Vol 16 No 4, April 2007
1009-1963/2007/16(04)/1159-08

Chinese Physics

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Breakup of spiral wave under different boundary conditions*

Zhao Ying-Kui(赵英奎)[a)b)†], Wang Guang-Rui(王光瑞)[b)], and Chen Shi-Gang (陈式刚)[b)]

[a)] *Graduate School of China Academy of Engineering Physics, Beijing* 100088, *China*
[b)] *Center for Nonlinear Studies, Institute of Applied Physics and Computational Mathematics, Beijing* 100088, *China*

(Received 11 July 2006; revised manuscript received 8 November 2006)

In this paper, we investigate the breakup of spiral wave under no-flux, periodic and Dirichlet boundary conditions respectively. When the parameter ε is close to a critical value for Doppler-induced wave breakup, the instability of the system caused by the boundary effect occurs in the last two cases, resulting in the breakup of spiral wave near the boundary. With our defined average order measure of spiral wave (AOMSW), we quantify the degree of order of the system when the boundary-induced breakup of spiral wave happens. By analysing the AOMSW and outer diameter R of the spiral tip orbit, it is easy to find that this boundary effect is correlated with large values of R, especially under the Dirichlet boundary condition. This correlation is nonlinear, so the AOMSW sometimes oscillates with the variation of ε.

Keywords: spiral, breakup, boundary, reaction-diffusion
PACC: 8220M, 0547, 0565

1. Introduction

The spiral waves in two spatial dimensions are the most paradigmatic examples of spatiotemporal self-organizing structures in excitable and oscillatory mediums. They have been the subject of extensive research in a large variety of systems, which include catalytic surface processes, Belousov–Zhabotinsky (BZ) reaction and the heart muscle.[1−3] The transition from a regular pattern to spatiotemporal chaos is a challenging phenomenon observed in nonlinear dynamics.[2] Among these phenomena, the instability of spiral waves in reaction diffusion systems is one of the most robust scenarios observed in experiments of chemical reactions[4−7] and in numerical simulations of various models.[8−20] Generally, stable spiral waves lose their stability via a Hopf bifurcation that leads to meandering spiral waves.[17] Two different breakup scenarios have been investigated both experimentally and numerically. The first one is the longitudinal Eckhaus instability,[6,18−20] which modulates the spiral waves so that the minimum of the local period in the system violates the dispersion relation and results in breakup. Depending on the behaviour of the Eckhaus mode, the spiral wave may break up far away from or near to the core. The second breakup is caused by a Doppler effect.[5,18] When the stable spiral tip meanders, the Doppler effect becomes so large that adjacent waves interact and break.

In this work, we investigate the breakup of spiral wave due to boundary effect, about which little has been researched before. The no-flux boundary condition, periodic boundary condition and a special Dirichlet boundary condition are considered here. This boundary effect can cause the instability of the system and induces the breakup of spiral waves near the boundary. If this boundary effect is weak, the spiral waves break up far away from the core; with such an effect being strong, the instability transfers inward and the breakup area of the system will increase. The breakup areas are dependent on parameters, so the patterns correspond to a series of degrees of order with different parameters. It is not appropriate to identify whether the system is just 'ordered' or 'disordered'. A qualified measure, which may be termed an average order measure of spiral wave (AOMSW), is introduced by us to characterize the degree of order for the spiral wave. This measure can give a clear 'score' for the spiral waves with different order degrees. The problems may be solved in the following steps: consider a 2D FitzHugh–Nagumo-type model and give the phenomena under different boundary conditions, and then

*Project supported by the Major Program of the National Natural Science Foundation for (Grant No 10335010) and the National Natural Science Foundation–the Science Foundation of China Academy of Engineering Physics (NSAF) (Grant No 10576005).
†E-mail: yingkui_zhao2001@yahoo.com

use our defined order measure AOMSW to investigate the properties of boundary-induced breakup of spiral wave.

2. Model and dynamical property under different boundary conditions

Consider a 2D FitzHugh–Nagumo-type model where each reaction unit in the lattice has two variables $u = u(x, y, t)$ and $v = v(x, y, t)$. These variables satisfy the following reaction–diffusion equations:[11]

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon} u(1-u)\left(u - \frac{b+v}{a}\right) + \nabla^2 u, \qquad (1)$$

$$\frac{\partial v}{\partial t} = f(u) - v,$$

$$f(u) = \begin{cases} 0, & 0 \le u < 1/3, \\ 1 - 6.75u(u-1)^2, & 1/3 \le u \le 1, \\ 1, & 1 < u. \end{cases}$$

