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Entanglement concentration and teleportation of multipartite entangled states in an ion trap*

Pan Chang-Ning(潘长宁) and Fang Mao-Fa(方卯发)†

College of Physics and Information Science, Hunan Normal University, Changsha 410081, *China*

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We propose an effective scheme for the entanglement concentration of a four-particle state via entanglement swapping in an ion trap. Taking the maximally entangled state after concentration as a quantum channel, we can faithfully and determinatively teleport quantum entangled states from Alice to Bob without the joint Bell-state measurement. In the process of constructing the quantum channel, we adopt entanglement swapping to avoid the decrease of entanglement during the distribution of particles. Thus our scheme provides a new prospect for quantum teleportation over a longer distance. Furthermore, the success probability of our scheme is 1.0.

Keywords: ion trap, concentration, teleportation
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Quantum entanglement[1] has many important applications in quantum information processing,[2−7] such as quantum teleportation, quantum secret key distribution, etc. Most of the applications of entanglement work perfectly only with the pure maximally entangled states. In practice, after the entanglement generation, it is impossible that one quantum system can be isolated from the environment absolutely. The entanglement of particles will inevitably decrease during the distribution process due to the coherence of these microscopic states being easily spoiled by the environment. So the quantum entangled states available practically are all non-maximally entangled states or the more general mixed states. Therefore, some schemes of entanglement distillation[8] or purification[9] (to convert the mixed entangled states into pure maximally entangled states) and entanglement concentration[10] (to convert the pure non-maximally entangled state into maximally entangled state) have been proposed to overcome this difficulty. And the concentration and purification of entangled photon states have been demonstrated experimentally.[8,11]

On the other hand, quantum teleportation, first proposed by Bennett *et al*,[2] has received great attention and a number of protocols have been suggested for its implementation in different ways. In original contributions,[1] the joint Bell-state measurement (BSM) is a key step in quantum teleportation. However, the joint BSM is very complex for experimental realization,[12] and thus becomes a bottle-neck for further development. Various schemes for teleportation have been proposed, which are different from the original protocol in that they do not require the Bell-state measurement. In Ref.[13] Vaidman considered both a spin state and a system with continuous variables, and presented a 'cross measurement' method, thus developing a two-way teleportation. In Ref.[14] de Almeida *et al* have teleported a superposition of coherent states without Bell-state measurement, with 50% of successful probability. Soon after that, Ye and Guo[3] treated the teleportation of an unknown atomic state in cavity quantum electrodynamics (QED); the successful probability of the scheme is 50% also. In Ref.[15] a proposal for teleporting an entanglement of zero- and one-photon states was presented with 100% of successful probability and 100% of fidelity. The entangled state of photons is relatively easy to create and teleport, but it is difficult to store for the future usage. Previously, Riebe *et al*[16] and Barrett *et al*[12] have implemented the quantum teleportation of atomic qubits with an average fidelity of 75% by using Ca^+ ions in a linear Paul trap and with an average fidelity of 78% using Be^+ ions in a segmented ion trap, respectively. The two schemes have not only realized the teleportation with atoms first in

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†Corresponding author. E-mail: mffang@hunnu.edu.cn

experiment, but also demonstrated the techniques of manipulating and transforming quantum states in an ion-trap system. Also as indicated in Refs.[3,17–19] the schemes for quantum teleportation are applicable probably only at a short distance. In these references, after maximally entangled states are generated, the entangled atoms are directly distributed to Alice (the sender) and Bob (the acceptor), who should be separated from each other by a large distance. During the distribution process, the entanglement of the atoms will decrease. The longer the distance, the larger the decrease. In the present study, in order to realize a long distance teleportation of an unknown ionic entangled internal state, we propose a feasible scheme in which both entanglement concentration via entanglement swapping[10,20,21] and quantum teleportation can be realized in an ion trap. Our scheme possesses some distinct advantages. (i) The teleportation can be successfully realized only by measuring ions in the trap but without the joint BSM. (ii) We adopt the trapped ions system in the constructing of the quantum channel, which is insensitive to the heating coming from a vibrational mode, So the coherent time with the quantum states is longer, thereby we can conveniently carry out the quantum information processing. (iii) The probability of successful teleportation in our scheme is 1.0. In addition, the quantum states that we concentrate and teleport are the entanglement states of ions which can be easily stored for the future usage, which has a potential significance for practical applications in quantum information processes.

