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Waveguide resonance of subwavelength metallic slits*

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The transmission characteristics of a metallic film with subwavelength periodic slits are investigated by using the two-dimensional finite-difference time-domain method (2D-FDTD). Two models are constructed to show the dependance of the transmission spectrum on the slit structure. A sandwiched structure is used to exhibit the contribution of the metallic wall inside slits to the extraordinary high transmission. And a filled slit structure is employed to reflect the relation between the average refractive index inside the slits and the transmission spectrum of the structure. The transmission characteristics of two structures can be explained well with the waveguide resonance theory.

Keywords: electromagnetic wave propagation, metallic subwavelength periodic slits, waveguide resonance

PACC: 4220, 7360D, 4280L

1. Introduction

Recently the extraordinary high transmission of light through metallic subwavelength slit arrays has aroused a great interest due to its potential applications and unclear physical mechanism.[1−3] When the size of slits is much smaller than the wavelength of the incident light, it can be found that the maximum transmission is about 2–3 times higher than that in the case of the hole porosity of the structure. Many theories have been proposed to interpret this phenomenon.[4−11] Although it is generally accepted that the enhanced transmission is mainly due to the resonant excitation of the plasma on the upper and bottom surfaces of the structure,[4,5,7] the contribution of the slit waveguide resonance cannot be ignored.[7−11]

In this paper, two models are investigated to show the role of the waveguide resonance in the extraordinary transmission phenomena. Firstly, the effect of the metallic wall inside the slit on the zero-order transmittance is explored. The waveguide resonance theory shows that the inner walls have an influence upon the average effective refraction and determine the resonance wavelength. A sandwiched metallic structure is introduced to show the effect of the inner wall on the transmission spectrum through replacing the middle layer with different kinds of metals (gold or aluminium). Then, considering the effect of the medium refraction in the slit on the waveguide resonance condition, the effect of medium inside slits is investigated by filling slits with other dielectric mediums. Filling the slits can change the optical-path which the propagating waves experience, while this has little effect on the evanescent waves which couple the surface plasma on both sides of the film. The zero-order transmission spectra of the two structures are calculated by using the two-dimensional finite-difference time-domain method (2D FDTD). The waveguide resonance theory is used to analyse these transmission spectra. It is found that the waveguide resonance theory can explain the simulation results well.

The remainder of this paper is organized as follows. In Section 2, the theoretical model is described. In Section 3, the transmission spectra of the sandwiched metallic structure are obtained and interpreted. In Section 4, the transmission spectra of the filled slit structure are presented and analysed. In Section 5, the dependence of the transmission spectra on the thickness of the film is investigated. Finally, a conclusion is drawn from the present study in Section 6.

2. Theoretical model

The dispersion properties of the metal must be considered here since the absorbtion and permittivity

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of the metallic material are frequency dependent. The Drude model is used here to describe the dependence of the metallic permittivity on the frequency, [12,13]

$$\epsilon_{\rm m}(\omega) = \epsilon_0\left(1 - \frac{\omega_{\rm p}^2}{\omega(\omega + {\rm i}\gamma)}\right), \tag{1}$$

where $\omega_{\rm p}$ is the plasma frequency of the metal and γ represents the damping rate which characters the ohmic absorbtion loss. Three kinds of metallic materials: silver (Ag), gold (Au), and aluminium (Al), are used in this paper, and their plasma frequency $\omega_{\rm p}$ and loss γ are shown in Table 1.

Table 1. Plasma frequency $\omega_{\rm p}$ and loss γ of several metals.

