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Remote preparation of a Greenberger–Horne–Zeilinger state via a two-particle entangled state*

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We present two schemes for realizing the remote preparation of a Greenberger–Horne–Zeilinger (GHZ) state. The first scheme is to remotely prepare a general N-particle GHZ state with two steps. One is to prepare a qubit state by using finite classical bits from sender to receiver via a two-particle entangled state, and the other is that the receiver introduces $N-1$ additional particles and performs $N-1$ controlled-not (C-Not) operations. The second scheme is to remotely prepare an N-atom GHZ state via a two-atom entangled state in cavity quantum electrodynamics (QED). The two schemes require only a two-particle entangled state used as a quantum channel, so we reduce the requirement for entanglement.

Keywords: remote state preparation, two-particle entangled state, Greenberger–Horne–Zeilinger (GHZ) state, controlled-not operation, cavity quantum electrodynamics (QED)
PACC: 0365, 0367

1. Introduction

Entanglement plays an important role in quantum information theory. One of the striking exhibitions of entanglement is quantum teleportation,[1−5] in which an unknown qubit state can be transmitted from a sender (Alice) to a spatially distant receiver (Bob) via an entangled quantum channel using only local operations and classical communication. Recently, Lo,[6] Pati,[7] and Bennett *et al*[8] have presented methods to transmit pure quantum states using entanglement and classical communication when the sender has the knowledge of the transmitted state. This quantum communication protocol is called remote state preparation (RSP). In RSP, Alice performs a measurement on her share of the entangled resource in a basis chosen in accordance with the state that she wishes to help Bob prepare in his laboratory. In Refs.[6–8], the authors have shown that, for some special ensemble of qubits, the classical communication cost is less in the RSP than in the standard teleportation, but for general states, RSP requires the classical communication cost as much as quantum teleportation does. Lo[6] and Bennett *et al*[8] have also studied the trade-off between entanglement cost and classical communication cost in RSP.

More recently, some authors[9−24] have investigated various protocols for the generalization of RSP in theory or in experiment. Shi and Tomita[12] have proposed a scheme to remotely prepare a two-particle entangled state by a three-particle Greenberger–Horne–Zeilinger (GHZ) state. Liu and Wang[18] have presented a scheme for remotely preparing a two-particle entangled state via two pairs of entangled particles. Zhan[22] has presented a scheme for remotely preparing a three-particle GHZ state by a two-particle entangled state and a three-particle entangled state. In this paper, we present two schemes for remotely preparing an N-particle GHZ state. The distinct advantage of the schemes is that remotely preparing an N-particle GHZ state only needs a two-particle entangled state, which reduces the degree of entanglement. The rest of this paper is organized as follows: Section 2 is devoted to the remote preparation of a general N-particle GHZ state; Section 3 is for the remotely preparing of an N-atom GHZ state via cavity quantum electrodynamics (QED); Section 4 is for the conclusion drawn from the present study.

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2. Remote preparation of a general N-particle GHZ state

In this section, we describe how to accomplish the remote preparation of an N-particle GHZ state via only one two-particle nonmaximally entangled state with two steps. Suppose that the sender Alice wishes to help the receiver Bob remotely prepare a general N-particle GHZ state

$$|\phi\rangle = \alpha_0|00\cdots0\rangle + \alpha_1 e^{i\varphi}|11\cdots1\rangle, \tag{1}$$

where $\alpha_i (i = 0, 1)$ and φ are real, $\alpha_i > 0$ and $\alpha_0^2 + \alpha_1^2 = 1$. We assume that Alice knows this state exactly, which means that she knows $\alpha_i (i = 0, 1)$ and φ completely, but Bob does not know them at all.

Step 1 Suppose that Alice and Bob share a two-particle entangled state

$$|\Psi\rangle_{AB_1} = \beta_0|00\rangle_{AB_1} + \beta_1|11\rangle_{AB_1},$$
$$\beta_0, \beta_1 > 0, \quad \beta_0^2 + \beta_1^2 = 1. \tag{2}$$

Alice and Bob transform their shared entangled state (2) into

$$|\Psi\rangle_{AB_1} = \alpha_0|00\rangle_{AB_1} + \alpha_1|11\rangle_{AB_1} \tag{3}$$

by using one bit from Alice to Bob.[23,25] As the final entangled state has the same Schmidt basis as the original one, Bob can only perform a permutative operation to accomplish the transformation, which indicates that Bob need not know the final state.[26] Alice performs the following unitary transformation on particle A

$$U = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}, \tag{4}$$

and the state (3) becomes

$$|\Psi\rangle_{AB_1} = \alpha_0|00\rangle_{AB_1} + \alpha_1 e^{i\varphi}|11\rangle_{AB_1}. \tag{5}$$

