathrm{i}k}{2}(r_1 - r_0)^{\mathrm{T}} Q_{\mathrm{e}}^{-1}(r_1 - r_0)\Big] \times \{\cos[(r_1 - r_0)^{\mathrm{T}} u_+] + \cos[(r_1 - r_0)^{\mathrm{T}} u_-]\}, \quad (4)$$

where u_+ and u_- are associated with the cosine parts and given by

$$u_+ = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \quad u_- = \begin{bmatrix} u_x \\ -u_y \end{bmatrix}. \quad (5)$$

In the following calculation, we choose the beam parameters as $\lambda = 0.6328$ μm, $w_{0x} = 1.0$ mm, $w_{0y} = 1.5$ mm, $w_{0xy} = 2.0$ mm.

Figure 1 shows the normalized intensity distributions and their contour graphs of off-axial ECosGBs with different displacement parameters of cosine parts. When u_x and u_y are small, we can find that the intensity profiles of off-axial ECosGBs are similar to the off-axial elliptical Gaussian beams as shown in Fig.1(a). When the parameters increase, the smaller lobe peaks appear on the two sides of the main peak in Fig.1(b), the number and the height increase as shown in Figs.1(c) and 1(d).

3. Propagation of off-axial elliptical cosine-Gaussian beams in nonsymmetrical optical systems

The propagation of an off-axial ECosGB passing through an axial nonsymmetrical optical system can be treated by the general Collins formula, the tensor form can be written as[18,19]

$$E_2(r_2) = \frac{\mathrm{i}}{\lambda}\exp(-\mathrm{i}kL_0)[\det(B)]^{-1/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1(r_1)\exp\Big[-\frac{\mathrm{i}k}{2}(r_1^{\mathrm{T}}B^{-1}Ar_1 - 2r_1^{\mathrm{T}}B^{-1}r_2 + r_2^{\mathrm{T}}DB^{-1}r_2)\Big]\mathrm{d}r_1\,, \quad (6)$$

where L_0 is optical path along the propagation axis, A, B, C and D are the sub-matrices of the optical system which are defined by[19]

$$\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}, \quad (7)$$

where r_1 and r_2 means position vector, r_1' and r_2' denotes slope vector.

Substituting Eq.(4) into Eq.(6) and after a straightforward vector integral operation, we obtain the analytical propagation expression of the off-axial ECosGB passing through an axially nonsymmetrical $ABCD$ optical system as follows:

$$E_2(r_2) = \frac{1}{4}\sum_{\pm u_\pm}\exp(-\mathrm{i}kL_0)[\det(BQ_{\mathrm{e}}^{-1} + A)]^{-1/2} \times \exp\Big\{-\frac{\mathrm{i}k}{2}r_2^{\mathrm{T}}[DB^{-1} - B^{-1\mathrm{T}}(Q_{\mathrm{e}}^{-1} + B^{-1}A)^{-1}B^{-1}]r_2\Big\}\exp[\mathrm{i}kr_0^{\mathrm{T}}(B + AQ_{\mathrm{e}})^{-1}r_2] \times \exp\Big[-\frac{\mathrm{i}k}{2}r_0^{\mathrm{T}}(Q_{\mathrm{e}} + A^{-1}B)^{-1}r_0\Big]\exp\Big[\frac{\mathrm{i}}{2k}u^{\mathrm{T}}(Q_{\mathrm{e}}^{-1} + B^{-1}A)^{-1}u\Big] \times \exp[\mathrm{i}(r_2^{\mathrm{T}}B^{-1\mathrm{T}} - r_0^{\mathrm{T}}B^{-1}A)(Q_{\mathrm{e}}^{-1} + B^{-1}A)^{-1}u], \quad (8)$$

where $\sum_{\pm u_\pm}$ denotes the sum for $+u_+$, $-u_+$, $+u_-$, $-u_-$ respectively.

Equation (8) is the main result of this paper, which can replace the usual method by using diffraction integral formula directly. In the derivation of Eq.(8), the following integral formula has been used:[20]

$$\int_{-\infty}^{\infty} \exp\Big[\frac{-(x-y)^2}{2a}\Big]\mathrm{d}x = \sqrt{2\pi a}\,. \tag{9}$$

4. Numerical simulations and discussions

In this section we will study the propagation properties of off-axial ECosGBs passing through nonsymmetrical optical systems by using the derived analytical propagation expression Eq.(8). First, for the sake of simplicity, we are going to illustrate the propagation of the off-axial ECosGBs through a free space. The sub-matrices for a free space of distance z are

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{10}$$

Substituting Eq.(10) into Eq.(8), we can obtain the three-dimensional intensity distributions of off-axial ECosGBs passing through a free space. In the following calculation, the propagation distance is represented by the equivalent Fresnel number $N_x = w_{0x}^2/(\lambda z)$.