The form of $f(u)$ describes a delayed production of the inhibitor and the equations have been used to model the patterns in a catalytic surface reaction in the CO oxidation on a Pt(110) surface. If we choose $f(u) = u$, the functions can be used to investigate the heart muscle. In simulations, we use Euler integration with a space step $\delta x = \delta y = 0.045$ s.u. and a time step $\delta t = 0.02$ t.u. We choose $a = 0.84$ and vary the parameter b, such that the corresponding local dynamics changes from oscillatory (if $b < 0$) to excitable (if $b > 0$). The parameter choice $a < 1$ yields excitable (oscillatory) behaviour for $b > 0$ ($b < 0$) and $0 < \varepsilon \ll 1$. In both cases, an unstable focus exists with $(u, v) = (u_0, v_0)$ in the local dynamics of Eqs. (1). In the excitable case, two more fixed points appear: $(u, v) = (0, 0)$ is the stable rest state and $(u, v) = (b/a, 0)$ is a saddle that marks the threshold of the excitable medium.

The definition of the Dirichlet boundary condition is as follows.[21] The no-normal flow boundary condition implies that $\hat{t} \cdot \nabla u = 0$ and $\hat{n} \cdot \nabla u \neq 0$, where $\hat{n}$ is a unit vector pointing outwards from the boundary, $\hat{t} = \hat{z} \times \hat{n}$ is a unit vector perpendicular to the path traversing the boundary, and $\hat{z}$ is a unit vector perpendicular to the lattice layer of the system. So the variables of the reaction units on the boundary are equal in value. We take u_b and v_b as boundary variables. In our numerical simulation, the system is divided into 200×200 lattice points (see Fig.1). The variables u_b and v_b can be obtained from the reaction-diffusion equation (Eq.1). The diffusion term is given by the diffusion of the points A, B, C and D. These points are middle points of the lines a, b, c and d respectively, and the lines are grid points inwards from the boundaries. For the Laplacian, we use 5-point prescription $\nabla_b^2 u_b = (u_A + u_B + u_C + u_D - 4u_b)/\delta^2 x$.

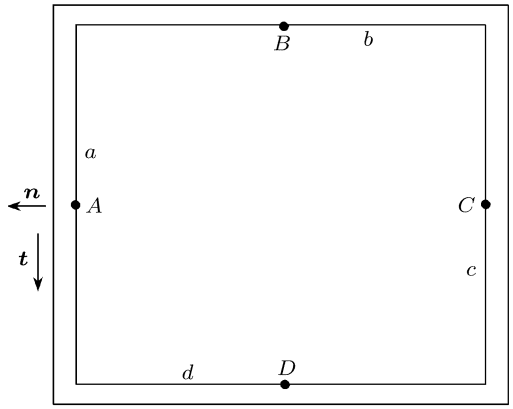


Fig.1. The definition of the special Dirichlet boundary condition.

In this definition of Dirichlet boundary condition, due to the variables being equal in value on the boundary, the topological structure of the system is of a spherical surface. So the boundary is one of the poles of the topological sphere. Because the boundary values u_b and v_b satisfy the reaction–diffusion equation (Eq.(1)), they vary constantly. Figures 2(a) and (b) are two typical snapshots of the system under the Dirichlet boundary condition at different times with $b > 0$. Figure 2(a) corresponds to the case in which the boundary units have not been excited, whereas the boundary units have been excited in Fig.2(b). These two cases appear alternately with wave fluctuating on the boundary. Thus a topological target wave transfers from the boundary to the centre. These boundary-excited waves collide with the inner spiral wave around the boundary. An unstable effect can be created within the collision area. Some little free wave tips are created near the boundary by this effect (see Fig.2(b)). With appropriate parameters, this effect can be amplified, and the breakup of spiral wave will occur. A similar effect can be found in the periodic boundary condition. In Fig.3 we give the breakup processes of spiral waves under different

boundary conditions. The figures in each column show the scenarios at different times but under the same parameters. The first column displays a propagation of spiral wave breakup caused by a typical Doppler effect under the no-flux boundary condition. The instability starts from the core of the wave. For a long run, the windows will be full of the absolute spatiotemporal chaos. While the second column shows the breakup of the spiral wave due to the periodic boundary effect. The spatiotemporal chaos appears round the boundary and then extends inward. Similarly, the third column shows that under the Dirichlet boundary condition, the instability effect starts around the boundary, and then spreads inward.