Then, Alice operates a local measurement on the single particle A with the base $\{|+\rangle, |-\rangle\}$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. If particle A is in the state $|+\rangle_A$ or $|-\rangle_A$, the state of particle B_1 will collapse into the following state with the aid of a rotational operation:

$$|\Psi\rangle_{B_1} = \alpha_0|0\rangle_{B_1} + \alpha_1 e^{i\varphi}|1\rangle_{B_1}. \tag{6}$$

Step 2 Alice informs Bob of the qubit number of the remotely prepared state via a classical channel. Bob introduces additional particles B_2, B_3, ..., B_N initially prepared in the state $|00\cdots0\rangle_{B_2B_3\ldots B_N}$ and performs $N-1$ C-Not operations with particle B_1 as the control bit and the $N-1$ additional particles as the target bit, thus transforming the state $(\alpha_0|0\rangle_{B_1} + \alpha_1 e^{i\varphi}|1\rangle_{B_1})|00\cdots0\rangle_{B_2B_3\ldots B_N}$ into the state $|\phi\rangle = \alpha_0|00\cdots0\rangle_{B_1B_2\ldots B_N} + \alpha_1 e^{i\varphi}|11\cdots1\rangle_{B_1B_2\ldots B_N}$, which indicates the success in the remote preparation of a general N-particle GHZ state. The probability of the successful preparing of the GHZ state approaches 1.

3. Remote preparation of an N-atom GHZ state via cavity QED

We note that the cavity QED system is a qualified candidate for demonstrating quantum information processing and atoms with long-lived internal states are well suited for storing qubits. However, little contribution has been made to the discussion about the remote preparation of atomic states.[27,28] Here we present a scheme for remotely preparing an N-atom GHZ state via a two-atom entangled state in cavity QED.

We first consider K ($K \geq 2$) identical two-level atoms that are simultaneously interacting with a single-mode cavity and driven by a classical field. In the strong driving regime $2\Omega \gg \delta, g$ and in the case of $\delta \gg g/2$ (Ω is the Rabi frequency of the classical field, g is the atom–cavity coupling strength and δ is the detuning between the atomic transition frequency and the cavity frequency), there exists no energy exchange between the atomic system and the cavity. Then in the interaction picture, the effective interaction Hamilton (let $\hbar = 1$) reads[29]

$$H_{\text{eff}} = 2(\Omega S_x + \lambda S_x^2), \tag{7}$$

where

$$\lambda = \frac{g^2}{2\delta}, \quad S_x = \frac{1}{2}\sum_{j=1}^{K}(S_j^+ + S_j^-),$$
$$S_j^+ = |e_j\rangle\langle g_j|, \quad S_j^- = |g_j\rangle\langle e_j|,$$

with $|e_j\rangle$ and $|g_j\rangle$ ($j = 1, 2, \cdots, K$) being the excited and ground states of the jth atom. Here we have assumed that the transition frequency of atoms is equal to the frequency of classical field. We note that the effective Hamilton is independent of the cavity field state, allowing it to be in a thermal state. Then the

evolution operator of the system is given by

$$U(t) = \mathrm{e}^{-\mathrm{i}H_{\mathrm{eff}}t} = \mathrm{e}^{-\mathrm{i}2(\Omega S_x + \lambda S_x^2)t}. \tag{8}$$

Using the representation of the operator S_z, the atomic states $|g_1g_2\cdots g_K\rangle$ and $|e_1e_2\cdots e_K\rangle$ can be expressed as $|K/2,-K/2\rangle$ and $|K/2,K/2\rangle$, respectively. On the other hand, such states can be expanded in terms of the eigenstates of S_x,[29]

$$|K/2,-K/2\rangle = \sum_{M=-K/2}^{M=K/2} C_M|K/2,M\rangle_x,$$

$$|K/2,K/2\rangle = \sum_{M=-K/2}^{M=K/2} C_M(-1)^{K/2-M}|K/2,M\rangle_x. \tag{9}$$

