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Generation of multi-atom cluster states via an unconventional geometric phase shift in cavity QED

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This paper proposes two schemes to generate the multi-atom cluster states. The first scheme is based on the interaction of atoms with a highly detuned cavity mode and a classical field, the second scheme is based on the interaction of atoms with a cavity mode, strongly driven by a resonant classical field.

Keywords: cluster state, three-level atom
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1. Introduction

Multiparticle entangled state, as an important resource in achieving quantum information processing, such as quantum cryptography,[1−3] quantum dense coding,[4−6] and quantum teleportation,[7−11] plays a fundamental role in testing quantum nonlocality.[12−14] It has been shown that there is only one class of entanglement[15] for two qubits, and there are two inequivalent classes of tripartite entangled states, i.e., the Greenberger–Horne–Zeilinger (GHZ) class[14] and the W class.[16] Recently, a particular multiqubit entangled state, called cluster state, was applied to the one-way quantum computation by Raussendorf and Briegel.[17] The cluster states have a high persistency of entanglement and can be regarded as an entanglement source for the GHZ states,[18] but are more immune to decoherence than GHZ states.[19] Therefore, a number of applications of cluster states to quantum computation have been proposed.[20−24] Recently, Zheng[25] proposed two schemes to generate the cluster states in ion-trap systems. Zou and Mathis proposed two schemes to generate the cluster states in resonant microwave cavity,[26] and in linear optics system.[27] Dong *et al*[28] proposed two schemes to generate the cluster states based on atom–cavity resonant interactions and atomic ensembles respectively. They also discussed the case of cavity decay and atomic spontaneous emission.

The original scheme[29] for generation of the geometric phase gate via the strongly driven Jaynes–Cummings interaction and the scheme[30] for generation of the entangled states for multi-level atoms via the same model, these two schemes were proposed by Zheng. Then Zheng[31] and Chen *et al*[32] proposed the schemes to realize the unconventional geometric phase gate. Inspired from the scheme proposed in Ref.[31] we propose the first scheme to generate the multi-atom cluster states via an unconventional geometric phase shift in cavity QED, which is based on the large detuning interaction of the atoms with the cavity mode. Our second scheme is inspired from the schemes proposed in Refs.[29,30,32], which is based on the resonant coupling of the atoms to the classical field. In our schemes the cavity mode is displaced along a closed path depending on the atomic states. The system of the atomic states acquires a conditional phase, producing the multi-atom cluster states.

The structure of our paper is as follows. In Section 2, we briefly review the unconventional geometric phase shift caused by the displacement operator along a closed path in phase space. In Sections 3 and 4, two schemes for generating the multi-atom cluster states are introduced respectively. The conclusions are given in Section 5.

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2. Unconventional geometric phase

We first review the definition of the standard unconventional geometric phase shift due to displacement along an arbitrary path.[31,33,34] The displacement operator is expressed as

$$D(\alpha) = \mathrm{e}^{\alpha a^\dagger - \alpha^* a}, \tag{1}$$

where $a^\dagger$ and a are the creation and annihilation operators of the harmonic oscillator respectively. They satisfy the following relationship:

$$D(\alpha_2)D(\alpha_1) = D(\alpha_2+\alpha_1)\mathrm{e}^{\mathrm{i}\mathrm{Im}(\alpha_2\alpha_1^*)}. \tag{2}$$

Consider a path consisting of N short straight sections $\Delta\alpha_j$, the total operator is

$$\begin{aligned} D_t &= D(\Delta\alpha_N)\cdots D(\Delta\alpha_1) \\ &= D\Big(\sum_{j=1}^{N}\Delta\alpha_j\Big)\exp\Big[\mathrm{i}\mathrm{Im}\Big(\sum_{j=2}^{N}\Delta\alpha_j\sum_{k=1}^{j-1}\Delta\alpha_k^*\Big)\Big]. \end{aligned} \tag{3}$$

When $N\to\infty$, an arbitrary path γ can be reached. Then

$$D_t = D\Big(\int_\gamma \mathrm{d}\alpha\Big)\mathrm{e}^{\mathrm{i}\,\Theta}, \tag{4}$$

where

$$\Theta = \mathrm{Im}\Big(\int_\gamma \alpha^*\mathrm{d}\alpha\Big). \tag{5}$$

For a closed path

$$D_t = D(0)\mathrm{e}^{\mathrm{i}\,\Theta} = \mathrm{e}^{\mathrm{i}\,\Theta}, \tag{6}$$

where the phase operator Θ is determined by the area of a loop in the phase space and independent of the quantized state of the harmonic oscillator.