Metal	ω_p/(rad/s)	γ/(rad/s)
Ag	1.37×10^{16}	3.11×10^{13}
Au	1.24×10^{16}	1.30×10^{14}
Al	2.27×10^{16}	1.00×10^{15}

The propagation of the light in the metal is described by the Maxwell equations which are coupled with the oscillations of quasifree electrons in the metal (Drude model) as follows:[13]

$$\begin{aligned} \nabla \times \boldsymbol{E} &= -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}, \\ \nabla \times \boldsymbol{H} &= \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J}, \\ \frac{\partial \boldsymbol{J}}{\partial t} &+ \gamma \boldsymbol{J} = \epsilon_0 \omega_{\rm p}^2 \boldsymbol{E}, \end{aligned} \tag{2}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field vectors, respectively. $\boldsymbol{J}$ is the current density (A/cm^2) and equal to the time derivative of the metal polarization, i.e. $\boldsymbol{J} = \partial \boldsymbol{P}/\partial t$. ϵ_0 and μ_0 are the permittivity and magnetic permeability of the vacuum, respectively.

The 2D FDTD method is employed to simulate the interaction between the metal and the incident wave. The structures are irradiated normally by a TM- polarized plane wave pulse ($E_x, E_y, H_z \neq 0$) centred at $\lambda = 1100\,$nm. The spectrum width of this pulse can cover the range desired in the calculation. The temporal response of the structure is saved after calculation. Through a simple Fourier transform, the transmission spectra of the structure can be obtained. The details of the 2D FDTD method can be found in Refs.[12–14].

3. Effect of the inner wall

In this section, a sandwiched structure is used to show the effect of the inner wall on the transmission spectrum. The sandwiched structure is constructed by inserting a different metallic film with a thickness d_1 into a silver film, as shown in Fig.1. The total thickness of the structure is h and the whole structure is surrounded by vacuum. In this work, the materials of both upper and bottom layers are Ag and the middle layer is of Au or Al. The slits with a width of $d = 100\,$nm are distributed in the film, and the period of the slits is $p = 0.9\,\mu$m and $h = 1800\,$nm.

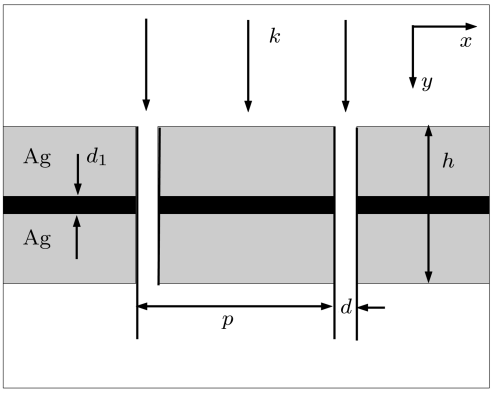


Fig.1. Schematic of the sandwiched structure.

The transmission spectra are shown in Fig.2 for a middle layer of gold. It can be found that the peak values decrease as the Au film thickness increases and the wavelengths corresponding to the peaks increase little. However, if the middle layer of Au is replaced by the Al film, the transmission peaks and the corresponding wavelengths decrease with thickness of the middle layer increasing. The shift of the transmission peak cannot be clearly explained by using the surface plasmon resonance theory. However, from the viewpoint of the waveguide resonance theory, the wave propagating in slits experiences an effective refractive index.[10] The transmission will reach peaks when the Fabry–Perot resonance condition is satisfied, i.e.

$$k_0 {\rm Re}(n_{\rm eff})h - \arg(r_{12}) = m\pi, \tag{3}$$

where k_0 is the incident wave vector in the vacuum, $n_{\rm eff}$ is the effective refractive index of the fundamental waveguide mode, and m is an integer. The first term on the left-hand side of Eq.(3) indicates that the resonance occurs only in the case that the cavity length is an integral multiple of the half-wavelength of the

waveguide mode. The second term is an additional phase shift when the wave is reflected at the grating interfaces. Here the effective refractive index n_{eff} is proportional to both the average refractive index of the medium in the slits and the average refractive index of the slit walls.

