Vol 16 No 5, May 2007
1009-1963/2007/16(05)/1239-07

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

The classification and analysis of dynamic networks*

Guo Jin-Li(郭进利)

Business School, University of Shanghai for Science and Technology, Shanghai 200093*, China*

(Received 12 April 2006; revised manuscript received 27 November 2006)

In this paper we, firstly, classify the complex networks in which the nodes are of the lifetime distribution. Secondly, in order to study complex networks in terms of queuing system and homogeneous Markov chain, we establish the relation between the complex networks and queuing system, providing a new way of studying complex networks. Thirdly, we prove that there exist stationary degree distributions of M–G–P network, and obtain the analytic expression of the distribution by means of Markov chain theory. We also obtain the average path length and clustering coefficient of the network. The results show that M–G–P network is not only scale-free but also of a small-world feature in proper conditions.

Keywords: complex network, lifetime of nodes, BA model, scale-free network, small-world network
PACC: 0540J, 8980H

1. Introduction

Recently, complex networks have become a very hot research topic among mathematical, physical and engineering fields.[1−10] In fact, as presented by Newman in Ref.[5], it may date back to 1965 when Price presented the original work on scale-free networks.[5,11,12] Price studied the citation network of scientific publications and found the degree distribution of citation network followed the power-law degree distribution. The main contribution of Price is that he generalized the idea of the 'rich-get-richer' phenomenon of Simon to the growth of networks.[5] It might be due to lack of the knowledge of network growth, such as the WWW (world-wide-web), but the work of Price is largely unknown in the scientific community, and cumulative advantage did not achieve currency as a model of network growth until its rediscovery some decades later by Barabási *et al*, who gave it the new name of 'preferential attachment'.[5] They proposed a network growth model of the Web that is very similar to Price's, but with one important difference. We recall the BA (Barabási and Albert) model below.

BA model

(1) *Growth*: Starting with a small number (m_0) of nodes, at every time step a new node is added with $m(\leq m_0)$ edges (that will be connected to the nodes already present in the system).

(2) *Preferential attachment*: When choosing the nodes to which the new node connects, the probability $\prod$ is assumed that a new node will be connected to node i depending on the degree k_i of node i, such that

$$\prod(k_i) = \frac{k_i}{\sum_j k_j}. \tag{1}$$

The study of BA model and its evolution form is focused on the analytic calculation of degree distributions. The main methods are 'mean-field approach',[2] 'rate-equation approach'[3] and the 'master-equation approach'.[6] In the viewpoint of analytic calculation, the BA model can well explain scale-free networks. The feature of the BA model is preferential attachment and growth, and the growth is never changed thereafter because it does not consider the lifetime distribution of nodes and it thinks that the lifetime of nodes is infinite. However, many networks in the real world have definite lifetimes. For instance, new companies being set up and old companies bankrupted every day are a supply of chain networks. Science collaboration network and movie actor collaboration network have lifetimes, so do the human sexual contact network and WWW network. Some new home pages join into the system and some old ones get out of the network, and so on. We wonder whether the degree distribution follows the power-law for a complex network with the lifetime of nodes.

Motivated by the feature of supply chains, in this paper we, firstly, classify the complex networks in which the nodes are of lifetime distribution. Sec-

*Project supported by the Shanghai Leading Academic Discipline Project, China (Grant No T0502) and by the Shanghai Municipal Education Commission Natural Science Foundation, China (Grant No 05EZ35).

ondly, in order to study complex networks in terms of queuing system and homogeneous Markov chain, we establish the relation between the complex networks and queuing system, thus, providing a new way of studying complex networks. Thirdly, we prove the stationary degree distribution of networks and obtain the analytic expression of the distribution and the average path length by means of Markov chain theory. The results show that the scale-free feature of complex networks not only depends on incoming nodes, but also on outgoing nodes after they are saturated.

2. Description and classification of the model

Based on the time interval of nodes arrival and the type of the lifetime distribution, we can classify the complex networks, as $M/M/\infty$, $M/G/\infty$, $G/M/\infty$ and $G/G/\infty$ in queuing theory.