Thus, if the K atoms are initially in the state $|g_1g_2\cdots g_K\rangle$, the evolution of the system will be

$$\sum_{M=-K/2}^{M=K/2} C_M\mathrm{e}^{-\mathrm{i}2(\Omega M+\lambda M^2)t}|K/2,M\rangle_x.$$

When K is even, then M is an integer. With the choice of $\lambda t = \pi/4$ and $\Omega t = n\pi$, the system will evolve into[29]

$$\frac{1}{\sqrt{2}}\sum_{M=-K/2}^{M=K/2} C_M[\mathrm{e}^{-\mathrm{i}\pi/4}+\mathrm{e}^{\mathrm{i}\pi/4}(-1)^M]|K/2,M\rangle_x = \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}|g_1g_2\cdots g_K\rangle + \mathrm{e}^{\mathrm{i}\pi/4}(-1)^{K/2}|e_1e_2\cdots e_K\rangle]. \tag{10}$$

On the other hand, for the case where K is odd, we choose $\lambda t = \pi/4$ and $\Omega t = (2n+3/4)\pi$, then the system will evolve into[29]

$$\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}(7/8)\pi}[\mathrm{e}^{-\mathrm{i}\pi/4}|g_1g_2\cdots g_K\rangle + \mathrm{e}^{\mathrm{i}\pi/4}(-1)^{(1+K)/2}|e_1e_2\cdots e_K\rangle]. \tag{11}$$

Similarly, we can obtain the following evolutions:

$$|m_1m_2\cdots m_K\rangle \xrightarrow{U(t)} \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\pi/4}|m_1m_2\cdots m_K\rangle + \mathrm{e}^{\mathrm{i}\pi/4}(-1)^{K/2}|\bar{m}_1\bar{m}_2\cdots\bar{m}_K\rangle] \tag{12}$$

with K being even ($\lambda t = \pi/4, \Omega t = n\pi$), and

$$|m_1m_2\cdots m_K\rangle \xrightarrow{U(t)} \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}(7/8)\pi}[\mathrm{e}^{-\mathrm{i}\pi/4}|m_1m_2\cdots m_K\rangle + \mathrm{e}^{\mathrm{i}\pi/4}(-1)^{(1+K)/2}|\bar{m}_1\bar{m}_2\cdots\bar{m}_K\rangle] \tag{13}$$

with K being odd ($\lambda t = \pi/4,\ \Omega t = 2n\pi + 3\pi/4$), where $m, \bar{m} \in \{g, e\}$, and $\langle m|\bar{m}\rangle = 0$.

If two identical two-level atoms simultaneously interact with a single-mode cavity and are driven by a classical field, i.e. $K = 2$, we can obtain the evolution with different initial states[29]

$$\begin{aligned}|m_1m_2\rangle \xrightarrow{U(t)} \mathrm{e}^{-\mathrm{i}\lambda t}[&\cos(\lambda t)(\cos(\Omega t)|m_1\rangle - \mathrm{i}\sin(\Omega t)|\bar{m}_1\rangle)(\cos(\Omega t)|m_2\rangle - \mathrm{i}\sin(\Omega t)|\bar{m}_2\rangle)\\ &- \mathrm{i}\sin(\lambda t)(\cos(\Omega t)|\bar{m}_1\rangle - \mathrm{i}\sin(\Omega t)|m_1\rangle)(\cos(\Omega t)|\bar{m}_2\rangle - \mathrm{i}\sin(\Omega t)|m_2\rangle)].\end{aligned} \tag{14}$$

Now we use the above system to implement the remote preparation of an N-atom GHZ state via only one two-atom entangled state. Suppose that the sender Alice wishes to help the receiver Bob remotely prepare an N-atom GHZ state

$$|\phi\rangle = \alpha|gg\cdots g\rangle + \beta|ee\cdots e\rangle, \tag{15}$$

where the parameters α and β are real numbers, satisfying

$$|\alpha|^2 + |\beta|^2 = 1.$$

The state in Eq.(15) is known completely to Alice but unknown to Bob. We also suppose that Alice and Bob share a two-atom entangled state as follows:

$$|\Psi\rangle_{12} = a|gg\rangle_{12} + b|ee\rangle_{12}, \tag{16}$$

where the parameters a and b are real, satisfying $|a| \le |b|$ and $|a|^2 + |b|^2 = 1$. The atom 1 belongs to Alice, and the atom 2 belongs to Bob.