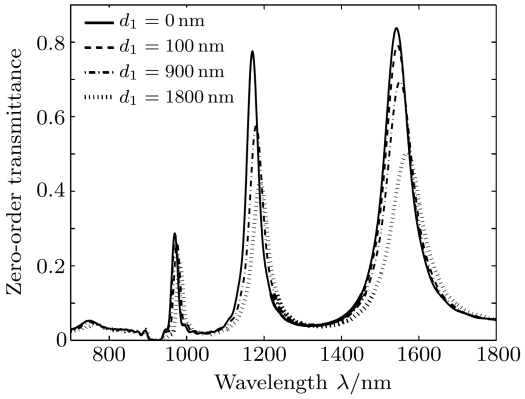


Fig.2. Transmission spectra for the sandwiched structure with a middle layer of Au.

For the middle layer of gold, the average refractive index of the slit walls has the following form:

$$n_{\text{wall}} = d_1 n_{\text{Au}} + (h - d_1) n_{\text{Ag}}, \tag{4}$$

where the n_{Au} and n_{Ag} are effective refractive indices for the Au section and Ag section, respectively. Due to ω_{p} of gold being smaller than that of silver, the effective refractive index of the slit wall is smaller than that of structure totally composed of silver, and the transmission peaks shift towards the longer wavelengths. When the Al film is inserted, the plasma resonance frequency of Al is bigger than that of Ag, thus the effective refractive index of the slit wall increases when the middle layer becomes thick. Therefore, the transmission peaks shift towards shorter wavelengths as shown in Fig.3.

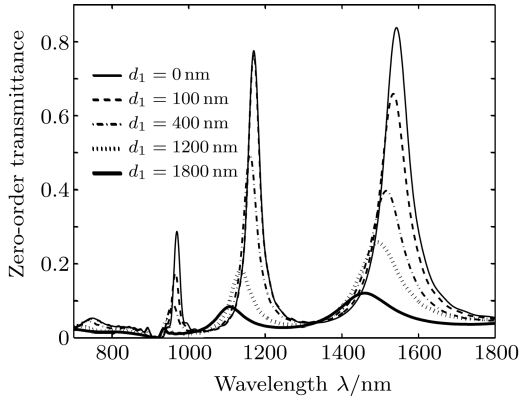


Fig.3. Transmission spectra for the sandwiched structure with a middle layer of Al.

The decrease of transmission peak values may be caused by the increase of the effective loss. Since the loss of Au or Al is bigger than that of silver, the effective conductivity decreases when the Al or Au layer is included, which leads to the fall of the transmission peaks in Fig.2 and Fig.3 as d_1 increases.

The properties of transmission spectra of sandwiched structure indicate that metallic walls of the slits are responsible for determining the position of transmission peaks. According to the waveguide resonance theory, this is due to the fact that the metallic walls are related to the average effective refraction and the transmission loss.

4. Effect of the average refractive index

The position of the transmission peak is also determined by the average refractive index in the slits. A filled slit model is constructed to show this effect. As shown in Fig.4, the slits are filled with the dielectric medium with a width of d_{in}, other parameters (the lattice constant p, slit width d, and the thickness h) are the same as those in the above section.

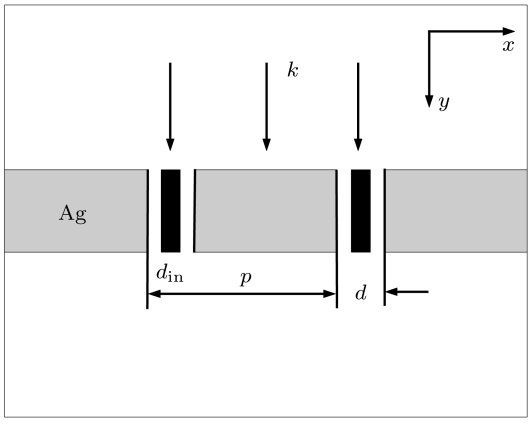


Fig.4. Schematic of the filled slit structure. A dielectric medium is inserted inside the slits.