2.1. M–M type network

The M–M type network is defined in two steps:
(i) *Lifetime of nodes:* the lifetimes of nodes are independent and identically distributed variables, and the lifetime distribution is the negative exponential distribution with parameter μ.
(ii) *Growth:* Starting with a small number (m_0) of nodes, each node has at least $m(\leq m_0)$ edges. The nodes arrival process is a homogeneous Poisson process with constant rate λ. At time t when a new node joins the network, it links to m different nodes already present in the system.

2.2. M–G type network

Assume that $B(t)$ is a continuous distribution function and

$$B(0+) = 0, \tag{2}$$

$$\frac{1}{\mu} = \int_0^\infty t \mathrm{d}B(t) < \infty. \tag{3}$$

Similarly, $A(t)$ is also a continuous distribution function and

$$A(0+) = 0, \tag{4}$$

$$\frac{1}{\lambda} = \int_0^\infty t \mathrm{d}A(t) < \infty. \tag{5}$$

The M–G type network is defined in two steps:
(i) *Lifetime of nodes:* the lifetimes of nodes are independent and identically distributed variables, and the lifetime distribution function is $B(t)$ satisfying Eqs.(2) and (3).
(ii) *Growth:* Starting with a small number (m_0) of nodes, each node has at least $m(\leq m_0)$ edges. The nodes arrival process is a homogeneous Poisson process with a constant rate λ. At time t when a new node joins the network, it links to m different nodes already present in the system.

2.3. G–M type network

The G–M type network is defined in two steps:
(i) *Lifetime of nodes:* the lifetimes of nodes are independent and identically distributed variables, and the lifetime distribution is the negative exponential distribution with parameter μ.
(ii) *Growth:* Starting with a small number (m_0) of nodes, each node has at least $m(\leq m_0)$ edges. The interarrival times between nodes are an independent and identically distributed variables, and the distribution function satisfies Eqs.(4) and (5). At time t when a new node joins the network, it links to m different nodes already present in the system.

2.4. G–G type network

The G–G type network is defined in two steps:
(i) *Lifetime of nodes:* the lifetimes of nodes are independent and identically distributed variables, and the lifetime distribution function is $B(t)$ satisfying Eqs.(2) and (3).
(ii) *Growth:* Starting with a small number (m_0) of nodes, each node has at least $m(\leq m_0)$ edges. The interarrival times between nodes is an independent and identically distributed variables, and the distribution function satisfies Eqs.(4) and (5). At time t when a new node joins the network, it links to m different nodes already present in the system.

2.5. M–G–P network

For the M–G type network, since the nodes arrival process is a homogeneous Poisson process at a constant rate λ. The lifetime of nodes is an independent and identically distributed variable, and the lifetime distribution function is $B(t)$ satisfying Eqs.(2) and (3). Once a node joins the system, we think that it accepts service. Meanwhile, nodes of stopping activities are thought to complete the service. Under such a situation, we can regard the M–G type network as a $M/G/\infty$ queuing system.

Denote

$$N(t) = \{\text{number of activity nodes at time } t\}, \quad N_2(t) = \{\text{number of stopping activity nodes at time } t\}.$$

We have

$$N(t) = \{\text{number of activity nodes at time } t\} = \{\text{number of accepting service nodes at time } t\},$$

where $N_2(t)$ is the output process of the system.

From $M/G/\infty$ queuing system theory,[13] at time t the average number of activity nodes in the network is

$$E[N(t)] = \lambda \int_0^t (1 - B(t))\mathrm{d}t.$$

As t goes to infinity, from $M/G/\infty$ queuing system theory,[14] we have

$$N = E[N(t)] \approx \frac{\lambda}{\mu}. \tag{6}$$

$N_1(t)$ stands for the input process of nodes, therefore, $N_1(t)$ is the Poisson process with parameter λ. By using $M/G/\infty$ queuing system theory,[13,14] after sufficiently long time, the limitation of output process $N_2(t)$ is also Poisson process with parameter λ. Next, we discuss the degree distribution of equilibration system after long time. Since output process $N_2(t)$ and input process $N_1(t)$ are Poisson process with parameter λ, that is, the rates of incoming and outgoing are λ. In such a situation, we can think the system is in saturation and equilibrium.