Firstly, Alice introduces an auxiliary atom a with the initial state of $\frac{1}{\sqrt{2}}(|g\rangle_a + |e\rangle_a)$. The atoms 1, 2, a are identical. Alice sends atoms 1 and a simultaneously across a single-mode cavity in a thermal state, and at the same time these atoms are driven by a classical field. The system evolution can be described by Eq.(8) as follows:

$$
\begin{aligned}
&(a|gg\rangle_{12}+b|ee\rangle_{12})\frac{1}{\sqrt{2}}(|g\rangle_a+|e\rangle_a)\\
&\rightarrow\frac{1}{\sqrt{2}}a\mathrm{e}^{-\mathrm{i}\lambda t}|g\rangle_2[\cos(\lambda t)(\cos(\Omega t)|g\rangle_1-\mathrm{i}\sin(\Omega t)|e\rangle_1)(\cos(\Omega t)|g\rangle_a-\mathrm{i}\sin(\Omega t)|e\rangle_a)\\
&\quad-\mathrm{i}\sin(\lambda t)(\cos(\Omega t)|e\rangle_1-\mathrm{i}\sin(\Omega t)|g\rangle_1)(\cos(\Omega t)|e\rangle_a-\mathrm{i}\sin(\Omega t)|g\rangle_a)]\\
&\quad+\frac{1}{\sqrt{2}}a\mathrm{e}^{-\mathrm{i}\lambda t}|g\rangle_2[\cos(\lambda t)(\cos(\Omega t)|g\rangle_1-\mathrm{i}\sin(\Omega t)|e\rangle_1)(\cos(\Omega t)|e\rangle_a-\mathrm{i}\sin\Omega t|g\rangle_a)\\
&\quad-\mathrm{i}\sin(\lambda t)(\cos(\Omega t)|e\rangle_1-\mathrm{i}\sin(\Omega t)|g\rangle_1)(\cos(\Omega t)|g\rangle_a-\mathrm{i}\sin(\Omega t)|e\rangle_a)]\\
&\quad+\frac{1}{\sqrt{2}}b\mathrm{e}^{-\mathrm{i}\lambda t}|e\rangle_2[\cos(\lambda t)(\cos(\Omega t)|e\rangle_1-\mathrm{i}\sin(\Omega t)|g\rangle_1)(\cos(\Omega t)|g\rangle_a-\mathrm{i}\sin(\Omega t)|e\rangle_a)\\
&\quad-\mathrm{i}\sin(\lambda t)(\cos(\Omega t)|g\rangle_1-\mathrm{i}\sin(\Omega t)|e\rangle_1)(\cos(\Omega t)|e\rangle_a-\mathrm{i}\sin(\Omega t)|g\rangle_a)]\\
&\quad+\frac{1}{\sqrt{2}}b\mathrm{e}^{-\mathrm{i}\lambda t}|e\rangle_2[\cos(\lambda t)(\cos(\Omega t)|e\rangle_1-\mathrm{i}\sin(\Omega t)|g\rangle_1)(\cos(\Omega t)|e\rangle_a-\mathrm{i}\sin(\Omega t)|g\rangle_a)\\
&\quad-\mathrm{i}\sin(\lambda t)(\cos(\Omega t)|g\rangle_1-\mathrm{i}\sin(\Omega t)|e\rangle_1)(\cos(\Omega t)|g\rangle_a-\mathrm{i}\sin(\Omega t)|e\rangle_a)]. \qquad (17)
\end{aligned}
$$

Secondly, Alice measures the states of atoms 1 and a. If her measurement is $|gg\rangle_{1a}$, the state of atom 2 will become

$$
\begin{aligned}
|\Psi\rangle_2=&\frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\lambda t}a(\mathrm{e}^{-\mathrm{i}\lambda t}\cos^2(\Omega t)+\mathrm{i}\sin(\lambda t))|g\rangle_2-\frac{\mathrm{i}}{2\sqrt{2}}\mathrm{e}^{-2\mathrm{i}\lambda t}a\sin(2\Omega t)|g\rangle_2\\
&-\frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\lambda t}b(\mathrm{e}^{-\mathrm{i}\lambda t}\sin^2(\Omega t)+\mathrm{i}\sin(\lambda t))|e\rangle_2-\frac{\mathrm{i}}{2\sqrt{2}}\mathrm{e}^{-2\mathrm{i}\lambda t}b\sin(2\Omega t)|e\rangle_2. \qquad (18)
\end{aligned}
$$

