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Internal mode of incoherent photovoltaic vector solitons*

Zhang Bing-Zhi(张冰志), Wang Hong-Cheng(王红成), and She Wei-Long(佘卫龙)†

State Key Laboratory of Optoelectronic Materials and Technologies, Sun Yat-sen University, Guangzhou 510275, *China*

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The internal modes of incoherent vector solitons (IVSs) in photovoltaic photorefractive materials are investigated in the framework of coupled nonlinear Schrödinger equations. It is found that there is a pair of internal modes corresponding to a bright–bright IVS. The propagation dynamics of the bright-bright IVS perturbed by the internal modes is simulated by numerical method.

Keywords: incoherent photovoltaic vector soliton, internal mode
PACC: 4265S, 4265J

1. Introduction

Optical spatial solitons (OSSs) have been the object of intensive theoretical and experimental researches during the last four decades.[1] When the confining effect of the refractive index exactly compensates for the effect of diffraction, the beam becomes self-trapped and a spatial soliton forms. Thus far, a lot of nonlinear effects have been found to be able to support OSSs, for example, cubic (Kerr) nonlinearity effect,[1] cubic-quintic competing nonlinearity,[2] photorefractive effect (supporting quasi-steady-state soliton, screening soliton, photovoltaic soliton, screening-photovoltaic soliton),[3−9] and photoisomerization nonlinearity effect,[10] etc. In all these kinds of OSSs, those forming in photorefractive materials and polymers are particularly interesting, because they can be formed in the optical power level of mW/cm^2.

The propagation dynamics of OSS can be described by the following generalized nonlinear Schrödinger equation (NLSE) in the paraxial approximation and the slowly varying amplitude approximation:[1]

$$\mathrm{i}2k_0n_0\frac{\partial E}{\partial z}+\nabla_\perp^2E+2n_0k_0^2\Delta n(|E|^2)E=0, \quad (1)$$

where $\Delta n(|E|^2)$ is a real function describing the photo-induced index perturbation, k_0 is the wave number of light in vacuum, and n_0 is the linear refractive index of the medium. The function E describes a complex envelope of the optical field. If the light beam is (1+1)D, then $\nabla_\perp^2=\dfrac{\partial^2}{\partial x^2}$; if it is (2+1)D, then $\nabla_\perp^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$. As is well known, the NLSE that supports a (1+1)D Kerr soliton is integrable.[11] However, a lot of OSSs require the non-integrable models of non-Kerr nonlinearities. It is generally believed that inelastic interaction between solitons due to emission of radiation is the major property, which differs from the nonlinear wave in integrable and non-integrable models. In a non-integrable model, a small perturbation may create an internal oscillation of a soliton.[12] Such oscillations displaying long-lived oscillations of the soliton amplitude are called internal modes. The existence of internal modes of solitons has been demonstrated for Klein–Gordon type models,[13] sine-Gordon solitons,[14] fundamental solitons in saturable nonlinear media,[15] and bright vortex solitons in cubic–quintic nonlinear media.[16] The internal modes play an important role in the dynamic evolution of solitons. If the internal modes exist, OSS can temporarily store some of the translational energy in these internal oscillations during collision and retrieve this energy after the collision. To the best of our knowledge, the internal modes of the incoherent photovoltaic vector solitons in photorefractive materials have not been reported.

The concept of vector solitons (multi-component solitons) was introduced by Christodoulides *et al*[17] for (1+1)D incoherently interacting optical beams. A

*Project supported by the National Natural Science Foundation of China (Grant No 10574167).
†Corresponding author. E-mail: stils02@zsu.edu.cn

single component OSS is a fundamental mode of the waveguide created in a nonlinearity medium, and a vector soliton can be taken as the higher-order mode of the self-induced waveguide. Recently, the concept of vector OSS has been extended to (2+1)D. In the (2+1)D case, the mutual trapping of incoherent optical beams leads to many novel OSSs, such as vortex and dipole vector solitons.[18,19] The linear stability of vector solitons is extremely important in soliton physics and has been investigated by the variational approach and numerical simulation.[20] Our aim in this paper is to investigate the internal modes of (1+1)D bright–bright incoherent vector solitons (IVSs) in photovoltaic photorefractive materials by the numerical method. We find that the (1+1)D bright–bright IVSs in photovoltaic photorefractive materials are linearly stable.