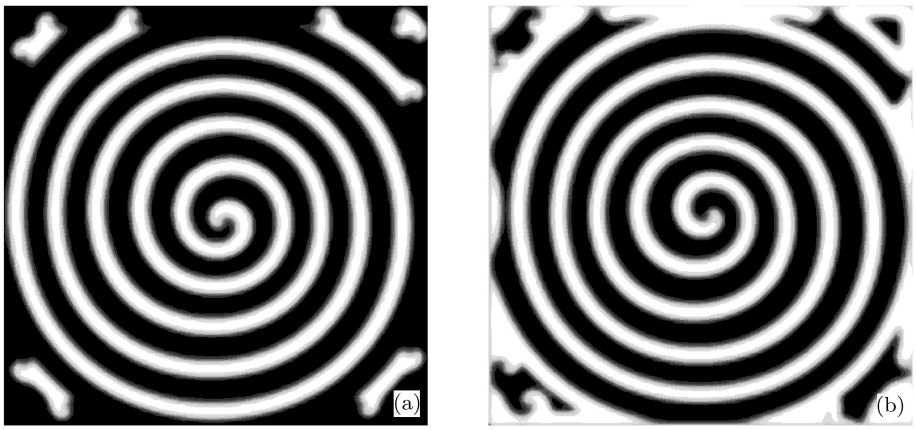


Fig.2. Stable spiral waves under Dirichlet boundary condition, without the boundary units excited (a) and with the units excited (b). These two states occur alternately.

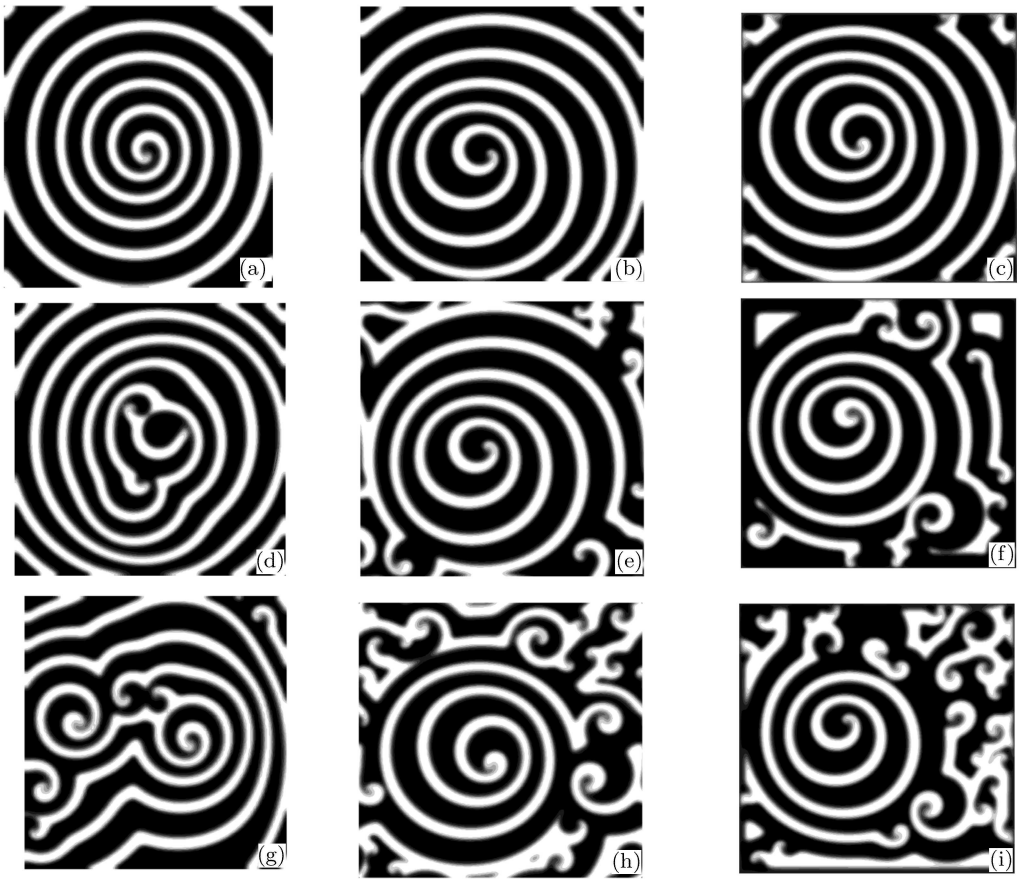


Fig.3. First column: propagation patterns of an initial stable spiral wave under no-flux boundary condition with $\varepsilon = 0.0685$. Breakup of spiral wave appears due to Doppler effect. The second column: propagation patterns of the same initial spiral wave under periodic boundary condition with $\varepsilon = 0.0672$. The instability of the spiral wave starts from the boundary. The third column: propagation figures of the same initial spiral wave under Dirichlet boundary condition with $\varepsilon = 0.0679$. Instability effect also starts from the boundary. These figures are in the excitable medium with $b = 0.05$ and 200×200 pixels.

