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One-step discrimination scheme on N-particle Greenberger–Horne–Zeilinger bases*

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We present an experimentally feasible one-step discrimination scheme on Bell bases with trapped ions, and then generalize it to the case of N-ion Greenberger–Horne–Zeilinger (GHZ) bases. In the scheme, all the orthogonal and complete N-ion GHZ internal states can be exactly discriminated only by one step, and thus it takes very short time. Moreover, the scheme is insensitive to thermal motion and dose not require the individual addressing of the ions. The Bell-state and GHZ-state one-step discrimination scheme can be widely used in quantum information processing based on ion-trap set-up.

Keywords: Bell-state analyser, GHZ-state analyser, trapped ions, quantum information processing
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The phenomenon of nonlocal correlations, or entanglement, between quantum-mechanical particles is a central topic in the growing field of quantum information science. Entangled states are required to realize diverse tasks in quantum information processing,[1−3] including quantum dense coding,[4−6] quantum teleportation,[7−10] entanglement swapping,[11] telecloning[12] and information concentration.[13,14] It is well known that the preparation and measurement of Bell states are essential to quantum dense coding and quantum teleportation. The extension to the Greenberger–Horne–Zeilinger (GHZ) states thus allows us to generalize the two-particle case to the multi-particle case.[15−22] Also, in a generalization of entanglement swapping,[15] a GHZ-state analyser can be actually used to prepare multi-particle GHZ states out of entangled pairs.

Theoretically, using only single-quantum-bit (qubit) operations and the controlled-NOT (CNOT) gates[23] one can construct a suitable quantum network to produce and identify any of the maximally entangled states for any number of particles.[1] For example, using a Hadamard (H) gate (operated on the first particle) and $N-1$ CNOT gates (operated on the first particle and any other of the particles, respectively), any one of the 2^N orthogonal and complete N-particle GHZ states (when $N = 2$, they are general Bell states) can be transformed into a corresponding product state of the N particles. Thus the complete deterministic N-particle GHZ bases measurement can be achieved via separate single-particle measurements on all the particles. However, when the number of particles N is large, such quantum networks are difficult to construct practically because of the complexity of manipulation and the limit of coherent time of the controlled systems.[1,24,25] In addition, the experimental process will be time-consuming and the efficiency must be low. In fact the time for it to take is proportional to the number of particles.[15]

In this paper, we present an experimentally feasible one-step discrimination scheme on Bell bases with trapped ions, and then generalize it to the case of N-ion Greenberger–Horne–Zeilinger (GHZ) bases. The scheme does not involve any complex quantum networks composed of single qubit operations and CNOT gates.[24−26] All the orthogonal and complete N-ion GHZ internal states can be exactly discriminated only by one step. Thus the time for it to take is irrespective of the number of ions N and is very short. Moreover, the scheme is insensitive to heating of vibrational

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mode and dose not require the individual addressing of the ions, which is very advantageous to experimental implementation.[27−29] The Bell-state and GHZ-state one-step-analyser are very useful in quantum information processing based on ion-trap set-up.[24,25,30,31]

We consider that $N(N \geq 2)$ two-level (denoted by $|e\rangle$ and $|g\rangle$) ions are confined in a linear Paul trap. And all the ions are simultaneously excited by two classical homogeneous lasers of frequencies $\omega_0 + \nu + \delta$ and $\omega_0 - \nu - \delta$, where ω_0 is the frequency of the transition $|e\rangle \leftrightarrow |g\rangle$ and ν is the frequency of one collective vibrational mode. Suppose $\delta \ll \nu$, we can neglect other vibrational modes. In these cases, the Hamiltonian of the system is given by (for simplicity, we let $\hbar = 1$)[27,28]

$$H = \nu a^+ a + \frac{1}{2}\omega_0 \sum_{j=1}^{N} \sigma_j^{\mathrm{z}} + \left\{ \Omega \mathrm{e}^{-\mathrm{i}\phi} \sum_{j=1}^{N} \sigma_j^+ \left[\mathrm{e}^{\mathrm{i}\eta_1 (a^+ + a)} \times \mathrm{e}^{-\mathrm{i}(\omega_0 + \nu + \delta)t} + \mathrm{e}^{\mathrm{i}\eta_2 (a^+ + a)} \mathrm{e}^{-\mathrm{i}(\omega_0 - \nu - \delta)t} \right] + \mathrm{H.c.} \right\}, \quad (1)$$