Since Alice has known the parameters α and β exactly, she can select the atomic velocity to meet the condition $\cos(\lambda t)=\alpha$ (or $\sin(\lambda t)=\beta$) and obtain a conclusive interaction time. In addition, she can make the Rabi frequency Ω satisfy the condition $\Omega t=\pi$ through modulating the classical field. After the interaction and measurement, Alice will inform Bob of her measurements trough a classical channel. The state of atom 2 on Bob's side will become

$$
|\Psi\rangle_2=\frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\lambda t}(\alpha a|g\rangle_2-\mathrm{i}\beta b|e\rangle_2). \qquad (19)
$$

After a rotation operation, the state in Eq.(19) will become

$$
\frac{1}{\sqrt{2}}(\alpha a|g\rangle_2+\beta b|e\rangle_2). \qquad (20)
$$

Thirdly, a cavity initially prepared in vacuum state $|0\rangle$ is introduced in Bob's location. Bob sends atom 2 into the cavity, and makes the resonant interaction (between atom 2 and the cavity mode) happen. According to Jaynes–Cummings model, the interaction Hamiltonian of the system in the rotation-wave approximation can be expressed as

$$
H=\omega(a^+a+S_z)+\varepsilon(aS_++a^+S_-), \qquad (21)
$$

where ω is the cavity frequency; ε is the coupling constant between cavity and atom; $S_+=|e\rangle\langle g|$, $S_-=|g\rangle\langle e|$ and $S_z=|g\rangle\langle g|-|e\rangle\langle e|$ are the atomic operators; a^+ and a are creation and annihilation operators for the cavity mode, respectively. After the interaction time t', the states of atom 2 and vacuum cavity evolve into the following form:

$$
\begin{aligned}
&\frac{1}{\sqrt{2}}(\alpha a|g\rangle_2+\beta b|e\rangle_2)|0\rangle\rightarrow\frac{1}{\sqrt{2}}[\alpha a|g\rangle_2|0\rangle\\
&\quad+\beta b(\cos(\varepsilon t')|e\rangle_2|0\rangle-\mathrm{i}\sin(\varepsilon t')|g\rangle_2|1\rangle)]. \qquad (22)
\end{aligned}
$$

Bob can select the atomic velocity to meet the condition $\cos(\varepsilon t')=a/b$, and the state of Eq.(22) will become

$$
\frac{1}{\sqrt{2}}a(\alpha|g\rangle_2+\beta|e\rangle_2)|0\rangle-\mathrm{i}\beta\sqrt{b^2-a^2}|g\rangle_2|1\rangle. \qquad (23)
$$

Then Bob detects the cavity by a photon detector. If $|0\rangle$ is obtained, Bob will obtain the state

$$
\frac{1}{\sqrt{2}}a(\alpha|g\rangle_2+\beta|e\rangle_2). \qquad (24)
$$

The remote preparation of the state $\alpha|g\rangle_2+\beta|e\rangle_2$ is successful with the probability of $\frac{1}{2}|a|^2$.

Through analyses, we find that the measurements $|ge\rangle_{1a}$, $|eg\rangle_{1a}$ and $|ee\rangle_{1a}$ also can successfully realize

the remote preparation of the state $\alpha|g\rangle_2 + \beta|e\rangle_2$ if appropriate operations are performed on atom 2. By considering the four cases, the total probability of the successfully preparing the state $\alpha|g\rangle_2 + \beta|e\rangle_2$ is $2|a|^2$.

Fourthly, in order to realize the remote preparation of an N-atom GHZ state, Bob introduces N additional atoms $\mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_N$ initially prepared in the state $|gg\cdots g\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N}$. Suppose atoms $2, \mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_N$ to be identical. Bob has atoms $2, \mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_N$, which are driven by a classical field, simultaneously cross a single-mode cavity in a thermal state. The system evolution is described by Eq.(8), with the choice of $\lambda t = \pi/4$ and $\Omega t = n\pi$ ($\Omega t = 2n\pi + 3\pi/4$) when N is odd (even), the whole state evolves into

$$\begin{aligned}|\Psi\rangle = \frac{1}{\sqrt{2}}[&|g\rangle_2 \mathrm{e}^{\mathrm{i}\varphi}(\alpha|gg\cdots g\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N} \\ &+ \mathrm{i}\beta d(N)|ee\cdots e\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N}) \\ &+ |e\rangle_2 \mathrm{e}^{\mathrm{i}\psi(N)}(\alpha|ee\cdots e\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N} \\ &- \mathrm{i}\beta d(N)|gg\cdots g\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N})], \end{aligned} \tag{25}$$