To study the spiral wave dynamics, we record the spiral tip trajectory,[20,22] then measure the outside diameter (R) of the tip orbit (defined by the isoline $u = 0.5$) when the spiral wave behaviour is stable or meandering (if the spiral tip has a stable circle orbit, R is the diameter of the circle; if the orbit of the spiral tip is meandering with a modulating frequency, R is the average diameter of the circle). If the medium is excitable ($b = 0.05$), the system first encounters a Hopf bifurcation around $\varepsilon = 0.0556$. Before that the spiral is stable; beyond that, the spiral wave meanders. Further increasing ε causes the spiral wave breakup. Figure 4 gives diagrams for R as a function of ε under different boundary conditions. We find that the curves are almost similar to each other in three cases except for different critical values for instability of the systems. In the case of the no-flux boundary condition (Fig.4(a)), there is a critical value $\varepsilon_0 = 0.0684$ for the Doppler effect instability. When $\varepsilon < \varepsilon_0$, the system is stable. Beyond ε_0 the Doppler effect causes the spiral wave breakup and spatiotemporal chaos state appearing in the system. This effect is the only instability ingredient in this case. Under the periodic boundary condition (Fig.4(b)), the spiral wave begins to break up at the point $\varepsilon = 0.0664$. When $0.0664 < \varepsilon < \varepsilon_0$, the system is unstable because of the boundary effect. The Doppler-induced instability also happens when $\varepsilon > \varepsilon_0$. In the Dirichlet boundary case (Fig.4(c)), the system properties are similar to those of the periodic case, but the critical value of system instability is smaller, or rather, $\varepsilon = 0.0656$. This means a relatively strong boundary effect under this condition.

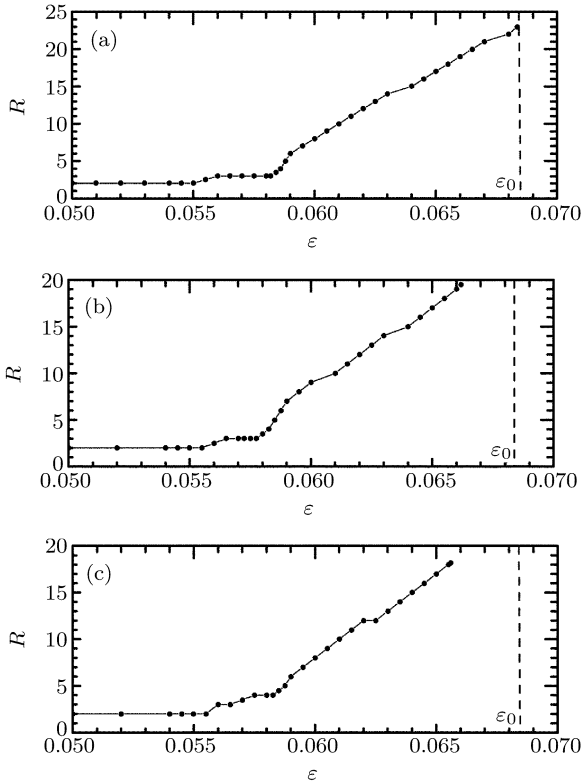


Fig.4. The diameter R of the spiral tip orbit as a function of ε at $b = 0.05$. The system is under (a) no-flux boundary condition, (b) periodic boundary condition and (c) Dirichlet boundary condition respectively.

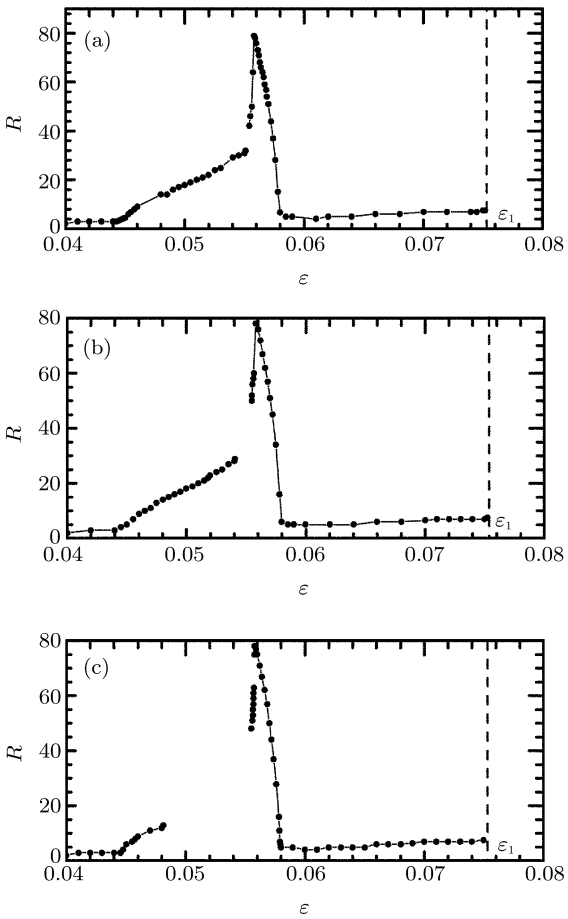


Fig.5. The diameter R of the spiral tip orbit vs ε for $b = -0.045$ with (a) no-flux boundary condition, (b) periodic boundary condition and (c) Dirichlet boundary condition respectively.