Firstly, slits are filled with the dielectric mediums with different widths. The relative permittivity of the filling medium is $\epsilon_2 = 1.4$ and three widthes are considered: $d_{\text{in}} = 50$ nm, $d_{\text{in}} = 70$ nm, and $d_{\text{in}} = 100$ nm, respectively. The corresponding transmission spectra are shown in Fig.5, and it can be seen that the transmission peaks shift to long wavelengths when the width of the filling medium increases. When $d_{\text{in}} = 100$ nm, there appears a new transmission mode

near to the wavelength $\lambda = 900\,\text{nm}$. As mentioned above, the effective refractive index is proportional to both the average refractive index of the metallic wall and the average refractive index of the medium in the slits. When the dielectric medium with a relative permittivity $\epsilon > 1$ is inserted into the slits, the average refractive index in the slits can be represented as $n_{\text{avg}} = 1 \times (d - d_{\text{in}}) + d_{\text{in}} n_{\text{in}}$, where n_{in} is the refractive index of the dielectric medium inserted into the slits. Thus the effective refractive index increases as the width d_{in} increases. According to Eq.(3), the positions of the resonance peaks will move towards the long wavelengths. When the increase of the effective refractive index is big enough, a new resonance mode will be produced, just as the case of $d_{\text{in}} = 100\,\text{nm}$ in Fig.5.

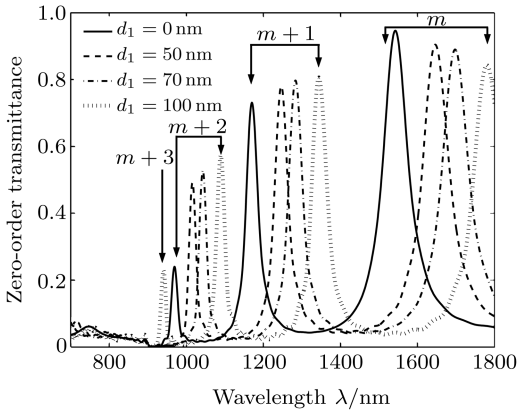


Fig.5. Transmission spectra for the filled slit structure. The relative permittivity of the filling medium is $\epsilon_2 = 1.4$ and the widths of the filling mediums are $d_{\text{in}} = 0$, $d_{\text{in}} = 50\,\text{nm}$, $d_{\text{in}} = 70\,\text{nm}$, and $d_{\text{in}} = 100\,\text{nm}$, respectively.

Then, the mediums with different permittivities are used to test this effect. The width of the filling medium is chosen as $d_{\text{in}} = 100\,\text{nm}$ and the values of relative permittivity are as follows: 1.00, 1.40, 1.77, 2.19, 2.34, and 2.66, respectively. The corresponding transmission spectra are shown in Fig.6. It can be clearly seen that the transmission peaks move towards long wavelengths and new transmission peaks appear obviously with ϵ_2 increasing. It can be also found that the spacing between two adjacent transmission peaks of high orders is smaller than that of lower orders. These phenomena can also be explained well by using the waveguide mode resonance theory. For the same order m, the increase of the effective refractive index will lead to the increasing of the resonance wavelength of the same order; when the increasing of n_{eff} is big enough, a new order occurs.

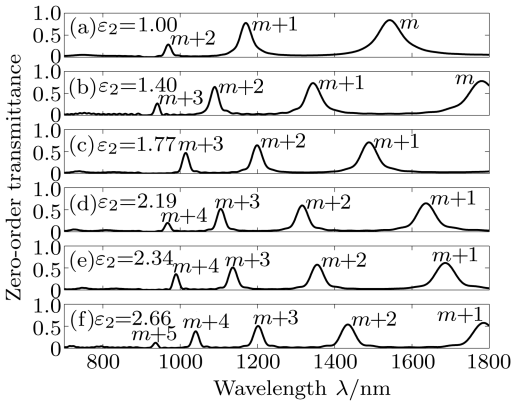


Fig.6. Transmission spectra for the filled slit structure. The width of the filled medium is $d_{\text{in}} = 100\,\text{nm}$ and the relative permittivities of the filling mediums are: 1.00, 1.40, 1.77, 2.19, 2.34, and 2.66, respectively.