where $a^+(a)$ denotes the creation (annihilation) operator for the collective vibrational mode, $\sigma_j^{\mathrm{z}} = |e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|$ is the general Pauli operator, $\sigma_j^+ = |e_j\rangle\langle g_j|$ and $\sigma_j^- = |g_j\rangle\langle e_j|$ are the raising and lowering operators, respectively, and $\eta_i = \sqrt{(k_i)^2/(2\nu N m)}(i = 1, 2)$ is the Lamb–Dicke parameter, with k_i being the effective wave vector of the ith laser field along the direction of the string of ions and m being the mass of each ion. We here have assumed that both fields have the same phase ϕ and intensity, and also assumed that the ion–laser interacting strength characterized by the Rabi frequency Ω is the same for all ions (In practical realization, one might use Raman transitions between low-lying states of the ions due to their long coherence time. By appropriately redefining the symbols, our formalism also describes this implementation[29]).

In practical realization the effective wave vector k_i can be nearly identical for the two fields[29] (in fact a total identity can be achieved by modulating the irradiation directions of the two lasers), and we assume $\eta_1 = \eta_2 = \eta$. When $\nu \gg \Omega$, we discard the rapidly oscillating terms and obtain the Hamiltonian in the interaction picture,

$$H_{\mathrm{I}} = \Omega \mathrm{e}^{-\mathrm{i}\phi} \mathrm{e}^{-\eta^2/2} \sum_{j=1}^{N} \sigma_j^+ \sum_{l=0}^{\infty} \frac{(\mathrm{i}\eta)^{2l+1}}{l!(l+1)!} \times \left[(a^+)^{l+1} a^l \mathrm{e}^{-\mathrm{i}\delta t} + (a^+)^l a^{l+1} \mathrm{e}^{\mathrm{i}\delta t} \right] + \mathrm{H.c.}. \quad (2)$$

Under the Lamb–Dicke regime (i.e. $\eta\sqrt{n+1} \ll 1$, with n being the vibrational quantum number), the Hamiltonian of Eq.(2) can approximated by the expansion to the first order in η (assuming $\phi = \pi/2$) as follows:

$$H_{\mathrm{I}} = \Omega\eta \sum_{j=1}^{N} (\sigma_j^+ + \sigma_j^-)(a^+ \mathrm{e}^{-\mathrm{i}\delta t} + a\mathrm{e}^{\mathrm{i}\delta t}). \quad (3)$$

In the case of $\delta \gg \Omega\eta$, the energy conserving transitions are $|e_j e_k n\rangle \leftrightarrow |g_j g_k n\rangle$ and $|g_j e_k n\rangle \leftrightarrow |e_j g_k n\rangle$. The transition $|e_j e_k n\rangle \leftrightarrow |g_j g_k n\rangle$ is mediated by $|g_j e_k n \pm 1\rangle$ and $|e_j g_k n \pm 1\rangle$. The contributions of $|g_j e_k n \pm 1\rangle$ are equal to those of $|e_j g_k n \pm 1\rangle$. The corresponding Rabi frequency is given by $2(\Omega\eta)^2/\delta$. Since the transition paths interfere destructively, there is no transfer of population to states with different vibrational excitation and thus the Rabi frequency is independent of vibrational quantum numbers.[27,28] The Rabi frequency for $|g_j e_k n\rangle \leftrightarrow |e_j g_k n\rangle$, mediated by $|e_j e_k n \pm 1\rangle$ and $|g_j g_k n \pm 1\rangle$, is also $2(\Omega\eta)^2/\delta$. Then the effective Hamiltonian is given by[27,28,32]

$$H_{\mathrm{e}} = \frac{\lambda}{2} \sum_{j=1}^{N} (|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|) + \lambda \sum_{j,k=1}^{N} (S_j^+ S_k^+ + S_j^+ S_k^- + \mathrm{H.c.}),\ j \neq k, \quad (4)$$

where $\lambda = 2\Omega^2\eta^2/\delta$. We define the new atomic bases as

$$|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle),\ |-_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle). \quad (5)$$