In what follows, we consider a collection of four identical two-level ions confined in a linear trap. The corresponding Hamiltonian in an interaction picture can be described as[22,23]

$$H_{\rm int} = \frac{\Omega}{2}\sum_{j\neq k,j,k=1}^{4}\sum_{m=1}^{2}(\sigma_j\sigma_k\exp\{-{\rm i}[\eta(a^2+a^{+2}) - \omega_{\rm L}^m t]\} + {\rm H.c.}), \quad (1)$$

where Ω is the usual Rabi frequency of laser–ion interaction, η is the Lamb–Dicke parameter and $\omega_{\rm L}^m$ is the frequency of the mth laser beam. $\sigma_{+i} = |e_i\rangle\langle g_i|$, $\sigma_i = |g_i\rangle\langle e_i|$, with $|e_i\rangle$ and $|g_i\rangle$ $(i = 1,2,3,4)$ being the excited and ground states of the ith ion respectively, a^+ and a are the creation and annihilation operators of the vibrational mode respectively. We consider an ion trap operating in the Lamb–Dicke limit, i.e. $\eta_i\sqrt{n+1} \ll 1$. The effective Rabi frequency $\widetilde{\Omega}$ can be determined in the second-order perturbation theory, $\widetilde{\Omega} = (2n+1)[(\Omega\eta)^2/(2\nu-\delta)]$,[23] with δ being the detuning, and ν being the trap frequency. Then the evolution of the four-particle system for different initial states can be given by

$$\begin{aligned}
|egeg\rangle &\to \cos\Big(\frac{\widetilde{\Omega}t}{2}\Big)|egeg\rangle - {\rm i}\sin\Big(\frac{\widetilde{\Omega}t}{2}\Big)|gege\rangle,\\
|gege\rangle &\to \cos\Big(\frac{\widetilde{\Omega}t}{2}\Big)|gege\rangle - {\rm i}\sin\Big(\frac{\widetilde{\Omega}t}{2}\Big)|egeg\rangle,\\
|egge\rangle &\to \cos\Big(\frac{\widetilde{\Omega}t}{2}\Big)|egge\rangle - {\rm i}\sin\Big(\frac{\widetilde{\Omega}t}{2}\Big)|geeg\rangle,\\
|geeg\rangle &\to \cos\Big(\frac{\widetilde{\Omega}t}{2}\Big)|geeg\rangle - {\rm i}\sin\Big(\frac{\widetilde{\Omega}t}{2}\Big)|egge\rangle. \quad (2)
\end{aligned}$$

Now suppose there are three spatially separated users: Alice, Bob and Charlie. Charlie is in the middle of the distance between Alice and Bob. They have shared the four-particle non-maximally entangled states as follows:

$$|\phi\rangle_{1234} = a|egeg\rangle_{1234} + b|gege\rangle_{1234}, \quad (3)$$

$$|\phi\rangle_{5678} = c|egeg\rangle_{5678} + d|gege\rangle_{5678}, \quad (4)$$

where a, b, c and d are the normalization coefficients, ions 1 and 2 belong to Alice and ions 7 and 8 belong to Bob. Charlie possesses ions 3, 4, 5 and 6. To realize the entanglement swapping, Charlie confines the ions 3, 4, 5 and 6 into an ion trap. Before the entanglement swapping, the state of ions 1, 2, 3, 4 and 5, 6, 7, 8 is

$$\begin{aligned}
|\phi\rangle_{1.2\cdots8} = {} & ac|egeg\rangle_{3456}|egeg\rangle_{1278}\\
& + ad|egge\rangle_{3456}|egge\rangle_{1278}\\
& + bc|geeg\rangle_{3456}|geeg\rangle_{1278}\\
& + bd|gege\rangle_{3456}|gege\rangle_{1278}. \quad (5)
\end{aligned}$$

After an interaction time t, the whole state evolves according to Eq.(2) into the form

$$\begin{aligned}
|\phi\rangle'_{1.2\cdots8} = {} & |egeg\rangle_{3456}\Big[ac\cos\Big(\frac{\widetilde{\Omega}t}{2}\Big)|egeg\rangle_{1278}\\
& - {\rm i}bd\sin\Big(\frac{\widetilde{\Omega}t}{2}\Big)|gege\rangle_{1278}\Big]\\
& + |gege\rangle_{3456}\Big[-{\rm i}ac\sin\Big(\frac{\widetilde{\Omega}t}{2}\Big)|egeg\rangle_{1278}\\
& + bd\cos\Big(\frac{\widetilde{\Omega}t}{2}\Big)|gege\rangle_{1278}\Big]\\
& + |egge\rangle_{3456}\Big[ad\cos\Big(\frac{\widetilde{\Omega}t}{2}\Big)|egge\rangle_{1278}\\
& - {\rm i}bc\sin\Big(\frac{\widetilde{\Omega}t}{2}\Big)|geeg\rangle_{1278}\Big]\\
& + |geeg\rangle_{3456}\Big[-{\rm i}ad\sin\Big(\frac{\widetilde{\Omega}t}{2}\Big)|egge\rangle_{1278}
\end{aligned}$$

$$+ bc\cos\Big(\frac{\widetilde{\Omega}t}{2}\Big)|geeg\rangle_{1278}\Big]. \quad (6)$$