3. Dynamic model of a cavity QED system assisted by a classical field

Consider two identical three-level atoms, each one has one excited state $|r\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The transition $|e\rangle\to|r\rangle$ is driven by the cavity mode with coupling constant g. The state $|g\rangle$ is not influenced during the interaction. The Hamiltonian is expressed as (assuming $\hbar=1$)

$$\begin{aligned} H = {} & \omega_0\sum_{j=1,2}S_{z,j} + \omega_a a^\dagger a + \sum_{j=1,2}[g(a^\dagger S_j^- + aS_j^+) \\ & + \Omega(\mathrm{e}^{-\mathrm{i}\omega t}S_j^+ + \mathrm{e}^{\mathrm{i}\omega t}S_j^-)], \end{aligned} \tag{7}$$

where $S_{z,j} = \frac{1}{2}(|r_j\rangle\langle r_j| - |e_j\rangle\langle e_j|)$, $S_j^+ = |r_j\rangle\langle e_j|$, and $S_j^- = |e_j\rangle\langle r_j|$, and $a^\dagger$ and a are the creation and annihilation operators for the cavity mode. ω_0, ω_a, ω and Ω are the $|e\rangle\to|r\rangle$ resonant transition frequency, cavity mode frequency, classical field frequency, and Rabi frequency respectively. In the rotating frame at the cavity frequency ω_a, the Hamiltonian becomes

$$\begin{aligned} H_{\mathrm{I}} = {} & \sum_{j=1,2}[\Delta S_{z,j} + g(a^\dagger S_j^- + aS_j^+) \\ & + \Omega(\mathrm{e}^{-\mathrm{i}\delta t}S_j^+ + \mathrm{e}^{\mathrm{i}\delta t}S_j^-)], \end{aligned} \tag{8}$$

where $\Delta = \omega_0 - \omega_a$, $\delta = \omega - \omega_a$. In the case of $\Delta \gg \Omega, g$, the atoms cannot exchange energy with the field. Then the Hamiltonian of Eq.(8) can be replaced by the effective Hamiltonian[31]

$$\begin{aligned} H_{\mathrm{eff}} = {} & \sum_{j=1,2}\frac{1}{\Delta}[(g^2a^\dagger a + \Omega^2 + \Omega g a\mathrm{e}^{\mathrm{i}\delta t} \\ & + \Omega g a^\dagger \mathrm{e}^{-\mathrm{i}\delta t})(|r_j\rangle\langle r_j| - |e_j\rangle\langle e_j|) \\ & + g^2|r_j\rangle\langle r_j|] + \frac{g^2}{\Delta}(S_1^+S_2^- + S_1^-S_2^+), \end{aligned} \tag{9}$$

where the term

$$\sum_{j=1,2}\frac{1}{\Delta}(g^2a^\dagger a + \Omega^2)(|r_j\rangle\langle r_j| - |e_j\rangle\langle e_j|)$$

describes the Stark shifts caused by the photons of the cavity field and classical field respectively. The term $\sum_{j=1,2}\frac{1}{\Delta}(\Omega g a\mathrm{e}^{\mathrm{i}\delta t} + \Omega g a^\dagger\mathrm{e}^{-\mathrm{i}\delta t})(|r_j\rangle\langle r_j| - |e_j\rangle\langle e_j|)$ describes the coupling between the atoms and cavity mode assisted by the classical field and $\sum_{j=1,2}\frac{g^2}{\Delta}|r_j\rangle\langle r_j|$ describes the Stark shift induced by the vacuum field. The term $\frac{g^2}{\Delta}(S_1^+S_2^- + S_1^-S_2^+)$ describes the dipole coupling between the two atoms caused by the cavity mode.