Feature of M–G type network: M–G type network is not only of input (growth), but also of output (see the following figure). When the network is in equilibrium after sufficiently long time, the average number of nodes is λ/μ, the scale of the network is about $N = \lambda/\mu$.

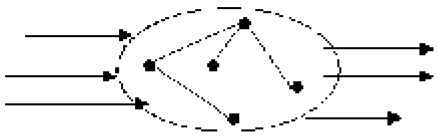


Fig.1. The input and the output process of the M–G type network.

According to Fig.1, the M-G type network is characterized by the following features.

(iii) *Preferential attachment*: When choosing the nodes to which the new node connects, the probability $\prod$ is assumed such that a new node will be connected to node i depending on the degree k_i of node i, satisfying Eq.(1);

(iv) After the system is saturated, the node stops activity if its lifetime is ended. At time t when a new node joins the network, the network deletes m_1 edges successively. Firstly, m_1 nodes are selected by the probability

$$\pi(k_i) = \frac{k_i + n}{\sum_j (k_j + n)}, \tag{7}$$

where $n \geq 0$ is called a saturated factor, and k_i is the degree of the selected node i. Secondly, the oldest edge (the edge linked to node i earliest) of node i is deleted. Then there are totally m_1 edges are deleted.

The network is called an M–G–P network (M–G preferential attachment network).

Remark 1 n in Eq.(7) can be regarded as a factor of saturation of the network.

3. Analysis of M–G–P network

3.1. The dynamic equilibration and degree distribution

We present an analysis of the M–G–P network below. As the M–G–P network grows, similarly to that in Ref.[3], $\sum_j k_j = 2E[N(t)]m$. As $t \to \infty$, we have $E[N(t)] \approx N = \lambda/\mu$, thus, $\sum_j k_j = 2Nm$. In the following, we discuss the degree distribution of the network after sufficiently long time. When a node is added to the network, the degree of node l increases from k to $k+1$ with probability

$$m \prod(k) = m \frac{k}{\sum_j k_j} = \frac{k}{2N}.$$

Moreover, by using item (iv) in Section 2, the degree of node l decreases from k to $k-1$ with probability

$$m_1 \pi(k) = m_1 \frac{k+n}{\sum_j (k_j + n)} = \frac{m_1(k+n)}{N(2m+n)}.$$

Otherwise, it remains unchanged.

Let

$$a = 2N, \quad h_k = \frac{2m_1}{2m+n}(k+n). \tag{8}$$

Then, the degree of node l increases from k to $k+1$ with probability k/a and decreases from k to $k-1$ with probability h_k/a. Otherwise it remains unchanged.

Let $K(t)$ denotes the degree of node l at time t in the network. Since $K(t)$ only changes at the renewal point of input process $N_1(t)$, without loss of generality, we assume that renewal points are $n = n_1$, n_1+1, $n_1+2, \ldots$. After sufficiently long time, the system is in saturation and equilibrium. Based on$N_1(t)$ and assumptions of the model, $\{K(n)|n = n_1, n_1+1, n_1+2, \ldots,\}$ is a homogeneous Markov chain with state space $\{m, m+1, \ldots, N\}$. The matrix of transition probabilities of the Markov chain is