Changing the width or refractive index of the inserted medium in the slit leads the transmission peaks to shift. The medium in the slit is related to the average effective refractive index which determines the positions of transmission peaks according to the waveguide resonance theory.

5. Effect of the film thickness

The dependence of the transmission spectrum on the thickness of film is also investigated. The transmission spectra for whole Ag films with different thicknesses are shown in Fig.7.

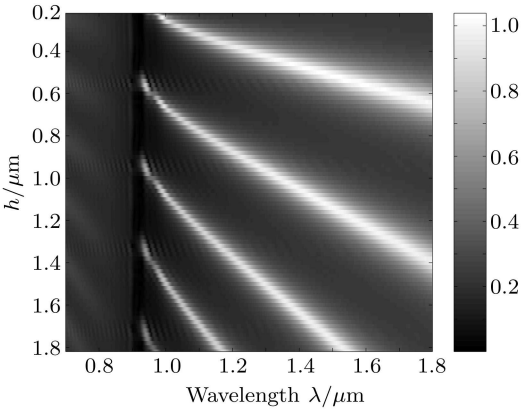
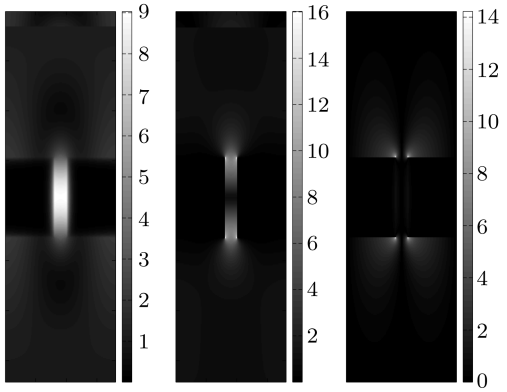


Fig.7. The dependence of transmission spectrum on the thickness of the whole silver film.

Other parameters are the same as those in the previous sections. The bright lines represent the high transmission. It can be seen that the highest transmission peak shifts away from the wavelength $\lambda = 900\,\text{nm}$ as the thickness increases. For a given wavelength, the difference in film thickness between two adjacent

transmission peaks is a constant. These phenomena accord well with the waveguide resonance condition.

In all the simulations, the zero-order transmittances are normalized by the period. These transmission peaks in value are greater than those in the case of the slit porosity of one period (100 nm/900 nm). This enhanced transmission is because of the surface plasmons which enhance the field intensity on both surface. For an Ag film of $h = 400$ nm with vacuum slits, a plane wave with $\lambda = 1284$ nm corresponding to the transmission peak is incident upon the structure, and the field amplitude distributions are shown in Fig.8.



(a) H_z (b) E_x (c) E_y

Fig.8. The amplitude distributions for the whole silver film with a thickness of $h = 400$ nm at transmission peak $\lambda = 1284$ nm. (a) H_z, (b) E_x, (c) E_y.

The magnetic fields are enhanced in the slit and form the first waveguide mode ($m = 1$) as shown in Fig.8(a). Figures 8(b) and 8(c) show that the electric fields at both ends of the slit are enhanced. This is due to the surface plasmon resonance. The cooperation of the waveguide resonance with the surface plasmons results in the energy transmission through the film.

6. Conclusion

Two structures have been modelled to investigate the transmission property of the subwavelength metallic slit arrays. For the sandwiched metallic structure, the transmission peaks can be shifted by inserting another kind of metal into the Ag film. This phenomenon reflects the effect of the metallic wall inside slits on the transmission spectra. According to the waveguide resonance theory, the average effective refraction changes when the middle metallic film is replaced, which leads the transmission peaks to shift. For the filled slit structure, the transmission peaks can also be adjusted by changing the width or the relative permittivity of the medium filled into slits. This changes the average refractive index inside the slits experienced by the propagating wave. These phenomena can be explained well by the waveguide resonance theory, and the transmission peaks are determined by the waveguide resonance condition. The two modes indicate that both the metallic walls and the mediums in the slits result in the average refractive index in the waveguide resonance condition.

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