For the oscillatory medium (see Fig.5), we take $b = -0.045$, and again study the relation between R and ε. Here two value ranges for spiral wave breakup are observed for the no-flux boundary condition in Fig.5(a). The first one, which is caused by the Doppler effect, occurs when $\varepsilon \in (0.0551, 0.0554)$, and the second one, which is caused by the Eckhaus instability, appears when $\varepsilon > \varepsilon_1 = 0.0752$. In Fig.5(b) there are also two breakup ranges, but the first one has a more

wide range with $\varepsilon \in (0.0541, 0.0554)$. Now the instability of the system mainly results from the effect of periodic boundary effect. The second range where the Eckhaus instability happens is similar to that of ε in Fig.5(a). For the Dirichlet boundary, there are three instability ranges. The first one begins from the point $\varepsilon = 0.0481$, and ends at $\varepsilon = 0.0554$. The boundary effect is the main reason for the breakup of spiral wave. Instability of the system in the second range ($\varepsilon \in (0.0558, 0.056)$) is totally caused by the boundary effect. In the third range there also exists Eckhaus instability with the same critical value ε_1. Actually, although we consider the system to be stable when $\varepsilon \in (0.0554, 0.0558)$, instability of the system still exists. But the spiral wave only partially breaks up, and the breakup area dose not spread.

3. Definition of AOMSW

If a spiral wave persists for 160 cycles without fragmentation we think of the breakup as being absent at these parameter values. With this definition, we can find an exact ε_0 as a critical transition point under the no-flux boundary condition in an excitable medium, such as $\varepsilon_0 = 0.0684$ in Fig.4(a). When $\varepsilon < \varepsilon_0$ the spiral wave is stable, and there is no fragmentation even after a very long run (1000 cycles). If $\varepsilon > \varepsilon_0$, the spiral wave is so unstable that the instability effect will make it break up after several cycles, and the system will be totally in a spatiotemporally chaotic state. In Section 2, though we have given the transition curve of spiral wave under different boundary conditions, the exact critical values are difficult to obtain for periodic and Dirichlet boundary conditions. For example, in an excitable medium (Figs.4(c)), although we have given the critical value $\varepsilon = 0.0656$ for the spiral wave breakup, a stable spiral wave can still persist without any fragmentation around some points of ε, such as $\varepsilon \in (0.06638, 0.06652)$. At the same time, around some other points, such as $\varepsilon \in (0.0666, 0.0668)$, the breakup of spiral wave easily appears. But wave fragments of spiral wave only appear in a very small area around the boundary, and the other parts are still stable even after quite a long time, while the system may be very easy to be full of the wave fragments within some parameter ranges, such as $\varepsilon \in (0.0677, 0.0679)$.

For the reason above, it is inappropriate to identify whether a spiral wave is 'ordered' or 'disordered'. So it needs a qualified measure to quantify it. We refer to it as AOMSW. The calculation of the AOMSW for a given parameter ε is divided into three major steps: first, calculate the order measure $O(t)$ of the snapshot on the lattice at each sampling time t, after some transient time t_{trans}; then average this order measure over a long time T to obtain the averaged order measure $O_r = \langle O(t) \rangle_t$ for this run; last, repeat these two steps and average O_r over 50 different runs under different initial conditions to acquire the AOMSW as follows:

$$\text{AOMSW} = \frac{1}{50} \sum_{r=1}^{50} O_r. \tag{2}$$

In our simulations, we choose $t_{\text{trans}} = 120$ cycles, and the measure time $T = 40$ cycles. However, in the first step, one encounters a difficulty in defining such a single measure $O(t)$ to characterize the degree of order for a snapshot at time t. From Fig.3, it is obviously found that the panels in the first row are more ordered than the corresponding panels in the second and third rows, and the spirals in the second row are more ordered than those in the third row correspondingly. One notes that each panel shows a total spiral with a free tip as seen from Figs.3(a)–(c). At the same time the excited units are connected except for several parts separated by the boundaries, while the fragmentized spirals in Figs.3(d)–(i) each contain at least two free wave tips, and the spirals break up into several independent excited pieces. So we can conclude that the more free wave tips and independent parts of the excited wave pieces the system contains, the more disordered the spiral will be. So the definition of $O(t)$ must obey the following items:

(1) $O(t)$ decreases with the number of the free wave tips N increasing.

(2) $O(t)$ decreases as the number of the independent excited wave pieces M increases.

(3) The fragmentized pieces correspond to the areas S_i $(i = 1, 2, \cdots, M)$. The larger the $\max(S_i)$ is, the less the $O(t)$ will be.