$$P=\begin{bmatrix}
1-\frac{m}{a} & \frac{m}{a} & & & & & & & & \\
\frac{h_{m+1}}{a} & 1-\frac{h_{m+1}}{a}-\frac{m+1}{a} & \frac{m+1}{a} & & & & & & & \\
0 & \frac{h_{m+2}}{a} & 1-\frac{h_{m+2}}{a}-\frac{m+2}{a} & \frac{m+2}{a} & & & & & & \\
0 & 0 & \frac{h_{m+3}}{a} & 1-\frac{h_{m+3}}{a}-\frac{m+3}{a} & \frac{m+3}{a} & & & & & \\
0 & 0 & 0 & \frac{h_{m+4}}{a} & 1-\frac{h_{m+4}}{a}-\frac{m+4}{a} & \frac{m+4}{a} & & & & \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & \frac{h_{N-1}}{a} & 1-\frac{h_{N-1}}{a}-\frac{N-1}{a} & \frac{N-1}{a} \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \frac{h_N}{a} & 1-\frac{h_N}{a}
\end{bmatrix}.$$

Let $v = (v_m, v_{m+1}, v_{m+2}, \ldots, v_N)$ be a stationary distribution of the Markov chain $\{K(n)|n = n_1, n_1+1, n_1+2, \ldots,\}$. Form Ref.[15], there exists the stationary distribution of the Markov chain, and it is the solution of the following equation:

$$v = vP. \tag{9}$$

From Eq.(9), we have

$$\begin{cases}
v_m = \Big(1-\dfrac{m}{a}\Big)v_m + \dfrac{h_{m+1}}{a}v_{m+1}, \\
v_k = \dfrac{k-1}{a}v_{k-1} + \Big(1-\dfrac{h_k}{a}-\dfrac{k}{a}\Big)v_k + \dfrac{h_{k+1}}{a}v_{k+1}, \quad m+1 \le k \le N-1, \\
v_N = \dfrac{N-1}{a}v_{N-1} + \Big(1-\dfrac{h_N}{a}\Big)v_N.
\end{cases} \tag{10}$$

Solving Eq.(10) , we can obtain

$$v_k = \frac{k-1}{h_k}v_{k-1} = \frac{(k-1)!}{(m-1)!\prod_{i=m+1}^{k} h_i}v_m \quad m+1 \le k \le N-1.$$

Since $v = (v_m, v_{m+1}, v_{m+2}, ..., v_N)$ is a probability distribution, it implies

$$1 = \sum_{k=m}^{N} v_k = v_m + \sum_{k=m+1}^{N} \frac{(k-1)!}{(m-1)!\prod_{i=m+1}^{k} h_i}v_m = \Big(1 + \sum_{k=m+1}^{N} \frac{(k-1)!}{(m-1)!\prod_{i=m+1}^{k} h_i}\Big)v_m.$$

Thus,

$$v_m = \frac{1}{1+\sum\limits_{k=m+1}^{N} \dfrac{(k-1)!}{(m-1)!\prod_{i=m+1}^{k} h_i}} = \frac{1}{1+\sum\limits_{k=m+1}^{N} \dfrac{(k-1)!\Big(\dfrac{2m+n}{2m_1}\Big)^{k-m}}{(m-1)!\prod_{i=m+1}^{k}(i+n)}}, \tag{11}$$

$$v_k = \frac{(k-1)!}{(m-1)!\prod_{i=m+1}^{k} h_i\Big(1+\sum\limits_{k=m+1}^{N} \dfrac{(k-1)!}{(m-1)!\prod_{i=m+1}^{k} h_i}\Big)}$$

$$= \frac{(k-1)!\Big(\frac{2m+n}{2m_1}\Big)^{k-m}}{(m-1)!\prod_{i=m+1}^{k}(i+n)\Big(1+\sum_{k=m+1}^{N}\frac{(k-1)!\Big(\frac{2m+n}{2m_1}\Big)^{k-m}}{(m-1)!\prod_{i=m+1}^{k}(i+n)}\Big)}, \quad m+1 \le k \le N. \tag{12}$$