After some numerical calculations we have found that the intensity distributions of off-axial ECosGBs on the planes of near field such as $N_x = 10$ and $N_x = 1$ are similar to the initial intensity distributions. But on the planes of far field such as $N_x = 0.1$, the distributions change obviously.

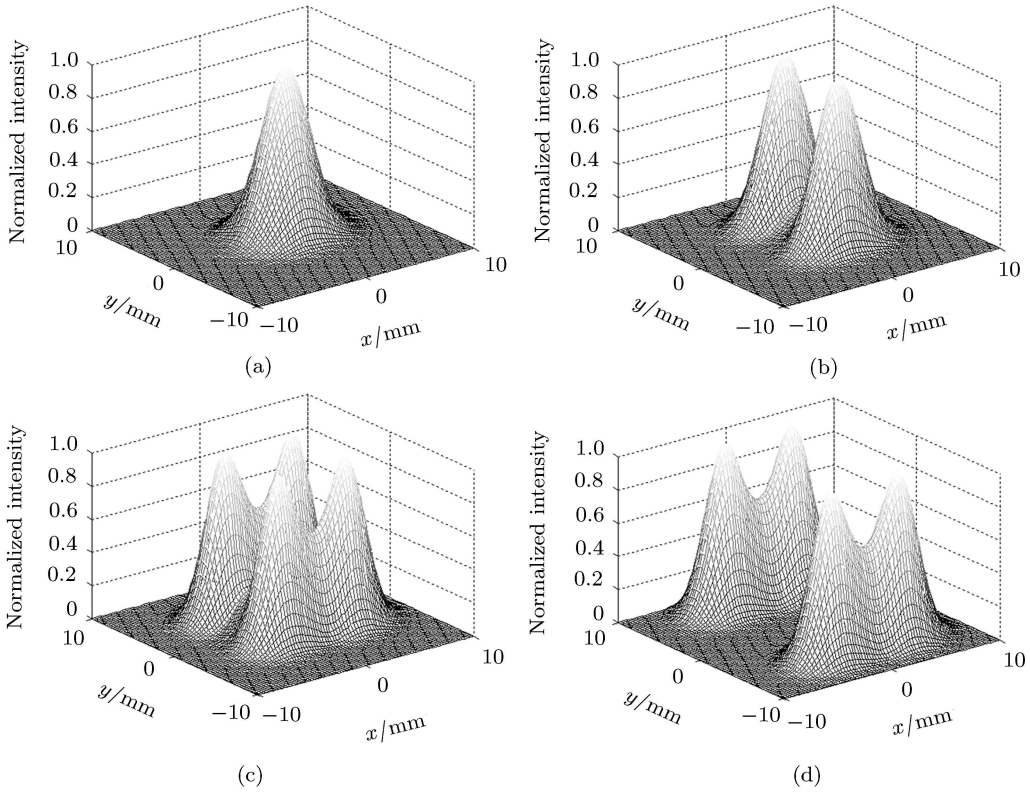


Fig.2. Three-dimension normalized intensity distributions graphs of the off-axial ECosGBs passing through a free space on the plane of $N_x = 0.1$, with $r_0^{\mathrm{T}} = (1, 1)$. The displacement parameters of cosine parts: (a) $u_x = 1$ mm^{-1}, $u_y = 1$ mm^{-1}; (b) $u_x = 1$ mm^{-1}, $u_y = 2$ mm^{-1}; (c) $u_x = 2$ mm^{-1}, $u_y = 2$ mm^{-1}; (d) $u_x = 2$ mm^{-1}, $u_y = 4$ mm^{-1}.

Figure 2 shows the normalized intensity distributions of off-axial ECosGBs passing through a free space on the output plane of $N_x = 0.1$. We can see only one lobe peak as shown in Fig.2(a) and it is similar to the elliptical Gaussian distribution. When the parameters ($u_x = 1$ mm^{-1}, $u_y = 1$ mm^{-1}) increase, the main peak becomes two lobe peaks as shown in Fig.2(b). Furthermore, a central hollow appears on each of the two lobe peaks and the distance between two lobe peaks becomes longer as shown in Figs.2(c) and 2(d). These profiles are similar to the intensity profiles of elliptical cosh-Gaussian beams on the input plane.[12]

Furthermore, we will study the transformation of off-axial ECosGBs passing through a focusing optical system. The sub-matrices for a focusing system consisting of a lens with focal lengths f_x and f_y in the x and y directions respectively, and a free space with distance of z are

$$A = \begin{bmatrix} 1 - z/f_x & 0 \\ 0 & 1 - z/f_y \end{bmatrix}, \quad B = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix},$$
$$C = \begin{bmatrix} -1/f_x & 0 \\ 0 & -1/f_y \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad (11)$$

and it can be regarded as an axially nonsymmetrical $ABCD$ optical system.