2. Theory

2.1. Incoherent photovoltaic vector solitons

Here we consider two incoherently interacting signal beams propagating along the direction z in a bulk photovoltaic photorefractive material. The direction x is the ferroelectric c axis and the beams are allowed to diffract only along this direction. Considering the effect of background illumination and neglecting the diffusion effect,[21,22] we can gain the expression of the space-charge field $E_{\rm sc}$ under open-circuit, based on the band-transport model developed by Kukhtarev *et al*[23] as follows

$$E_{\rm sc}(x) = -E_{\rm p}\frac{1+r_1|u_1|^2+r_2|u_2|^2}{1+|u_1|^2+|u_2|^2}. \tag{2}$$

The index change is directly proportional to the space-charge field via the Pockels effect, which is

$$\Delta n_j(x) = \frac{1}{2}n_j^3 r_{\rm eff}E_{\rm sc}(x), \quad (j=1,2), \tag{3}$$

where $E_{\rm p} = \kappa_b^{\rm o}\gamma N_{\rm A}/q\mu$, $r_j = \kappa_j^{\rm e}/\kappa_{\rm b}^{\rm o}$, $|u_j|^2 = s_jI_j/s_{\rm b}I_{\rm b}$; n_j is the linear index of the signal beam j; $\kappa_j^{\rm e}$ and $\kappa_{\rm b}^{\rm o}$ are the Glass coefficients of the signal and background beams, respectively; γ is the recombination rate coefficient; $N_{\rm A}$ is the number density of the negatively charged acceptors that compensate for the ionized donors; q is the charge on the carrier; μ is the mobility; I_j and $I_{\rm b}$ are the intensities of the signal and background beams, respectively; s_j and $s_{\rm b}$ are the photoionization cross sections of the signal and background beams, respectively.

Substituting Eqs.(2) and (3) into Eq.(1), we obtain the coupled NLSEs for the IVS in photovoltaic photorefractive materials

$$\begin{aligned}\frac{\partial u_1}{\partial\zeta} &= \frac{\rm i}{L_1}\left(\frac{1}{2}\frac{\partial^2 u_1}{\partial\xi^2}+b_1\frac{1+r_1|u_1|^2+r_2|u_2|^2}{1+|u_1|^2+|u_2|^2}u_1\right),\\ \frac{\partial u_2}{\partial\zeta} &= \frac{\rm i}{L_2}\left(\frac{1}{2}\frac{\partial^2 u_2}{\partial\xi^2}+b_2\frac{1+r_1|u_1|^2+r_2|u_2|^2}{1+|u_1|^2+|u_2|^2}u_2\right),\end{aligned} \tag{4}$$

where $\zeta = z/z_0$, $\xi = x/x_0$, $L_j = k_jx_0^2/z_0$, $b_j = \frac{1}{2}k_j^2x_0^2n_j^2\gamma_{\rm eff}E_{\rm p}$, x_0 and z_0 are arbitrary spatial scales, and k_j is the vacuum wave number of the jth signal beam.

We now look for the bright–bright vector soliton solution of the following form

$$\begin{aligned}u_1 &= f(\xi)\exp({\rm i}\Gamma_1\zeta),\\ u_2 &= g(\xi)\exp({\rm i}\Gamma_2\zeta),\end{aligned} \tag{5}$$

where Γ_1 and Γ_2 are two soliton propagation constants. Then the functions $f(\xi)$ and $g(\xi)$ satisfy the following ordinary differential equations

$$\begin{aligned}&\frac{{\rm d}^2 f}{{\rm d}\xi^2}+2b_1\frac{1+r_1f^2+r_2g^2}{1+f^2+g^2}f-2L_1\Gamma_1 f=0,\\ &\frac{{\rm d}^2 g}{{\rm d}\xi^2}+2b_2\frac{1+r_1f^2+r_2g^2}{1+f^2+g^2}g-2L_2\Gamma_2 g=0,\end{aligned} \tag{6}$$

The boundary condition is

$$f(\pm\infty)=g(\pm\infty)=0. \tag{7}$$

Generally, there are infinite sets of solutions for Eq.(6) and Eq.(7), which represent multi-humped vector solitons. However, we consider only the single-humped vector solitons because of the evidence that only they are stable.[24] Because their intensity profiles are symmetric about x axis, another limiting condition for single-humped vector solitons is required, which is

$$f'(0)=g'(0)=0. \tag{8}$$

Then, we can compute the eigenvalues (Γ_1,Γ_2) with the shooting method. And for a pair of fixed (Γ_1,Γ_2), we can obtain the envelope amplitude of the optical fields.