From Markov chain theory,[15] the stationary degree distribution of the network is

$$p(k) = \lim_{n\to\infty} P\{K(n)=k\} = v_k, \quad m \le k \le N-1.$$

If the saturated factor $n < 3$ is a positive integer, and $m_1 \ge m + n/2$, from Eqs.(8), (11) and (12), we immediately obtain the stationary degree distribution of the M–G–P network, expressed as follows:

$$p(k) = \frac{(k-1)!\Big(\frac{2m+n}{2m_1}\Big)^{k-m}}{(k+n)!\sum_{k=m}^{N}\frac{(k-1)!}{(k+n)!}\Big(\frac{2m+n}{2m_1}\Big)^{k-m}}, \quad k = m, m+1, \ldots, N. \tag{13}$$

If $n = 2$, $m_1 = m + 1$, when N is sufficiently large, by using Eq.(13), we can obtain

$$p(k) = \frac{(k-1)!}{(k+2)!\sum_{k=m}^{N}\frac{(k-1)!}{(k+2)!}} = \frac{1}{k(k+1)(k+2)\Big(\frac{1}{2m(m+1)} - \frac{1}{2(N+1)(N+2)}\Big)}$$

$$\approx \frac{2m(m+1)}{k(k+1)(k+2)}, \quad m \le k \le N.$$

Thus, if $n = 2$, $m_1 = m + 1$, N is sufficiently large , the stationary degree distribution of M–G–P network is

$$p(k) \approx \frac{2m(m+1)}{k(k+1)(k+2)}, \quad m \le k \le N. \tag{14}$$

Remark 2 Barabási and Mendes used the mean-field approach[1−3] to estimate the stationary degree distribution of BA model as follows:

$$p(k) = \frac{2m^2}{k^3}.$$

Using the master equation approach[6] Dorogovtsev and Mendes obtained the exact solution of the average degree distribution of BA model

$$p(k) = \frac{2m(m+1)}{k(k+1)(k+2)}. \tag{15}$$

This is the same as Eq.(14). Although Dorogovtsev and Mendes obtained Eq.(15), they did not prove the existence of the stationary average degree distribution.

If $n = 0$, $m_1 = m$, when N is sufficiently large, by using Eq.(13), we can obtain

$$p(k) = \frac{1}{k\sum_{k=m}^{N}\frac{1}{k}}, \quad m \le k \le N. \tag{16}$$

Remark 3 In Refs.[1–6], the stationary degree distribution of growing networks with preferential attachment is a power law distribution with exponent $\gamma \ge 2$. However, the exponent of HowNet network[16] is about 1. It shows that Eq.(16) is reasonable.

3.2. The average path length and clustering coefficient

In this section, we estimate the average path length and the clustering coefficient of the M–G–P network. Let

$$b = \frac{2m+n}{2m_1} < 1, \quad \kappa = -\frac{1}{\ln b}, \quad C = \frac{1}{\sum_{k=m}^{N}\frac{(k-1)!}{(k+n)!}b^{k-m}}.$$

Using Eq.(13), we have

$$p(k)=C\frac{\mathrm{e}^{-(k-m)/\kappa}(k-1)!}{(k+n)!}\approx Ck^{-(n+1)}\mathrm{e}^{-(k-m)/\kappa},\quad m+1\le k\le N. \tag{17}$$

From Eq.(61) in Ref.[3] and Eq.(17), we can obtain the average path length by using Eq.(63) in Ref.[3]

$$l=\frac{\ln N+\ln[\mathrm{Li}_{n+1}(\mathrm{e}^{-1/\kappa})/\mathrm{Li}_n(\mathrm{e}^{-1/\kappa})]}{\ln[\mathrm{Li}_{n-1}(\mathrm{e}^{-1/\kappa})/\mathrm{Li}_n(\mathrm{e}^{-1/\kappa})-1]}+1=\frac{\ln N+\ln\Big[\mathrm{Li}_{n+1}\Big(\frac{2m+n}{2m_1}\Big)\Big/\mathrm{Li}_n\Big(\frac{2m+n}{2m_1}\Big)\Big]}{\ln\Big[\mathrm{Li}_{n-1}\Big(\frac{2m+n}{2m_1}\Big)\Big/\mathrm{Li}_n\Big(\frac{2m+n}{2m_1}\Big)-1\Big]}+1, \tag{18}$$

where $\mathrm{Li}_n(x)$ is an nth polylogarithm of x.