where $\varphi = -\pi/4$ ($\varphi = 5\pi/8$) when N is odd (even), $\psi(N)$ and $d(N)$, which are functions of N, are determined as

$$\psi(N) = \begin{cases} 5\pi/4, & \text{if } N = 4k-3, \\ \pi/4, & \text{if } N = 4k-1, \\ 9\pi/8, & \text{if } N = 4k-2, \\ \pi/8, & \text{if } N = 4k, \end{cases} \tag{26}$$

$$d(N) = \begin{cases} -1, & \text{if } N = 4k-3, \\ 1, & \text{if } N = 4k-1, \\ 1, & \text{if } N = 4k-2, \\ -1, & \text{if } N = 4k, \end{cases} \tag{27}$$

where k is a natural number. We perform the following unitary transformations on atoms B_1 and B_2, respectively:

$$R = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\pi/4} & 0 \\ 0 & 1 \end{pmatrix}.$$

The transformation can be achieved by using the Stark effects in a static electric field.[30] Then we have

$$\begin{aligned}|\Psi\rangle = \frac{1}{\sqrt{2}}[&|g\rangle_2 \mathrm{e}^{\mathrm{i}\varphi}\mathrm{e}^{-\mathrm{i}\pi/2}(\alpha|gg\cdots g\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N} - \beta d(N)|ee\cdots e\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N}) \\ &+ |e\rangle_2 \mathrm{e}^{\mathrm{i}\psi(N)}(\alpha|ee\cdots e\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N} - \beta d(N)|gg\cdots g\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N})]. \end{aligned} \tag{28}$$

Table 1. Corresponding unitary transformations on the atoms $\mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_N$ according to the result of measurement on atom 2.

Result of measurement on atom 2	Corresponding unitary transformations on the atoms $\mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_N$
$\|g\rangle_2$	$(I_{\mathrm{B}_1})^{[1-d(N)]/2}[\sigma_{z\mathrm{B}_2}]^{[1+d(N)]/2} \otimes I_{\mathrm{B}_3} \otimes \cdots \otimes I_{\mathrm{B}_N}$
$\|e\rangle_2$	$[\sigma_{x\mathrm{B}_1}]^{[1-d(N)]/2}[\mathrm{i}\sigma_{y\mathrm{B}_2}]^{[1+d(N)]/2} \otimes \sigma_{x\mathrm{B}_3} \otimes \cdots \otimes \sigma_{x\mathrm{B}_N}$

Bob carries out a local measurement on the single atom 2 with the base $\{|g\rangle, |e\rangle\}$ and performs corresponding unitary transformations on the atoms $\mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_N$ (see Table 1), which can be easily implemented. Then we obtain

$$\alpha|gg\cdots g\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N} + \beta|ee\cdots e\rangle_{\mathrm{B}_1\mathrm{B}_2\ldots\mathrm{B}_N}, \tag{29}$$

which indicates the success in the remote preparation of the N-atom GHZ state in Eq.(15). If the result of detecting the cavity is $|1\rangle$, the remote state preparation fails. The total probability of the successful remote preparation of the N-atom GHZ state is $2|a|^2$. If $|a| = |b| = \frac{1}{\sqrt{2}}$, i.e. for a quantum channel composed of a two-atom maximally entangled state, the state in Eq.(20) will become

$$\frac{1}{2}(\alpha|g\rangle_2 + \beta|e\rangle_2). \tag{30}$$

Under this condition, it is unnecessary for Bob to introduce a vacuum cavity field. And the total successful probability can reach 1.

4. Summary

Especially, we should point out that the two schemes of RSP have their own advantages. The first scheme has the following advantages compared with the previous schemes indicated in Refs.[12,18,22]. First, our scheme for remotely preparing a general