Then we can rewrite H_{e} as

$$H_{\mathrm{e}} = \frac{1}{2}\lambda \left[\sum_{j=1}^{N} (|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|) \right]^2. \quad (6)$$

We note that the ionic internal state evolution operator

$$U(t) = \mathrm{e}^{-\mathrm{i}H_{\mathrm{e}}t} \quad (7)$$

is independent of the phonon number of the vibrational mode, allowing it to be in a thermal state.

We first consider the case where $N = 2$, and obtain the following evolutions:

$$\begin{aligned} U(t)|+_1\rangle|+_2\rangle &= \mathrm{e}^{-\mathrm{i}2\lambda t}|+_1\rangle|+_2\rangle, \\ U(t)|+_1\rangle|-_2\rangle &= |+_1\rangle|-_2\rangle, \\ U(t)|-_1\rangle|+_2\rangle &= |-_1\rangle|+_2\rangle, \\ U(t)|-_1\rangle|-_2\rangle &= \mathrm{e}^{-\mathrm{i}2\lambda t}|-_1\rangle|-_2\rangle. \end{aligned} \tag{8}$$

If choosing $t = \dfrac{\pi}{4\lambda}$, it is easy to prove that

$$\begin{aligned} U\left(\frac{\pi}{4\lambda}\right)\frac{1}{\sqrt{2}}(|g_1g_2\rangle + \mathrm{i}|e_1e_2\rangle) &= \mathrm{e}^{-\mathrm{i}\pi/4}||g_1g_2\rangle, \\ U\left(\frac{\pi}{4\lambda}\right)\frac{1}{\sqrt{2}}(|g_1g_2\rangle - \mathrm{i}|e_1e_2\rangle) &= \mathrm{e}^{\mathrm{i}5\pi/4}||e_1e_2\rangle, \\ U\left(\frac{\pi}{4\lambda}\right)\frac{1}{\sqrt{2}}(|g_1e_2\rangle + \mathrm{i}|e_1g_2\rangle) &= \mathrm{e}^{-\mathrm{i}\pi/4}||g_1e_2\rangle, \\ U\left(\frac{\pi}{4\lambda}\right)\frac{1}{\sqrt{2}}(|g_1e_2\rangle - \mathrm{i}|e_1g_2\rangle) &= \mathrm{e}^{\mathrm{i}5\pi/4}||e_1g_2\rangle. \end{aligned} \tag{9}$$

Hence, the complete deterministic Bell-state measurement can be achieved by detecting ions 1 and 2 separately.[24,25]

When $N > 2$, using the representation of operator σ^z, the ionic states $|g_1g_2\cdots g_N\rangle$ and $|e_1e_2\cdots e_N\rangle$ can be expressed as $|N/2, -N/2\rangle$ and $|N/2, N/2\rangle$, respectively. On the other hand, such states can be expanded in terms of the eigenstates of Pauli operator σ^x[28,32,33] as follows:

$$\begin{aligned} |N/2,-N/2\rangle &= \sum_{M=-N/2}^{M=N/2} C_M|N/2,M\rangle_x, \\ |N/2,N/2\rangle &= \sum_{M=-N/2}^{M=N/2} C_M(-1)^{N/2-M}|N/2,M\rangle_x. \end{aligned} \tag{10}$$

We first consider even N; the case of odd N will be discussed later. If the N ions are initially in the state $|g_1g_2\cdots g_N\rangle$, the evolution of the system will be

$$|N/2,-N/2\rangle \xrightarrow{U(t)} \sum_{M=-N/2}^{M=N/2} C_M\mathrm{e}^{-\mathrm{i}2\lambda M^2 t}|N/2,M\rangle_x. \tag{11}$$