The clustering coefficient $C_i^{[17]}$ of a single node i describes the density of connections in the neighbourhood of node i. It is given by the ratio of the number E_i of links between the nearest neighbours of i and the potential number of such links $E_{\max}=k_i(k_i-1)/2$, that is, $C_i=E_i/E_{\max}=2E_i/k_i(k_i-1)$. The clustering coefficient of the network is then the average of all individual C_i. Based on the above analysis of the M–G–P network, the output process $N_2(t)$ and the input process $N_1(t)$ are Poisson processes with parameter λ. From the assumptions of item (iii) and item (iv) of M–G–P network, the clustering character of M–G–P network emerges during the growth phase. Assuming that $m\gg 1$, and N is sufficiently large.

Assume that node i joins the system at time t_i, and at time t its degree is $k_i(t)$. By using continuum theory, $k_i(t)$ satisfies the dynamic equation

$$\frac{\partial k_i}{\partial t}=\lambda m\frac{k_i}{\sum_j k_j}.$$

Since the arrival process of nodes is the homogeneous Poisson process with parameter λ, at the growth phase, the average number of nodes is λt at time t. Hence,

$$\frac{\partial k_i}{\partial t}=\frac{k_i}{2t}.$$

The solution of the above equation is

$$k_i(t)=m\Big(\frac{t}{t_i}\Big)^{1/2}.$$

Similar to Ref.[18], the probability of node $j(>i)$ connected with node i is

$$p_{ij}=p(t_i,t_j)=\frac{m}{2}\frac{1}{\sqrt{t_it_j}}.$$

Without loss of generality, we assume that the arrival time is $t_1=1$ for the first node joining the network. Note that the network is undirected, therefore, we have

$$\begin{aligned}2E_l(t)&=\int_1^t p(t_l,t_i)\mathrm{d}t_i\int_1^t p(t_l,t_j)p(t_i,t_j)\mathrm{d}t_j\\&=\frac{m^3}{16t_l}\ln^2 t,\\C_l(t)&=\frac{2E_l}{k_l(t)(k_l(t)-1)}=\frac{m^2\ln^2 t}{8\sqrt{t}(m\sqrt{t}-\sqrt{t_l})}.\end{aligned}$$

Consequently, the clustering coefficient in the growth phase is expressed as follows:

$$\begin{aligned}C&=\frac{1}{\lambda t}\int_1^t C_l(t)\mathrm{d}t_l=\frac{1}{\lambda t}\int_1^t\frac{m^2\ln^2 t}{8\sqrt{t}(m\sqrt{t}-\sqrt{t_l})}\mathrm{d}t_l\\&=\frac{m^2\ln^2 t}{4\lambda t}\Big(\frac{1}{\sqrt{t}}+m\ln\Big(1+\frac{\sqrt{t}-1}{(m-1)\sqrt{t}}\Big)-1\Big)\\&\approx\frac{m^2\ln^2 t}{4\lambda t}\Big(\frac{1}{\sqrt{t}}+\frac{m}{m-1}\Big(1-\frac{1}{\sqrt{t}}\Big)-1\Big)\\&=\frac{m^2\ln^2 t}{4(m-1)\lambda t}\Big(1-\frac{1}{\sqrt{t}}\Big)\\&=\frac{m^2\ln^2(\lambda t/\lambda)}{4(m-1)\lambda t}\Big(1-\frac{\sqrt{\lambda}}{\sqrt{\lambda t}}\Big).\end{aligned} \tag{19}$$

After a sufficiently long time, the network is in equilibrium and the average number of nodes in the network changes from λt into N. From Eq.(19), we can obtain the approximate clustering coefficient for M–G–P network

$$\begin{aligned}C&\approx\frac{m^2\ln^2(N/\lambda)}{4(m-1)N}\Big(1-\frac{\sqrt{\lambda}}{\sqrt{N}}\Big)\\&=\frac{m^2\mu\ln^2\mu}{4(m-1)\lambda}(1-\sqrt{\mu}).\end{aligned} \tag{20}$$