As an example, we consider the IVS in a Cu:KNSBN crystal. We chose two linearly polarized signal beams with different wavelengths in vacuum, which are $\lambda_1 = 488\,{\rm nm}$ and $\lambda_2 = 514.5\,{\rm nm}$. The signal beams are extraordinary ray (e ray) and are narrow in the direction parallel to the c axis. The uniform background beam is an ordinary ray (o ray)

with a vacuum wavelength $\lambda_b = 488\,\mathrm{nm}$. The parameters of the Cu:KNSBN crystal are $n_1^{\mathrm{e}} = 2.25$, $n_1^{\mathrm{o}} = 2.33$, $n_2^{\mathrm{e}} = 2.23$, $n_2^{\mathrm{o}} = 2.31$, $\gamma_{\mathrm{eff}} = 120\,\mathrm{pm/V}$, $E_{\mathrm{p}} = 20\,\mathrm{kV/cm}$, $r_1 = 2.5$, $r_2 = 1.6$.[25] If choosing the spatial scales $z_0 = 1.578\,\mathrm{mm}$, $x_0 = 11.362\,\mu\mathrm{m}$ we will have $b_1 = 65.817$, $b_2 = 57.135$, $L_1 = 2.370$, $L_2 = 2.228$.

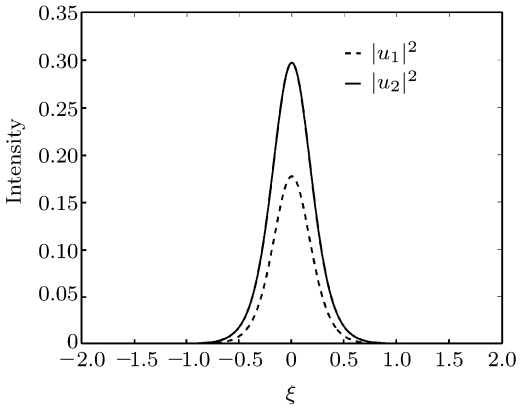


Fig.1. The intensity profiles of the vector solitons for $\Gamma_1 = 32.56$ and $\Gamma_2 = 29.89$.

The eigenvalues (Γ_1, Γ_2) can now be calculated from Eq.(6) by the shooting method and therefore infinite sets of vector soliton solutions can be obtained. Figure 1 shows the intensity profiles of the signal beams for $\Gamma_1 = 32.56$ and $\Gamma_2 = 29.89$. The intensity amplitudes of the two signal beams at the centre are $f^2(0) = 0.177$ and $g^2(0) = 0.297$, respectively.

2.2. Internal modes of bright-bright incoherent photovoltaic vector solitons

Since single-humped vector solitons have been known to be stable, we now investigate their internal modes. To determine these internal modes, we write the perturbed soliton solutions as

$$\begin{aligned} u_1 &= [f(\xi) + \phi_+(\xi)\exp(\mathrm{i}\sigma\zeta) + \phi_-^*(\xi)\exp(-\mathrm{i}\sigma^*\zeta)]\exp(\mathrm{i}\Gamma_1\zeta), \\ u_2 &= [g(\xi) + \Psi_+(\xi)\exp(\mathrm{i}\sigma\zeta) + \Psi_-^*(\xi)\exp(-\mathrm{i}\sigma^*\zeta)]\exp(\mathrm{i}\Gamma_2\zeta), \end{aligned} \tag{9}$$

where $f(\xi)$ and $g(\xi)$ are the envelope amplitudes of the optical fields without perturbation, $(\phi_+, \phi_-, \Psi_+, \Psi_-)$ are small perturbations, σ is the eigenvalue, and the asterisk represents complex conjugation. Substituting Eq.(9) into Eq.(4) and dropping higher-order terms in $(\phi_+, \phi_-, \Psi_+, \Psi_-)$, we can obtain the following linearized equations:

$$\begin{aligned} &\phi_+'' + 2L_1(-\sigma - \Gamma_1)\phi_+ + 2b_1V\phi_+ + 2b_1V_{11}(\phi_+ + \phi_-) + 2b_1V_{12}(\Psi_+ + \Psi_-) = 0, \\ &\phi_-'' - 2L_1(-\sigma + \Gamma_1)\phi_- + 2b_1V\phi_- + 2b_1V_{11}(\phi_+ + \phi_-) + 2b_1V_{12}(\Psi_+ + \Psi_-) = 0, \\ &\Psi_+'' + 2L_2(-\sigma - \Gamma_2)\Psi_+ + 2b_2V\Psi_+ + 2b_2V_{22}(\Psi_+ + \Psi_-) + 2b_2V_{21}(\phi_+ + \phi_-) = 0, \\ &\Psi_-'' - 2L_2(-\sigma + \Gamma_2)\Psi_- + 2b_2V\Psi_- + 2b_2V_{22}(\Psi_+ + \Psi_-) + 2b_2V_{21}(\phi_+ + \phi_-) = 0, \end{aligned} \tag{10}$$