(4) $O(t) = 1$ for the whole spiral wave.

These considerations finally lead to the following definition of $O(t)$:

$$O(t) = \sum_{i=1}^{M} \left(\frac{S_i}{S_0} \right)^{\beta}, \tag{3}$$

where $S_0 = \sum_{i=1}^{M} S_i$ is the total area of the excited units, and β is in direct proportion to N. We choose $\beta = 1 + \dfrac{N-1}{\alpha}$, where α is a given integer. The wave

tip is a point defect. The variables of this point satisfy $\partial_t u = 0$. Here we consider the points that satisfy $u = 0.5$ and $\partial_t u = 0$ as wave tips. Now, in numerical simulation, we can calculate the number of the free wave tips N for a panel given by the distribution of u at each sampling time t. Then, for a given value u_e, if $u > u_e$, we think of the lattice point as being excited. Now we can count the number of the separately excited pieces M, and obtain the area S_i of the pieces by calculating the number of the jointly excited lattice points. In Fig.2(b), it is easy to find that there are several free wave tips on the boundary. But the spiral have not broken up yet. We do not count this kind of free wave tips. We take $\alpha = 10$ for the excitable medium and $\alpha = 5$ for the oscillatory medium. In Fig.3, the corresponding $O(t)$ for Figs.3(a)–(i) are 1, 1, 1, 0.8812, 0.7864, 0.7485, 0.5573, 0.3274, 0.2713. Obviously, the degree of order of the spiral wave is perfectly reflected by $O(t)$. As the time and ensemble average of $O(t)$, the AOMSW can reflect the property of the system.

4. Property of spiral wave with AOMSW

Now we use the definition of AOMSW to discuss the properties of the spiral wave. The AOMSW versus ε is plotted in Fig.6 with $b = 0.05$. The dashed line represents the no-flux boundary condition. Varying with ε, AOMSW is 1 when ε reaches 0.0684. Apparently, when $\varepsilon < 0.0684$, the spiral wave is stable without any fragmentation. When $\varepsilon > 0.0684$, AOMSW is close to zero, which shows that the system is in the spatiotemporally chaotic state, and is full of small wave fragments, while in the periodic boundary case (solid line in Fig.6(a)), we cannot find a distinct critical value for instability of spiral wave. Within the range $\varepsilon \in (0.065, 0.0664)$, AOMSW ≈ 1. This implies that the spiral wave breaks up only partially, and this breakup happens occasionally. Varying ε from 0.0664 to 0.0684, AOMSW becomes smaller gradually. At the same time, AOMSW fluctuates with ε. Several peaks of AOMSW occur during this process. These peaks correspond to relatively ordered spiral states. It means that the system encounters a weak boundary effect in the local range of ε. If $\varepsilon > 0.0684$, the status is the same as that in the no-flux boundary condition. The AOMSW is close to zero and the spiral wave breaks up due to the doppler effect instability. In Fig.6(b), the solid line shows the AOMSW varying with ε under the Dirichlet boundary condition. Comparing it with the curve of the periodic case, we find that relatively strong boundary effect appears when $\varepsilon > 0.0655$ and the AOMSW also fluctuates with the varying of ε. But the peaks are much steeper than those in the periodic boundary cases. And most of peak maxima are close to 1, even AOMSW = 1 at some points ($\varepsilon = 0.0666, 0.06692$, etc) and in some ranges ($\varepsilon \in (0.06638, 0.06652)$, $\varepsilon \in (0.06682, 0.06686)$, etc). When $\varepsilon > \varepsilon_0$, the AOMSW is close to zero, and the system comes to an absolute instability state too.

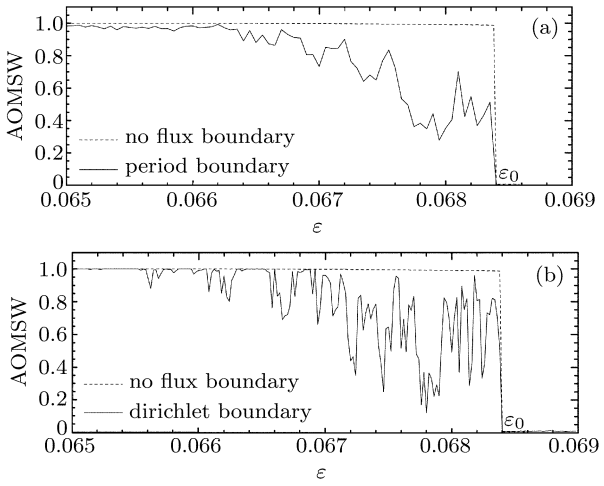


Fig.6. The order measure AOMSW as a function of ε at $b = 0.05$. (a) The dashed line corresponds to no-flux boundary condition and the solid line is for periodic boundary one. (b) The dashed line is also for no-flux boundary condition and the solid line for Dirichlet boundary condition.