The time evolution of this system is described by the Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H_{\mathrm{eff}}|\psi(t)\rangle. \tag{10}$$

After performing the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\,H_0 t}|\psi(t')\rangle \tag{11}$$

with

$$\begin{aligned} H_0 = {} & \sum_{j=1,2}\frac{1}{\Delta}[(g^2a^\dagger a + \Omega^2)(|r_j\rangle\langle r_j| \\ & - |e_j\rangle\langle e_j|) + g^2|r_j\rangle\langle r_j|], \end{aligned} \tag{12}$$

we obtain

$$\mathrm{i}\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t} = H_{\mathrm{i}}|\psi'(t)\rangle, \tag{13}$$

where

$$H_{\rm i}=\sum_{j=1,2}\frac{\Omega g}{\Delta}\big\{\big[a{\rm e}^{{\rm i}(\delta-g^2/\Delta)t}+a^{\dagger}{\rm e}^{-{\rm i}(\delta-g^2/\Delta)t}|r_j\rangle\langle r_j|\big] - \big[a{\rm e}^{{\rm i}(\delta+g^2/\Delta)t}+a^{\dagger}{\rm e}^{-{\rm i}(\delta+g^2/\Delta)t}|e_j\rangle\langle e_j|\big]\big\} + \frac{g^2}{\Delta}(S_1^+S_2^-+S_1^-S_2^+). \quad (14)$$

After the infinitesimal interval $[t,t+{\rm d}t]$, the evolution operator of the states in our qubit subspace is

$$U({\rm d}t)=D({\rm d}\alpha)={\rm e}^{a^{\dagger}{\rm d}\alpha-a{\rm d}\alpha^*}={\rm e}^{-{\rm i}H_{\rm i}{\rm d}t}, \quad (15)$$

where

$${\rm d}\alpha={\rm i}\frac{\Omega g}{\Delta}{\rm e}^{-{\rm i}(\delta+g^2/\Delta)t}{\rm d}t.$$

During an interaction time t', the displacement parameter α can be expressed as

$$\alpha(t')={\rm i}\int_0^{t'}\frac{\Omega g}{\Delta}{\rm e}^{-{\rm i}(\delta+g^2/\Delta)t}{\rm d}t =-\frac{\Omega g}{\Delta\delta+g^2}[{\rm e}^{-{\rm i}(\delta+g^2/\Delta)t'}-1], \quad (16)$$

and the geometric phase shift Θ is

$$\Theta={\rm Im}\Big(\int_\gamma\alpha^*{\rm d}\alpha\Big) ={\rm Im}\Big(\int_0^t{\rm i}\frac{(\Omega g)^2}{\Delta(\Delta\delta+g^2)}(1-{\rm e}^{-{\rm i}(\delta+g^2/\Delta)t'}){\rm d}t'\Big) =-\frac{(\Omega g)^2}{\Delta(\Delta\delta+g^2)}\Big[t-\frac{\Delta}{\Delta\delta+g^2}\sin\Big(\frac{g^2+\delta\Delta}{\Delta}t\Big)\Big] \quad (17)$$

and

$$\Theta_\xi={\rm Im}\Big(\int_\gamma\xi\alpha^*{\rm d}\xi\alpha\Big)=\xi^2\Theta. \quad (18)$$

In order to generate a multi-atom cluster state, we assume that each atom and the cavity mode initially are in the state $(|g_j\rangle+|e_j\rangle)/\sqrt{2}(j=1,2,...,N)$ and the vacuum state $|0\rangle$ respectively. Thus the initial state of the atomic system is

$$|\phi\rangle_{12\cdots N}=\frac{1}{2^{N/2}}(|g_1\rangle+|e_1\rangle)\otimes_{j=2}^{N}(|g_N\rangle+|e_N\rangle). \quad (19)$$

Under the condition $(\delta+g^2/\Delta)t=2\pi$, the displacement is along a closed path, returning to the original point in phase space and acquiring a geometric phase conditional upon the electronic states. Therefore, after the interaction time t by $H_{\rm eff}$, the system evolves to

$$|\phi\rangle_{12\cdots N}=\frac{1}{2^{N/2}}({\rm e}^{{\rm i}(4\Theta+2\Omega^2t/\Delta)}|e_1\rangle|e_2\rangle + {\rm e}^{{\rm i}(\Theta+\Omega^2t/\Delta)}|e_1\rangle|g_2\rangle + {\rm e}^{{\rm i}(\Theta+\Omega^2t/\Delta)}|g_1\rangle|e_2\rangle+|g_1\rangle|g_2\rangle) \otimes_{j=3}^{N}(|g_N\rangle+|e_N\rangle). \quad (20)$$

Performing the one-qubit operation

$$|e_j\rangle\to{\rm e}^{-{\rm i}(\Theta+\Omega^2t/\Delta)}|e_j\rangle\quad(j=1,2),$$

the state of the atomic system becomes

$$|\phi\rangle_{12\cdots N}=\frac{1}{2^{N/2}}({\rm e}^{{\rm i}2\Theta}|e_1\rangle|e_2\rangle+|e_1\rangle|g_2\rangle+|g_1\rangle|e_2\rangle + |g_1\rangle|g_2\rangle)\otimes_{j=3}^{N}(|e_N\rangle+|g_N\rangle). \quad (21)$$