where

$$\begin{aligned} V &= \frac{1 + r_1f^2 + r_2g^2}{1 + f^2 + g^2}, \\ V_{11} &= \frac{(r_1 - 1)f^2}{(1 + f^2 + g^2)^2},\ V_{22} = \frac{(r_2 - 1)g^2}{(1 + f^2 + g^2)^2}, \\ V_{12} &= \frac{(r_2 - 1)fg}{(1 + f^2 + g^2)^2},\ V_{21} = \frac{(r_1 - 1)fg}{(1 + f^2 + g^2)^2}. \end{aligned} \tag{11}$$

If the eigenvalue σ is real, the vector soliton is linearly stable with respect to internal mode. In contrast, if σ is purely imaginary, the vector soliton is linearly unstable. We note that if $(\sigma, \phi_+, \phi_-, \Psi_+, \Psi_-)$ is a solution of Eq.(10), so is $(-\sigma, \phi_-, \phi_+, \Psi_-, \Psi_+)$ when σ is real. Therefore, the internal modes always come in pairs.

According to the boundary conditions of single-humped solitons, we can obtain the asymptotic solutions of Eq.(10) at infinity

$$\begin{aligned} \phi_+(\xi \to +\infty) &= c_1\exp(-\sqrt{2L_1(\Gamma_1 + \sigma) - 2b_1}\xi), \\ \phi_+(\xi \to -\infty) &= c_2\exp(\sqrt{2L_1(\Gamma_1 + \sigma) - 2b_1}\xi), \end{aligned} \tag{12}$$

where c_1 and c_2 are two constants. We can also obtain the similar asymptotic expression of other perturbation function at infinity.

Our task is to find the eigenvalues of Eq.(10). First, we discretize the Eq.(10) at finite intervals, and then use finite difference method to solve Eq.(10) with

two-point Dirichlet boundary conditions ($\phi_+(\xi \to \pm\infty) = 0$). Although there are a lot of discrete eigenvalues, only a pair of them satisfies the asymptotic behaviour of Eq.(10) at infinity, which is $\sigma = \pm 0.392$. The corresponding eigenvectors ($\phi_+, \phi_-, \Psi_+, \Psi_-$), which correspond to the amplitude perturbation functions, are the internal modes of the incoherent photovoltaic vector soliton. The amplitude perturbation functions are shown in Fig.2. Other 'eigenvalues' cannot satisfy Eq.(12): e.g., $\sigma = \pm 4.382$. Its corresponding eigenvectors are plotted in Fig.3. Obviously, their asymptotic behaviour at infinity cannot satisfy Eq.(12), and therefore, they are not the internal modes of the vector soliton.

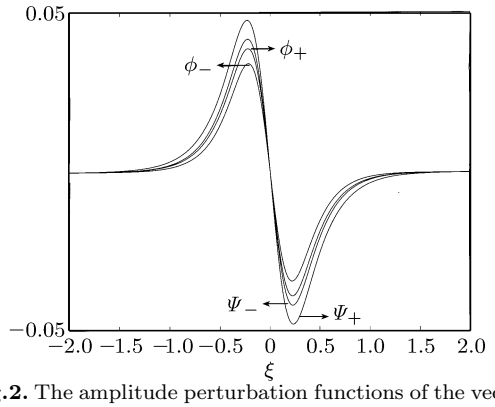


Fig.2. The amplitude perturbation functions of the vector soliton with respect to $\sigma = 0.392$.

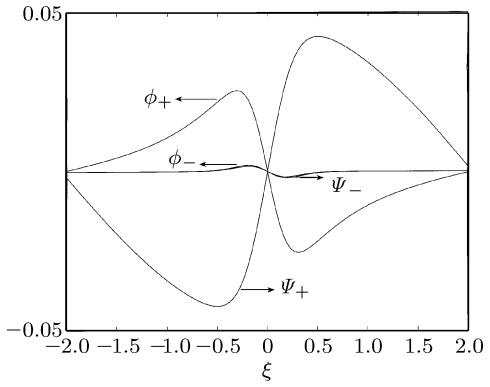


Fig.3. The amplitude perturbation functions corresponding to the 'eigenvalue' $\sigma = 4.382$. It cannot satisfy Eq.(12).