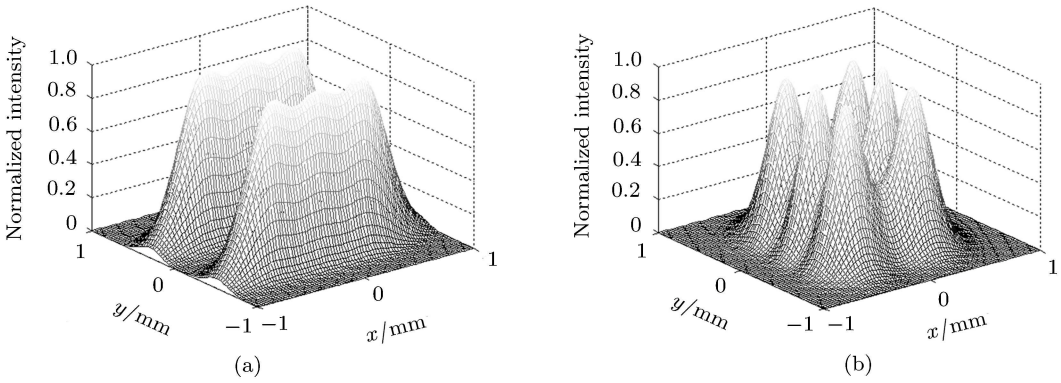


Fig.3. Three-dimension normalized intensity distributions graphs of the ECosGBs with $r_0^{\mathrm{T}} = (0, 0)$ passing through a focusing system consisted of a focal lens with focal lengths $f_x = 1580$ mm and $f_y = 2000$ mm. The same displacement parameters of cosine parts $u_x = 2$ mm^{-1} and $u_y = 2$ mm^{-1}. The different propagation distances: (a) $z = 2000$ mm, (b) $z = 1580$ mm.

In the calculation, the focal lengths are $f_x = 1580$ mm and $f_y = 2000$ mm. Figure 3 shows that the normalized intensity distributions of ECosGBs on the two focal planes of the lens. From Fig.3 we can find that the intensity distributions are compressed in x and y directions respectively. The two profiles on the two focal planes are different and this is because the source beam of Fig.1(c) experiences different mergence of neighbouring beam lobes in the x and y directions. However, their profiles are still similar to the intensity profiles of elliptical cosh-Gaussian beams.

Figure 4 shows the normalized intensity distributions and their contour graphs of off-axial ECosGBs on the focal plane of the lens by choosing the parameters as $z = f_x = f_y = 1580$ mm. Comparing Figs.2(b), 2(c) and 2(d) with Figs.4(b), 4(c) and 4(d), we can find that the profiles on the focal plane are similar to the intensity distributions on the plane of $N_x = 0.1$ and the intensity distributions are compressed in x and y directions respectively. The profile in Fig.4(a) becomes flat-topped. These profiles on the focal plane are also similar to the profiles of elliptical cosh-Gaussian beams on the input plane in some special cases.[12] That is to say, an ECosGB on the input plane corresponds to an elliptical cosh-Gaussian beam in the far field or on the focal plane. These observations are consistent with elliptical cosh-Gaussian beams[12] and the reciprocity property of cosine-Gaussian and cosh-Gaussian beams in a turbulent atmosphere.[14,15]