In an oscillatory medium, Fig.5 shows that, if $\varepsilon > \varepsilon_1$, the system undergoes the Eckhaus instability, the critical values ε_1 are the same under different boundary conditions. So we only plot the AOMSW versus ε with $\varepsilon \in (0.0480, 0.0580)$, where the outer diameter of the spiral wave tip orbit is relatively large. Now the system contains some complex phenomena including Doppler-induced and boundary-induced instabilities. In Fig.7, the dashed line also corresponds to the no-flux boundary condition. Apparently, there is a narrow range of ε, where AOMSW< 0.2. It is the result of Doppler effect instability, and the spiral wave breaks up into pieces. Under the periodic boundary condition, there is a relatively wide range of ε ($\varepsilon \in (0.0544, 0.0555)$) where the AOMSW < 0.2. Out of this range, $0.6 <$ AOMSW < 1 and the AOMSW varies slowly. It is obvious that with this relatively

large value of AOMSW, the breakup of spiral wave only occurs partially near the boundary, while under the Dirichlet boundary condition, the breakup of spiral occurs within an even wide range of ε. When $\varepsilon < 0.048$, AOMSW= 1; when $\varepsilon > 0.048$, AOMSW begin to be less than 1. With the increasing of ε, the AOMSW decreases gradually. During this process the instability of spiral wave appears near the boundary and transfers to the core. Once $\varepsilon > 0.049$, the AOMSW decreases dramatically. Within $\varepsilon \in (0.05, 0.0555)$, the AOMSW$< 0.2$, and the system is in an absolute chaotic state. Varying ε from 0.0555 to 0.058, the curve is somehow similar to the corresponding part of periodic boundary one. Two peaks of AOMSW appear near the points $\varepsilon = 0.056$ and $\varepsilon = 0.0573$. It seems that it is more difficult for the breakup of spiral wave to happen at these values. This relatively stable effect in the local area is somehow correlated with the resonance between the spiral wave and inner-transferred waves from boundary. A simple explanation for the resonance and breakup of collided spiral waves may be found in Ref.[23], where resonant frequencies are always integral multiples of the intrinsic frequencies of the spiral waves. The problem becomes slightly complex in the present work. We know that ε determines the frequency of the spiral wave. Once the scale L of the system is fixed, ε will determine the difference in phase between the spiral waves and the inner-transferred waves. An appropriate phase difference can induce an instability of the wave. This instability effect will be reflected by the boundary and will cause an additional instability on the wave during the next collision. Then this instability effect will be amplified again and again through the sequential collisions, and induce the breakup of the spiral wave near the boundary finally.

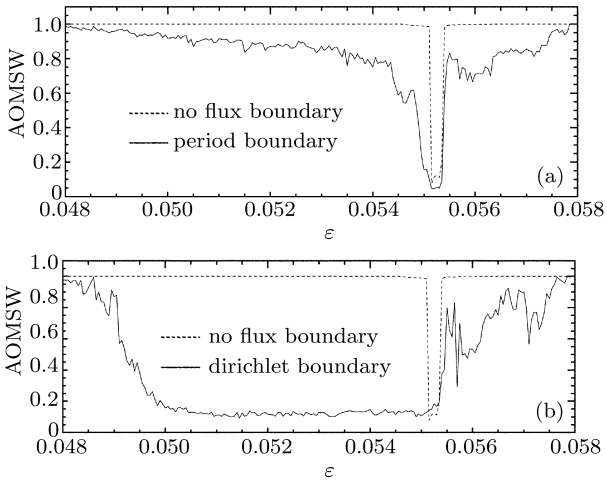


Fig.7. The order measure AOMSW vs ε with $b = -0.045$. The dash line corresponds to no-flux boundary condition in (a) and (b). The solid line represents periodic boundary condition in (a), and Dirichlet boundary condition in (b).