With the choice $t = \dfrac{\pi}{4\lambda}$, we obtain

$$\frac{1}{\sqrt{2}}\sum_{M=-N/2}^{M=N/2} C_M[\mathrm{e}^{-\mathrm{i}\pi/4} + \mathrm{e}^{\mathrm{i}\pi/4}(-1)^M]|N/2,M\rangle_x = \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}|g_1g_2\cdots g_N\rangle + \mathrm{e}^{\mathrm{i}\pi/4}(-1)^{N/2}|e_1e_2\cdots e_N\rangle]. \tag{12}$$

Similarly, we can obtain

$$\begin{aligned} |e_1\cdots e_je_{j+1}\cdots e_N\rangle &\xrightarrow{U(\frac{\pi}{4\lambda})} \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}|e_1\cdots e_je_{j+1}\cdots e_N\rangle + \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}(-1)^{N/2}|g_1\cdots g_jg_{j+1}\cdots g_N\rangle], \\ |e_1\cdots e_jg_{j+1}\cdots g_N\rangle &\xrightarrow{U(\frac{\pi}{4\lambda})} \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}|e_1\cdots e_jg_{j+1}\cdots g_N\rangle + \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}(-1)^{N/2}|g_1\cdots g_je_{j+1}\cdots e_N\rangle], \\ |g_1\cdots g_jg_{j+1}\cdots g_N\rangle &\xrightarrow{U(\frac{\pi}{4\lambda})} \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}|g_1\cdots g_jg_{j+1}\cdots g_N\rangle + \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}(-1)^{N/2}|e_1\cdots e_je_{j+1}\cdots g_N\rangle]. \end{aligned} \tag{13}$$

According to the evolutions given above, we can obtain the following results:

$$\begin{aligned} &\frac{1}{\sqrt{2}}(|g_1j_2j_3\cdots j_N\rangle + \mathrm{i}|e_1\bar{j}_2\bar{j}_3\cdots\bar{j}_N\rangle) \xrightarrow{U(\frac{\pi}{4\lambda})} \mathrm{e}^{-\mathrm{i}\pi/4}|g_1j_2j_3\cdots j_N\rangle, \\ &\frac{1}{\sqrt{2}}(|g_1j_2j_3\cdots j_N\rangle - \mathrm{i}|e_1\bar{j}_2\bar{j}_3\cdots\bar{j}_N\rangle) \xrightarrow{U(\frac{\pi}{4\lambda})} \mathrm{e}^{\mathrm{i}5\pi/4}|e_1\bar{j}_2\bar{j}_3\cdots\bar{j}_N\rangle, \end{aligned} \tag{14}$$

where N is even and $N/2$ is odd;

$$\begin{aligned} &\frac{1}{\sqrt{2}}(|g_1j_2j_3\cdots j_N\rangle + \mathrm{i}|e_1\bar{j}_2\bar{j}_3\cdots\bar{j}_N\rangle) \xrightarrow{U(\frac{\pi}{4\lambda})} \mathrm{e}^{\mathrm{i}\pi/4}|e_1\bar{j}_2\bar{j}_3\cdots\bar{j}_N\rangle, \\ &\frac{1}{\sqrt{2}}(|g_1j_2j_3\cdots j_N\rangle - \mathrm{i}|e_1\bar{j}_2\bar{j}_3\cdots\bar{j}_N\rangle) \xrightarrow{U(\frac{\pi}{4\lambda})} \mathrm{e}^{-\mathrm{i}\pi/4}|g_1j_2j_3\cdots j_N\rangle, \end{aligned} \tag{15}$$

where N is even and $N/2$ is also even, $j \in \{g, e\}$, and $\bar{j}$ is the counterpart of the binary number j. So 2^N orthogonal and complete N-particle GHZ states can be exactly discriminated by separate single-ion measurements.[24,25]