N-particle GHZ state reduces the requirements for the quantum channel. The quantum channel is only composed of a two-particle entangled state. But, in the schemes for remotely preparing a three-particle GHZ state, given in Refs.[18,22], the quantum channel is composed of three two-particle entangled states, a two-particle entangled state and a three-particle entangled GHZ state. The scheme in Ref.[12] achieves the RSP of an N-particle entangled state by means of a $(N+1)$-particle entangled GHZ state, and the scheme in Ref.[18] is generalized to the RSP of an N-particle entangled state by using N-biparticle entangled pairs. Second, the total probability of the RSP of a general N-particle GHZ state equals 1 in our scheme, whereas in the schemes presented in Refs.[12,18,22], the probability of success in the RSP of a general GHZ state reaches 50% at most, and only for the specially chosen states, can the RSP be achieved with certainty. The second scheme requires only a two-atom entangled state used as a quantum channel to remotely prepare an N-atom GHZ state, whereas the previous scheme in Ref.[28] for remotely preparing a two-atom entangled state requires two two-atom entangled states, so we reduce the requirement for entanglement. We should emphasize that the prepared GHZ state in our schemes is owned only by a user, but the prepared state in the previous schemes[12,18,22] is shared by N users.

In conclusion, we present two schemes for realizing the remote preparation of a GHZ state. The distinct advantage of the schemes is that remotely preparing an N-particle GHZ state only needs a two-particle entangled state, which reduces the degree of entanglement. In the first scheme, the N-particle GHZ state can be remotely prepared with certainty if a qubit state is prepared by using finite classical bits from a sender to a receiver, and then the receiver introduces $N-1$ additional particles and performs $N-1$ C-Not operations. The other scheme is to remotely prepare an N-atom GHZ state via a two-atom entangled state in cavity QED. The successful probability of the remote preparation is $2|a|^2$. If the quantum channel is composed of a two-atom maximally entangled state, the successful probability of the remote preparation reaches 1. In this case, there is no need to introduce any other vacuum cavity field, so the influences of cavity decay and thermal field are all eliminated.

References

[1] Bennett C H, Brassard G, Grépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[2] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* (London) **390** 575
[3] Zhan Y B 2004 *Chin. Phys.* **13** 1801
[4] Lin X and Li H C 2005 *Chin. Phys.* **14** 1724
[5] Yuan H C and Qi K G 2005 *Chin. Phys.* **14** 1716
[6] Lo H K 2000 *Phys. Rev.* A **62** 012313
[7] Pati A K 2001 *Phys. Rev.* A **63** 014302
[8] Bennett C H, Di Vincenzo D P, Shor P W, Smolin J A, Terhal B M and Wootters W K 2001 *Phys. Rev. Lett.* **87** 077902
[9] Devetak I and Berger T 2001 *Phys. Rev. Lett.* **87** 197901
[10] Zeng B and Zhang P 2002 *Phys. Rev.* A **65** 022316
[11] Zheng Y Z, Gu Y J and Guo G C 2002 *Chin. Phys. Lett.* **19** 14
[12] Shi B S and Tomita A 2002 *J. Opt.* B: *Quantum Semiclass. Opt.* **4** 380
[13] Abeyesinghe A and Hayden P 2003 *Phys. Rev.* A **68** 062319
[14] Berry D W and Sanders B C 2003 *Phys. Rev. Lett.* **90** 057901
[15] Yu Y F, Feng J and Zhan M S 2003 *Phys. Lett.* A **310** 329
[16] Leung D W and Shor P W 2003 *Phys. Rev. Lett.* **90** 127905
[17] Hayashi A, Hashimoto T and Horibe M 2003 *Phys. Rev.* A **67** 052302
[18] Liu J M and Wang Y Z 2003 *Phys. Lett.* A **316** 159
[19] Paris M G A, Cola M and Bonifacio R 2003 *J. Opt.* B: *Quantum Semiclass. Opt.* **5** S360
[20] Babichev S A, Brezqer B and Lvovsky A I 2004 *Phys. Rev. Lett.* **92** 047903
[21] Liu J M and Wang Y Z 2004 *Chin. Phys.* **13** 147
[22] Zhan Y B 2005 *Commun. Theor. Phys.* **43** 637
[23] Ye M Y, Zhang Y S and Guo G C 2004 *Phys. Rev.* A **69** 022310
[24] Peng X H, Zhu X W, Fang X M, Feng M, Liu M L and Gao K L 2003 *Phys. Lett.* A **306** 271
[25] Nielsen M A 1999 *Phys. Rev. Lett.* **83** 436
[26] Jensen J G and Schack R 2001 *Phys. Rev.* A **63** 062303
[27] Huang Y Z and Zhan M S 2004 *Phys. Lett.* A **327** 404
[28] Zhang Y Q, Jin X R and Zhang S 2005 *Chin. Phys.* **14** 1732
[29] Zheng S B 2003 *Phys. Rev.* A **68** 035801
[30] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392