Choosing $2\Theta=-\pi$ (namely $2\Omega g/(\Delta\delta+g^2)=1$), $(\delta+g^2/\Delta)t=2\pi$, we obtain the cluster state of the atoms 1 and 2

$$|\phi\rangle_{12\cdots N}=\frac{1}{2^{N/2}}(|g_1\rangle+|e_1\rangle\sigma_z^2)(|g_2\rangle+|e_2\rangle) \otimes_{j=3}^{N}(|g_N\rangle+|e_N\rangle), \quad (22)$$

where $\sigma_z^2=|g_2\rangle\langle g_2|-|e_2\rangle\langle e_2|$. Then we drive the atoms 2 and 3 through the second cavity. After an interaction time $t=2\pi/(\delta+g^2/\Delta)$, choosing $2\Theta=-\pi$ and performing the one-qubit operation $|e_j\rangle\to{\rm e}^{-{\rm i}(\Theta+\Omega^2t/\Delta)}|e_j\rangle(j=2,3)$, we obtain the state

$$|\phi\rangle_{12\cdots N}=\frac{1}{2^{N/2}}(|g_1\rangle+|e_1\rangle\sigma_z^2)(|g_2\rangle +|e_2\rangle\sigma_z^3)(|g_3\rangle+|e_3\rangle)\otimes_{j=4}^{N}(|g_N\rangle+|e_N\rangle). \quad (23)$$

From the form of the above state, we can conclude that if we send two atoms through a vacuum cavity for an interaction time $t=2\pi/(\delta+g^2/\Delta)$, choosing $2\Theta=-\pi$ and performing the one-qubit operation on the two atoms, step by step, then we can obtain a multi-atom cluster state easily. Namely, firstly we send atoms 1 and 2 through a vacuum cavity for an appropriate time, and perform the one-qubit operation on the atoms 1 and 2. Then we send atoms 2 and 3 through another vacuum cavity, and perform the one-qubit operation on the atoms 2 and 3, and so on. Finally, then we can obtain the multi-atom cluster state

$$|\phi\rangle_N=\frac{1}{2^{N/2}}\otimes_{j=1}^{N}(|g_j\rangle+|e_j\rangle\sigma_z^{j+1}), \quad (24)$$

where $\sigma_z^{N+1}\equiv1$.

4. Dynamic model of a cavity QED system assisted by a strong resonant classical field

We consider two identical three-level atoms, each one has one excited state $|r\rangle$ and two ground states

$|e\rangle$ and $|g\rangle$. The qubits are encoded in the ground state $|g\rangle$ and the state $|-\rangle = 1/\sqrt{2}(|e\rangle - |r\rangle)$. The transition $|e\rangle \rightarrow |r\rangle$ is driven by the cavity mode with the coupling constant g and the detuning Δ, assisted by a resonant classical field with Rabi frequency Ω. The state $|g\rangle$ is not influenced in the interaction, the Hamiltonian[35] is expressed as (assuming $\hbar = 1$)

$$H' = \omega_1 \sum_{j=1,2} S_{z,j} + \omega_{\rm b} a^\dagger a + \sum_{j=1,2} [g(a^\dagger S_j^- + a S_j^+) + \Omega(\mathrm{e}^{-\mathrm{i}\omega_{\rm L} t} S_j^+ + \mathrm{e}^{\mathrm{i}\omega_{\rm L} t} S_j^-)], \quad (25)$$

where ω_1, $\omega_{\rm b}$ and $\omega_{\rm L}$ are the $|e\rangle \rightarrow |r\rangle$ resonant transition frequency, cavity mode frequency, and classical field frequency respectively. Atomic operators $S_{z,j} = \frac{1}{2}(|r_j\rangle\langle r_j| - |e_j\rangle\langle e_j|)$, $S_j^+ = |r_j\rangle\langle e_j|$, and $S_j^- = |e_j\rangle\langle r_j|$. $a^\dagger$ and a are the creation and annihilation operators of the cavity mode respectively. In the rotating frame at the driving field frequency $\omega_{\rm L}$ (assuming $\omega_{\rm L} = \omega_1$), the Hamiltonian is given by

$$H^{\rm L} = \Delta a^\dagger a + \sum_{j=1,2} [g(a^\dagger S_j^- + a S_j^+) + \Omega(S_j^+ + S_j^-)] = H_0^{\rm L} + H_i^{\rm L}, \quad (26)$$

with

$$H_0^{\rm L} = \Delta a^\dagger a + \sum_{j=1,2} \Omega(S_j^+ + S_j^-),$$
$$H_{\rm i}^{\rm L} = \sum_{j=1,2} g(a^\dagger S_j^- + a S_j^+).$$