5. Conclusion and discussion

In this paper, we have studied the breakup of spiral wave under different boundary conditions. Using the 'order' measure AOMSW that we define for a spiral wave, we give the property of the system with the varying of ε. In an excitable medium ($b = 0.05$), the spiral wave can break up due to a Doppler effect when ε is larger than the critical value ε_0 under different boundary conditions. For the periodic boundary condition, the instability of the spiral wave occurs with $\varepsilon < \varepsilon_0$ because of collision between the spiral wave and the wave transferring inward from the boundary. This instability effect increases gradually with the increasing of ε. But there are several peaks in the curve of AOMSW vs ε, and these peaks correspond to relatively stable status of the system. For the Dirichlet boundary condition, the property of the system is similar to that of the periodic boundary case. But the system shows a more complex phenomenon. The system can still maintain perfect spiral waves (AOMSW ≈ 1) even when ε is very close to ε_0. In an oscillatory medium ($b = -0.045$), the property of the system is the same under different boundary conditions when ε is near the Eckhaus instability range. When the system encounters a Doppler effect instability, we give a narrow parameter range of ε for the breakup of spiral wave under the no-flux boundary condition, and a relatively wide range of ε under the periodic boundary condition, and an even much wider range of ε under the Dirichlet boundary condition. The instability of the wave in the first boundary condition is caused by a Doppler effect, and the last two conditions are caused mainly by the boundary effect. Comparing Fig.4 with Fig.6, and Fig.5 with Fig.7, we find that the breakup of spiral waves caused by boundary effects exhibit several common properties. (1) ε is near the range where the system encounters a Doppler effect instability. (2) The tip of the spiral wave meanders, and the outside diameter (R) of the spiral tip is relatively large. (3) System stability under the Dirichlet boundary condition is more sensitive to the outer diameter of spiral

tip orbit. (4) There always exist several narrow ranges of ε, where the spiral wave is inactive to the boundary effect instability. So the AOMSW is relatively large in the local area, which corresponds to the peaks in Fig.6 and Fig.7. This phenomenon correlates with resonance between the spiral wave and inner-transferred wave.

It is easy to find that the data shown here are robust. We have chosen smaller step sizes of spatial and temporal discretization of the partial differential equations. The basic property is similar to as indicated by the data here. In Fig.6 and Fig.7, the parameter ε is also robust. So the curve of AOMSW is not smooth. Considering computing speed, we have chosen these robust discrete steps.

Acknowledgment

We are grateful to Professor Li Jing-Hui and Dr Yuan Guo-Yong for valuable discussion.

References

[1] Winfree A T 1987 *When Time Breaks Down* (Princeton, NJ: Princeton University Press)
[2] Cross M C and Hobenberg P C 1993 *Rev. Mod. Phys.* **65** 851
Zhou T S and He X L 2002 *Chin. Phys.* **12** 1234
[3] Kapral R and Showalter K 1995 *Chemical Waves and Patterns* (Dordrecht: Kluwer)
[4] Park J S and Lee K J 1999 *Phys. Rev. Lett.* **83** 5393
[5] Ouyang Q, Swinney H L and Li G 2000 *Phys. Rev. Lett.* **84** 1047
[6] Ouyang Q and Flesselles J M 1996 *Nature* (London) **379** 143
[7] Zhou L Q and Ouyang Q 2000 *Phys. Rev. Lett.* **85** 1650
[8] Coullet P, Gil L and Lega J 1989 *Phys. Rev. Lett.* **62** 1619
[9] Courtemanche M and Winfree A T 1991 *Int. J. Bifurcation Chaos Appl. Sci. Eng.* **1** 431
[10] Panfilov A V and Hogeweg P 1993 *Phys. Rev.* A **176** 295
Shao X, Ren Y and Ouyang Q 2005 *Chin. Phys.* **15** 0513
[11] Bär M and Eiswirth M 1993 *Phys. Rev.* E **48** R1635
[12] Karma A 1994 *Chaos* **4** 461
[13] Marée A F M and Panfilov A V 1997 *Phys. Rev. Lett.* **78** 1819
[14] Zykov V S, Mikhailov A S and Müller S C 1998 *Phys. Rev. Lett.* **81** 2811
[15] Tobias S M and Knobloch E 1998 *Phys. Rev. Lett.* **80** 4811
[16] Qu Z, Xie F, Garfinkel A and Weiss J N 2000 *Ann. Biomed. Eng.* **28** 755
[17] Barkley D 1994 *Phys. Rev. Lett.* **72** 164
[18] Bär M, Hildebrand M, Eiswirth M, Falcke M, Engle H and Neufeld 1994 *Chaos* **4** 499
[19] Bär M and Or-Guil M 1999 *Phys. Rev. Lett.* **82** 1160
[20] Yang J Z, Xie F G, Qu Z L and Garfinkel A 2003 *Phys. Rev. Lett.* **91** 148302
[21] Noble D R, Chen S, Georgiadis J G and Buckius R O 1995 *Physics of Fluids* **7** 203
Morse P M and Feshbach H 1953 *Methods of Theoretical Physics*, Part I (New York: McGraw-Hill) p679
[22] Bär M, Brusch L and Michal O G 2004 *Phys. Rev. Lett.* **92** 119801
[23] Yan G Y, Sun P, Zhang G C, Wang G R and Chen S G 2005 *Euro. Phys. Lett.* **72** 908
Yan G Y, Zhang G C, Wang G R, Chen S G and Sun P 2005 *Chin. Phys. Lett.* **22** 291