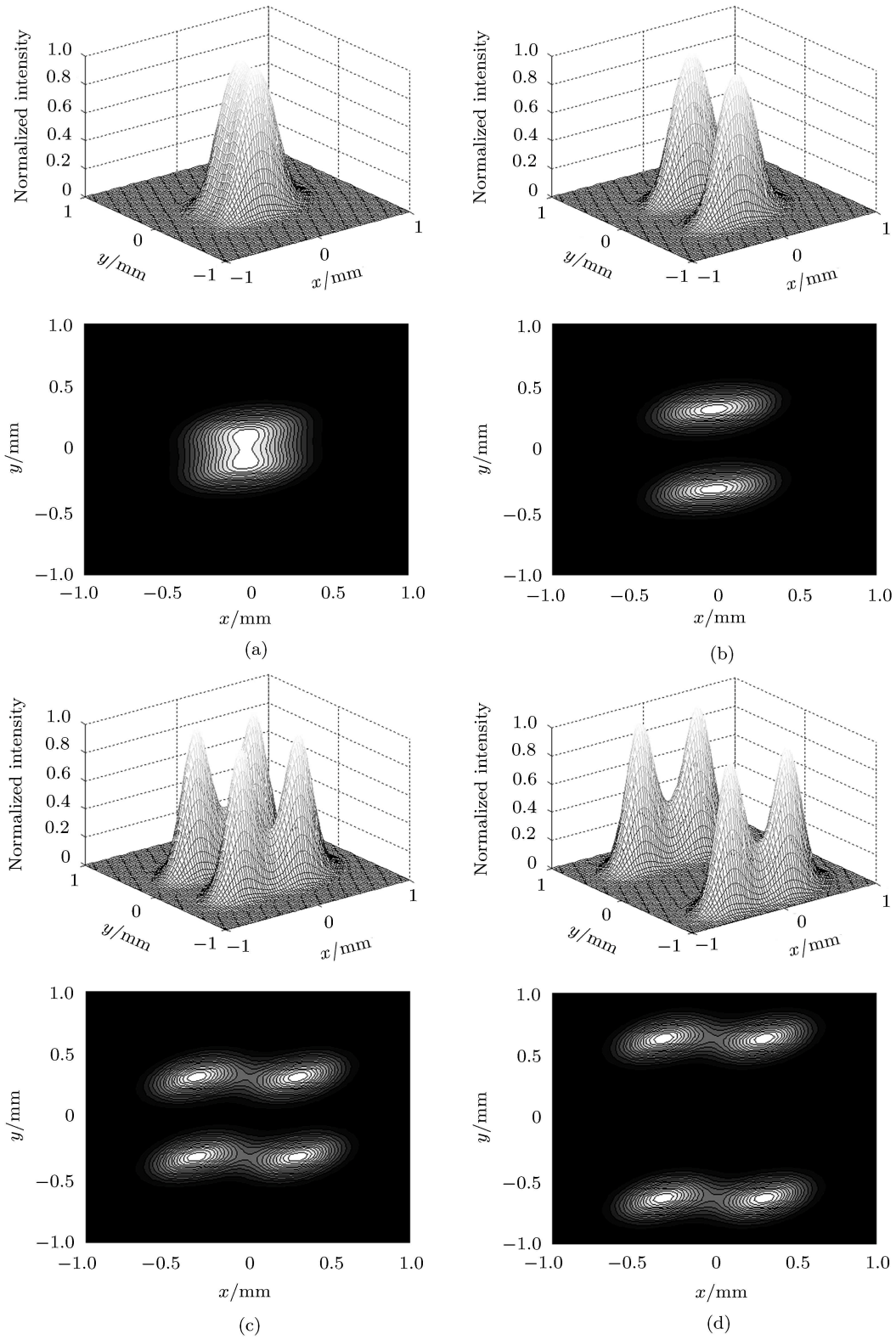


Fig.4. Three-dimensional normalized intensity distributions graphs of the ECosGBs passing through a focusing system consisted of a focal lens on the focal plane of $z = f_x = f_y = 1580$, with $r_0^{\mathrm{T}} = (0, 0)$. (a) $u_x = 1$ mm^{-1}, $u_y = 1$ mm^{-1}; (b) $u_x = 1$ mm^{-1}, $u_y = 2$ mm^{-1}; (c) $u_x = 2$ mm^{-1}, $u_y = 2$ mm^{-1}; (d) $u_x = 2$ mm^{-1}, $u_y = 4$ mm^{-1}.

5. Conclusions

In this paper, we have introduced a new kind of light beams called the off-axial ECosGBs by using tensor method. The analytical expression is obtained as a useful method in treating the propagation and transformation of off-axial ECosGBs passing through axially nonsymmetrical optical systems. By virtue of numerical simulations, the propagation properties of off-axial ECosGBs through a free space and a focusing system are illustrated respectively. The results indicate that an ECosGBs in the far field or on the focal plane corresponds to an elliptical cosh-Gaussian beam on the input plane. It can be concluded that a beam with flat-topped profile can be obtained by choosing suitable beam parameters of cosine part and the result may be of practical interest because flat-topped light beams are needed in many applications.

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