As for odd N, the evolutions of Eqs.(12–15) cannot be obtained directly from the Hamiltonian of Eq.(6). In this case, however, we can use another resonant laser to excite all the ions after, before or simultaneously applying Eq.(6), and then the total ionic internal state evolution operator can be described by[28,32]

$$U(t) = \exp\left\{-\mathrm{i}2gt\sum_{j=1}^{N}(|+_j\rangle\langle+_j| - |-_j\rangle\langle-_j|)\right\} \times \exp\left\{-\mathrm{i}\frac{\lambda t}{2}\left[\sum_{j=1}^{N}(|+_j\rangle\langle+_j| - |-_j\rangle\langle-_j|)\right]^2\right\}, \qquad (16)$$

where g is Rabi frequency characterizing the interaction strength between the ions and the resonant laser field. It is easy to verify that any of the GHZ states $\frac{1}{\sqrt{2}}(|g_1 j_2 j_3 \cdots j_N\rangle \pm \mathrm{i}|e_1 \bar{j}_2 \bar{j}_3 \cdots \bar{j}_N\rangle)$ can also be transformed into a corresponding product state with the choice $\lambda t = \pi/4$ and $gt = 3\pi/4$ for odd N. In the same way, the evolution operator shown in Eq.(16) can produce the same results of Eqs.(12–15) for even N with the choice $gt = \pi$ and $\lambda t = \pi/4$.

We briefly discuss the feasibility of the current scheme. With the vibrational frequency $\nu = 1.2\,\mathrm{MHz}$,[24] we can choose $\delta = 0.1\nu, \eta = 0.1$, and $\Omega = 0.1\nu$, thus the conditions $\delta \ll \nu$, $\nu \gg \Omega$ and $\nu \gg \Omega\eta$ can be satisfied. The next eigenfrequency is $\nu_\mathrm{r} = \sqrt{3}\nu$, then the detuning between the Rabi frequency and this vibrational frequency is $\delta_\mathrm{r} = |\Omega - \nu_\mathrm{r}| \approx 1.632\nu \gg \delta$. So it is reasonable to neglect other vibrational modes. In the experiment reported in Ref.[24] the lifetime of the metastable $D_{5/2}$ state of ${}^{40}C_\mathrm{a}^+$ is $\tau_l \approx 1.16\,\mathrm{s}$. However, with the parameters given above, the duration of our scheme is only $\tau = \pi\delta/(8\eta^2\Omega^2) \approx 52\,\mu\mathrm{s}$ (as for odd N, we can set $g = 8\Omega^2\eta^2/\delta$) which is much shorter than τ_l. Meanwhile the scheme is insensitive to thermal motion and dose not require the individual addressing of the ions, which is very advantageous to experimental implementation.[27−29] Therefore, based on the current ion-trap techniques[24,25,29−31,34,35] the proposed scheme might be realizable.

It deserves being pointed out that the time for our scheme to take to make the Bell-state measurement is much shorter than that required by the schemes in Refs.[24–26] which use the way of quantum network. Because the difference between times required by our scheme and by theirs is proportional to the number of ions N, when $N > 2$ our scheme will be greatly faster than theirs.

The Bell-state or GHZ-state discrimination is essential to quantum dense coding and quantum teleportation. The key step in quantum dense coding protocol is the Bell state or GHZ state measurement (discrimination). If Bob (receiver) can completely and deterministically discriminate the Bell bases[4] or GHZ bases[15] after possessing all particles, dense coding successes with certainty, otherwise, it fails.[36] The Bell-state or GHZ-state analysis is also required in the quantum teleportation protocol,[7,21] which is the foundation of entanglement swapping[11] (which is usually used for implementing entanglement concentration[37−40]), telecloning[12], information concentration[13,14] and teleportation-based quantum computation.[31,41,42] So our scheme has great potential applications in quantum information processing based on ion-trap set-up.[24,25,30,31]

In summary, We have presented a one-step-analyser of N-particle GHZ-state with trapped ions. The analyser does not involve any complex quantum networks composed of single qubit operations and CNOT gates. Thus it is much faster than previous schemes.[24−26] Moreover, the scheme is insensitive to thermal motion and dose not require the individual addressing of the ions. The scheme can be realized with the ion-trap techniques available presently, and has great potential applications in quantum information processing based on ion-trap set-up.

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