Perform the transformation

$$H_{\rm i}' = \mathrm{e}^{\mathrm{i}H_0^{\rm L} t} H_{\rm i}^{\rm L} \mathrm{e}^{-\mathrm{i}H_0^{\rm L} t} = \frac{g}{2} \sum_{j=1,2} [|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| + \mathrm{e}^{\mathrm{i}2\Omega t} |+_j\rangle\langle -_j| - \mathrm{e}^{-\mathrm{i}2\Omega t} |-_j\rangle\langle +_j|] a \mathrm{e}^{-\mathrm{i}\Delta t} + \mathrm{H.c.}, \quad (27)$$

where $|\pm_j\rangle = (|e_j\rangle \pm |r_j\rangle)/\sqrt{2}$, and $S_{jx}|\pm_j\rangle = (S_j^+ + S_j^-)|\pm_j\rangle = \pm|\pm_j\rangle$. In the strong driving region $\Omega \gg \Delta, g$, we discard the rapidly oscillating term, and the effective Hamiltonian in the rotating wave approximation is

$$H_{\rm i\ eff}' = \frac{g}{2} \sum_{j=1,2} (|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|) \times (a\mathrm{e}^{-\mathrm{i}\Delta t} + a^\dagger \mathrm{e}^{\mathrm{i}\Delta t}). \quad (28)$$

After the infinitesimal interval $[t, t + \mathrm{d}t]$, the corresponding evolution of the states in our qubit subspace can be described as

$$U(\mathrm{d}t) = D(\mathrm{d}\alpha) = \mathrm{e}^{a^\dagger \mathrm{d}\alpha - a \mathrm{d}\alpha^*} = \mathrm{e}^{-\mathrm{i}H_{\rm ieff}' \mathrm{d}t}, \quad (29)$$

where $\mathrm{d}\alpha = \mathrm{i}(g/2)\mathrm{e}^{\mathrm{i}\Delta t}\mathrm{d}t$. After an interaction time t', the displacement parameter α can be expressed as

$$\alpha(t') = \mathrm{i} \int_0^{t'} \frac{g}{2} \mathrm{e}^{\mathrm{i}\Delta t} \mathrm{d}t = \frac{g}{2\Delta} [\mathrm{e}^{\mathrm{i}\Delta t'} - 1], \quad (30)$$

and the geometric phase shift Θ is

$$\Theta = \mathrm{Im}\Big(\int_\gamma \alpha^* \mathrm{d}\alpha\Big) = \frac{g^2}{4\Delta^2} [\Delta t - \sin(\Delta t)]. \quad (31)$$

In order to generate a multi-atom cluster state, we prepare $N (N \geq 2)$ atoms initially in the state $(|g_j\rangle + |e_j\rangle)/\sqrt{2} (j = 1, 2, ..., N)$. The cavities are all prepared in the vacuum state $|0\rangle$. The total initial atomic state is

$$|\phi\rangle_{12\cdots N} = \frac{1}{2^{N/2}} (|g_1\rangle + |e_1\rangle) \otimes_{j=2}^N (|g_j\rangle + |e_j\rangle). \quad (32)$$

Firstly, we perform the transformation $|e_j\rangle \rightarrow |-_j\rangle$ $(j = 1, 2)$, then drive the atoms 1 and 2 through a vacuum cavity. Under the condition $\Delta t = 2\pi$, the displacement is along a closed path, returning to the original point in phase space and acquiring a geometric phase conditional upon the electronic states. After the interaction time t, the system of atoms evolves to

$$|\phi\rangle_{12\cdots N} = \frac{1}{2^{N/2}} (|g_1\rangle|g_2\rangle + \mathrm{e}^{\mathrm{i}(\Omega t + \Theta)} |g_1\rangle|-_2\rangle + \mathrm{e}^{\mathrm{i}(\Omega t + \Theta)} |-_1\rangle|g_2\rangle + \mathrm{e}^{\mathrm{i}(2\Omega t + 4\Theta)} |-_1\rangle|-_2\rangle) \otimes_{j=3}^N (|g_j\rangle + |-_j\rangle). \quad (33)$$