3. Nonlinear evolution of perturbed vector soliton

In the following, we study numerically the dynamics of the IVS under the perturbation of the internal modes. Firstly, we simulate the propagation of the IVS without perturbation. The evolution of IVS is plotted in Fig.4. One can see that each component of the IVS does not change their amplitude shape during the propagation.

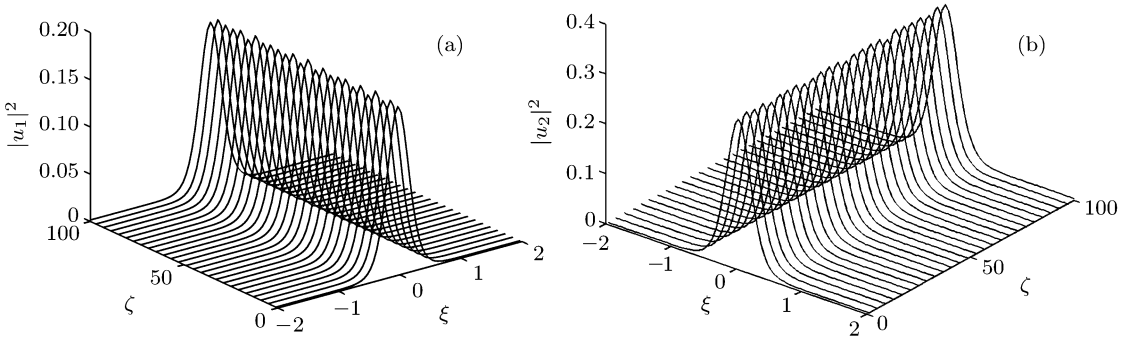


Fig.4. Dynamical evolution of an incoherent vector soliton without perturbation. (a) The normalized intensity of u_1 component; (b) the normalized intensity of u_2 component.

Next, we add the perturbation of the internal mode to the IVS, and the initial condition can be written as

$$\begin{aligned} u_1 &= [f(\xi) + \phi_+(\xi) + \phi_-^*(\xi)] \exp(\mathrm{i}\Gamma_1\zeta), \\ u_2 &= [g(\xi) + \Psi_+(\xi) + \Psi_-^*(\xi)] \exp(\mathrm{i}\Gamma_2\zeta). \end{aligned} \tag{13}$$

When the IVS is perturbed by the amplitude perturbation functions ($\phi_+, \phi_-, \Psi_+, \Psi_-$) for $\sigma = 0.392$ (see Fig.2), the evolution of them behaves like that shown in Fig.5. As mentioned above, the same result will be obtained when the IVS is perturbed by the amplitude perturbation functions for $\sigma = -0.392$. We note that the amplitudes of IVSs under the perturbation of the internal modes show the characteristics of quasi-periodical oscillation.

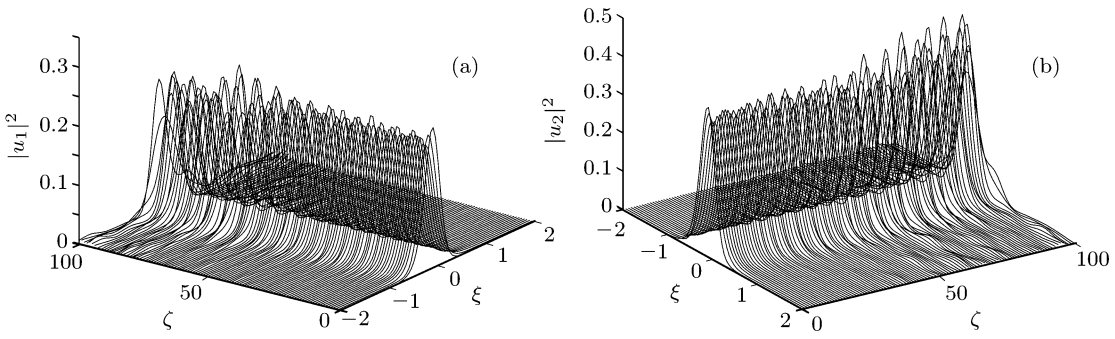


Fig.5. Evolution of the incoherent vector soliton under the perturbation of the internal mode for $\sigma = 0.392$.

4. Conclusion

In conclusion, we have obtained the IVSs in the framework of coupled NLSE and investigated the internal modes of the IVSs in photovoltaic photorefractive materials. We have found that there is a pair of internal modes corresponding to fixed propagation constants Γ_1 and Γ_2 of IVS. Numerical results show that the IVSs in a Cu: KNSBN crystal present quasi-periodical oscillation characteristics, under the perturbation of the internal mode.

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