With the choice of $\widetilde{\Omega}t = \pi/2$, Charlie detects the ions 3, 4, 5 and 6, if ions 3, 4, 5 and 6 are in the state $|egeg\rangle_{3456}$ or $|gege\rangle_{3456}$, then the system state vector collapses into the following state:

$$|\phi\rangle_{1278} = \frac{ac|egeg\rangle_{1278} - \mathrm{i}bd|gege\rangle_{1278}}{\sqrt{2}}, \quad (7)$$

or

$$|\phi\rangle'_{1278} = \frac{-\mathrm{i}ac|egeg\rangle_{1278} + bd|gege\rangle_{1278}}{\sqrt{2}}. \quad (8)$$

Suppose that we choose the initial states $|\phi\rangle_{1234}$ and $|\phi\rangle_{5678}$ satisfying $a = d$ and $b = c$, i.e. the ions 1, 2, 3, 4 and 5, 6, 7, 8 being initially in the same state. Thus ions $1, 2$ and $7, 8$ become a maximally entanglement state of four-particle, that is the so-called entanglement concentration via entanglement swapping,[20,21] through corresponding rotation operation

$$|\phi\rangle''_{1278} = ab\frac{|egeg\rangle_{1278} + |gege\rangle_{1278}}{\sqrt{2}}. \quad (9)$$

If the measurement result is the state $|egge\rangle_{3456}$ or $|geeg\rangle_{3456}$, through calculation, the degree of entanglement of the ion pairs (1, 2) and (7, 8) becomes smaller than the initial one. Therefore the successful probability is $P = 2|ab|^2$. For the condition $a = c$, and $b = d$, if there exists a small error, i.e. $a = d + kd$, (k being a small constant) for the above case, with $d = 0.7$ and $k = 0.1$ chosen in the calculation, the fidelity of the obtained maximally entangled state of the ions 1, 2 and 7, 8 can reach 0.989.[21] Evidently, the small error only result in slight effect on the fidelity. Now particles 1, 2 and particles 7, 8 are in the maximally entangled state as shown in Eq.(9). Taking it as a quantum channel, an entangled state can be teleported from Alice to Bob. Next, we explicitly present the protocol of teleportation. The ions 9 and 10 to be teleported are assumed to be initially in the state

$$|\varphi\rangle_{9,10} = \alpha|eg\rangle_{9,10} + \beta|ge\rangle_{9,10}, \quad (10)$$

where α and β are unknown coefficients. Now Alice wants to teleport this state to Bob. Similarly, Alice sends ions 1, 2, 9 and 10 simultaneously into another ion trap described by Eq.(1). After an interaction time T, the state of the ions 1, 2, 9 and 10 evolves according to Eq.(2) and can be expressed as

$$\begin{aligned}|\Psi\rangle_{1278910} = {} & \frac{\alpha}{\sqrt{2}}|eg\rangle_{78}\Big[\cos\Big(\frac{\widetilde{\Omega}T}{2}\Big)|egeg\rangle_{12910} \\ & - \mathrm{i}\sin\Big(\frac{\widetilde{\Omega}T}{2}\Big)|gege\rangle_{12910}\Big] \\ & + \frac{\beta}{\sqrt{2}}|eg\rangle_{78}\Big[\cos\Big(\frac{\widetilde{\Omega}T}{2}\Big)|geeg\rangle_{12910} \\ & - \mathrm{i}\sin\Big(\frac{\widetilde{\Omega}T}{2}\Big)|egge\rangle_{12910}\Big] \\ & + \frac{\alpha}{\sqrt{2}}|ge\rangle_{78}\Big[\cos\Big(\frac{\widetilde{\Omega}T}{2}\Big)|egge\rangle_{12910} \\ & - \mathrm{i}\sin\Big(\frac{\widetilde{\Omega}T}{2}\Big)|geeg\rangle_{12910}\Big] \\ & + \frac{\beta}{\sqrt{2}}|ge\rangle_{78}\Big[\cos\Big(\frac{\widetilde{\Omega}T}{2}\Big)|gege\rangle_{12910} \\ & - \mathrm{i}\sin\Big(\frac{\widetilde{\Omega}T}{2}\Big)|egeg\rangle_{12910}\Big]. \end{aligned} \quad (11)$$