From Ref.[18], the clustering coefficient of the BA model is

$$C=\frac{m^2\ln^2 t}{4(m-1)t}\Big(\ln\Big(1+\frac{1}{m}\Big)-\frac{1}{m+1}\Big)\underset{t\to\infty}{\to}0.$$

In other words , when time is sufficiently long, the clustering coefficient of the BA model goes to 0, that

is, the BA model is not of the small-world feature . This is not in agreement with the empirical results.[3,5] From Eq.(20) we realize that the clustering coefficient of M–G–P network is not equal to 0. The result shows that M–G–P network is not only a scale-free network but also of the small-world feature.

4. Discussion and conclusion

Although Barabási *et al* applied the continuum theory to the analysis of complex networks, their analysis about the BA model is defective,[19] due to their incorrect assumption that the distribution of the nodes arrival time is uniform.[1−3] However, from Ref.[20], we know that it is a Γ-distribution. For modifying the defectiveness, a continual-time growth model has been studied in Ref.[19]. From Remark 4.4 of Ref.[19], we know that, for any given integer $k > m$,

$$\lim_{t\to+\infty} P\{k_i(t) = k\} = 0, \quad i = 1, 2, ...,$$

where $k_i(t)$ is the degree of node i at time t in the BA model. Therefore, the stationary degree distribution of node i in the BA model is not power-law.

To sum up, the BA model can well describe the mechanism of forming the scale-free network. But it is not a small-world network. Many complex networks in real world are not only scale-free but also of small-world. On increasing and deleting the proportion of edges, the M–G–P network with sufficiently large number of nodes is not only a scale-free network but also of a small-world feature. The whole period of life completes through three phases, the growth phase, the dynamic equilibrium and the decay phase, and there may be different incoming parameter λ and lifetime parameter μ in different phases.

References

[1] Barabási A L and Albert R 1999 *Science* **286** 509
[2] Barabási A L, Albert R and Jeong H 1999 *Physica* A **272** 173
[3] Albert R and Barabási A L 2002 *Rev. Mod. Phys.* **74** 47
[4] Krapivsky P L and Redner S 2001 *Phys. Rev.* E **63** 066123
[5] Newman M E J 2003 *SIAM Review* **45** 167
[6] Dorogovtsev S N and Mendes J F F 2002 *Advances in Physics* **519** 107
[7] Li Y, Shan X M and Ren Y 2004 *Acta Phys. Sin.* **53** 3695 (in Chinese)
[8] Li Y, Liu Y, Shan X M Ren Y, Jiao J and Qiu B 2005 *Chin. Phys.* **14** 2153
[9] Guo J L 2006 *Acta Phys. Sin.* **55** 3916 (in Chinese)
[10] Guo J L 2006 *Systems Engineering—Theory & Practice* **26**(7) 33 (in Chinese)
[11] de Price D S 1965 *Science* **149** 510
[12] de Price D S 1976 *J. Amer. Soc. Inform. Sci.* **27** 292
[13] Ross S M 1983 *Stochastic Processes* (New York: Wiley)
[14] ≪Modern Mathematics Handbook≫ editor committee 2000 *Modern Mathematics Handbook* (Wuhan, China: The Publisher of Hua Zhong University of Science and Technology) (in Chinese)
[15] Wang Z K 1996 *Random Processes* (Beijing: Beijing Normal University Press) (in Chinese)
[16] Tang L, Zhang Y G and Fu X 2006 *Journal of Southeast University* (English Edition) **22**(3) 353
[17] Watts D J and Strogatz S H 1998 *Nature* **393** 440
[18] Fronczak A, Fronczak P and Holyst J A 2003 *Phys. Rev.* E **68** 046126
[19] Guo J L and Bai Y Q 2006 *DCDIS* Ser. B **13**(3) 523
[20] Guo J L and Wang C P 2006 *Impulsive Dynamical Systems and Applications* **4** 970