Then we perform the one-qubit transformation $|-_j\rangle \rightarrow \mathrm{e}^{-\mathrm{i}(\Omega t + \Theta)} |-_j\rangle (j = 1, 2)$, the state of atomic system becomes

$$|\phi\rangle_{12\cdots N} = \frac{1}{2^{N/2}} (|g_1\rangle|g_2\rangle + |g_1\rangle|-_2\rangle + |-_1\rangle|g_2\rangle + \mathrm{e}^{\mathrm{i}2\Theta} |-_1\rangle|-_2\rangle) \otimes_{j=3}^N (|g_N\rangle + |-_N\rangle). \quad (34)$$

After performing the transformation $|-_j\rangle \rightarrow |e_j\rangle (j = 1, 2)$, choosing $2\Theta = \pi$ (namely $g/\Delta = 1$), and $\Delta t = 2\pi$, we obtain the cluster state of the atoms 1 and 2

$$|\phi\rangle_{12\cdots N} = \frac{1}{2^{N/2}} (|g_1\rangle + |e_1\rangle\sigma_z^2)(|g_2\rangle + |e_2\rangle) \otimes_{j=3}^N (|g_N\rangle + |e_N\rangle), \quad (35)$$

where $\sigma_z^j = |g_j\rangle\langle g_j| - |e_j\rangle\langle e_j| (j = 2)$. Secondly, we drive the atoms 2 and 3 through the second vacuum cavity after performing the transformation $|e_j\rangle \rightarrow |-_j\rangle (j = 2, 3)$. After an interaction time $t = 2\pi/\Delta$, we perform the one-qubit transformation $|-_j\rangle \rightarrow \mathrm{e}^{-\mathrm{i}(\Theta + \Omega t)} |-_j\rangle (j = 2, 3)$. By choosing $2\Theta = \pi$

and performing the transformation $|-_j\rangle \rightarrow |e_j\rangle (j = 2, 3)$, we obtain the state of atomic system

$$|\phi\rangle_{12\cdots N} = \frac{1}{2^{N/2}}(|g_1\rangle + |e_1\rangle\sigma_z^2)(|g_2\rangle + |e_2\rangle\sigma_z^3)(|g_3\rangle + |e_3\rangle)\otimes_{j=4}^{N}(|g_N\rangle + |e_N\rangle). \quad (36)$$

From the form of the above state, we can conclude that if we perform the transformation $|e_j\rangle \rightarrow |-_j\rangle$ and send two atoms through a vacuum cavity for an interaction time $t = 2\pi/\Delta$, thence by choosing $2\Theta = \pi$ and performing the one-qubit transformation $|-_j\rangle \rightarrow \mathrm{e}^{-\mathrm{i}(\Omega t+\Theta)}|-_j\rangle$ and the transformation $|-_j\rangle \rightarrow |e_j\rangle$ on the two atoms, step by step, we can obtain a multi-atom cluster state, as

$$|\phi\rangle_N = \frac{1}{2^{N/2}} \otimes_{j=1}^{N} (|g_j\rangle + |e_j\rangle\sigma_z^{j+1}), \quad (37)$$

where $\sigma_z^{N+1} \equiv 1$.

5. Conclusion

In summary, we have proposed two schemes to generate the multi-atom cluster state via an unconventional geometric phase shift in cavity QED. In the two schemes, the cavity mode is displaced along a closed path depending on the atomic states. The system of the atomic states acquires a conditional phase, producing a cluster state of the multi-atom. The first scheme is based on the large-detuning interaction of the two atoms with the cavity mode. According to interaction time $t = 2\pi/(\delta + g^2/\Delta)$, and $2\Theta = -\pi$, we obtained the time for generating a cluster state $t_\mathrm{f} = \Delta\pi/(\Omega g)(\Delta \gg \Omega, g)$. The second scheme is based on the resonant coupling of the atoms to the classical field. According to interaction time $t = 2\pi/\Delta$, and $2\Theta = \pi$, we obtained the time for generating a cluster state $t_\mathrm{s} = 2\pi/g$. Thus the time for generating a cluster state in the second scheme is shorter than that in the first one. In the second scheme, the detuning $\Delta = g$ is satisfied because of $\Delta t = 2\pi$ and $2\Theta = g^2\pi/\Delta^2 = \pi$. The two schemes can be realized with the present cavity QED techniques.

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