Choose $\widetilde{\Omega}T = \pi/2$, then Eq.(11) will become

$$\begin{aligned}|\Psi\rangle'_{1278910} = {} & N[(\alpha|eg\rangle_{78} - \mathrm{i}\beta|ge\rangle_{78})|egeg\rangle_{12910} \\ & + (-\mathrm{i}\alpha|eg\rangle_{78} + \beta|ge\rangle_{78})|gege\rangle_{12910} \\ & + (-\mathrm{i}\alpha|ge\rangle_{78} + \beta|eg\rangle_{78})|geeg\rangle_{12910} \\ & + (\alpha|ge\rangle_{78} - \mathrm{i}\beta|eg\rangle_{78})|egge\rangle_{12910}], \end{aligned} \quad (12)$$

where N is a normalization factor. Then Alice will detect the ions 1, 2, 9 and 10. If the result is $|egeg\rangle_{12910}$, the state of the ions 7 and 8 will collapse into the following state:

$$|\Psi\rangle_{78} = N[\Big(\alpha|eg\rangle_{78} - \mathrm{i}\beta|ge\rangle_{78})]. \quad (13)$$

After he receives Alice's measurement result, Bob only needs to make correlative rotations R_8 on the 8th ion,

$$R = \begin{bmatrix} \mathrm{i} & 0 \\ 0 & 1 \end{bmatrix} \quad (14)$$

to reconstruct the initial entangled state of ions 9 and 10, i.e.

$$|\varphi\rangle_{7,8} = \alpha|eg\rangle_{7,8} + \beta|ge\rangle_{7,8}, \quad (15)$$

and the probability of successful teleportation is 1/4. Similarly, if the measurement results are $|gege\rangle_{12910}, |geeg\rangle_{12910}$ or $|egge\rangle_{12910}$, with this information Bob can faithfully recover the original state through a corresponding unitary operation according to Table 1 and can obtain the same probability of teleportation, i.e. 1/4. That is to say, the total successful probability of teleportation is 1.0.

Here we briefly discuss the experimental feasibility of the proposed scheme. If the detuning from the sideband is very large during the interaction between trapped ions and bichromatic light, the heating coming from a vibrational mode can be eliminated; only the radiative time of ions needs considering in

this case. For the Rydberg ions with principal numbers of 50 and 51 respectively, the radiative time is $T_{\rm r} = 3 \times 10^{-2}$ s. For the interaction between trapped ions and the laser field, no particular assumption is required for the experimental parameters. For example, we choose $\delta = 0.90\upsilon$, $\eta = 0.10$, $\Omega = 0.10\upsilon$ and the vibrational frequency $\nu = 4.0\pi \times 10^5$Hz, then the interaction time is 5×10^{-3}s, which is much shorter than the atomic radiative time $T_{\rm r}$, so our scheme is realizable.

Table 1. The unitary transformations that Bob must perform on his qubits, are conditional on Alice's measurement result, to complete the teleportation protocol. Here R_i is the rotation operation as indicated in Eq.(14) on particle i, and σ_i^x is a Pauli operation on particle i.

Alice's result	$\|\phi\rangle_{78}$	Bob's operation
$\|egeg\rangle_{12910}$	$\alpha\|eg\rangle_{78} - {\rm i}\beta\|ge\rangle_{78}$	R_8
$\|gege\rangle_{12910}$	$-{\rm i}\alpha\|eg\rangle_{78} + \beta\|ge\rangle_{78}$	R_7
$\|geeg\rangle_{12910}$	$-{\rm i}\alpha\|ge\rangle_{78} + \beta\|eg\rangle_{78}$	$R_7\sigma_8^x\sigma_7^x$
$\|egge\rangle_{12910}$	$\alpha\|ge\rangle_{78} - {\rm i}\beta\|eg\rangle_{78}$	$R_8\sigma_8^x\sigma_7^x$

In conclusion, we have proposed a protocol to concentrate four-particle states by entanglement swapping with trapped ions, and a scheme for teleporting the bipartite unknown ionic entangled states from Alice to Bob by using maximally entanglement after concentration, where the BSM is not needed. The distinct advantages in our scheme are that our scheme is not only realizable for the teleportation of unknown ionic entangled states in a longer distance, but also insensitive to heating coming from a vibrational mode and the successful probability for the teleportation can reach 1.0. The created maximally entangled states can be more easily stored for future usage than the entangled photons, which is another advantage. With the development of the ionic technology, our scheme has a potential significance for the practical applications in the future.

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