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Entanglement swapping between atom and cavity and generation of entangled state of cavity fields*

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This paper proposes a scheme where entanglement swapping between atom and cavity can be realized. Λ-type three-level atoms interacting resonantly with cavity field are considered. By detecting atom and cavity field, it realizes entanglement swapping between atom and cavity. It uses the technique of entanglement swapping to generate an entangled state of two cavity fields by measuring on atoms. It discusses the experimental feasibility of the proposed scheme and application of entangled state of cavity fields.

Keywords: entanglement swapping, cavity QED, entangled state
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Recently, much attention has been paid to entanglement. It is well known that the study of entanglement not only helps to deeply discuss the fundamental theory of quantum mechanics, but also extends its potential application in quantum information processing. An important entangled state of two qubits is the Bell state or EPR state. For the Bell state, we know that there is strong correlation between two particles. A measurement of the first qubit can give information about the second qubit. That is, the measurement outcomes are correlated. The measurement correlations in the Bell state are stronger than those which could ever exist between classical systems. Due to this striking feature, quantum entanglement has been the key ingredient in quantum information processing. Entangled states, being important resources in the field of quantum information, have been extensively used, such as for quantum teleportation,[1] quantum computation,[2] quantum communication[3,4] and remote state preparation.[5] Recently, many schemes for the generation of an entangled state have been proposed.[6−8]

Entanglement swapping is a striking phenomenon in the field of study of entanglement. Taking a four-particle system as an example, we suppose that particles 1 and 2 are in the Bell state, and particles 3 and 4 are also in a two-particle maximally entangled state. Perform a Bell state measurement on particles 2 and 3. As a result, the measurement of qubits 2 and 3 makes particles 1 and 4 be in the maximally entangled state. This phenomenon is called entanglement swapping, because there is not strong correlation between particles 1 and 4 before Bell state measurement. Recently, much attention has been paid to entanglement swapping.[9] By using entanglement swapping, some schemes about quantum teleportation and quantum secrecy have been proposed.[10]

Along with the development of quantum information techniques, all kinds of physical systems[11−14] have been proposed for quantum information processing. The cavity QED system is a qualified candidate.[15−17] In this paper, we use the cavity QED techniques[18−20] to finish information swapping between atom and cavity field. We investigate two separate cavity field which are σ^- polarized. Let two Λ-type three-level atoms interact resonantly with the cavity. The cavity 1 and atom 1 and the cavity 2 and atom 2 are in an entangled state respectively. Appropriate joint detection on atom 2 and cavity 1, atom 1 and cavity 2 will be in the maximally entangled state. Entanglement swapping between atom and cavity is successfully realized. Furthermore, we can generate an entangled state of two cavity field by the technique of entanglement swapping. This kind of technique of information swapping provides potential application for quantum information processing.

Firstly, we study entanglement swapping between atom and cavity. Consider two identical Λ-type three-level atoms having one excited $|e\rangle$ and two less excited states $|g^+\rangle$ and $|g^-\rangle$ with different magnetic

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quantum numbers[21] and two separate single-mode cavity fields. We assume that the two cavity fields are σ^- polarized so that transition between atom $|e\rangle$ and $|g^-\rangle$ only occurs when the atom interacts with the cavity. That is, σ^- polarized field does not affect the atomic state $|g^+\rangle$. Suppose that the atom interacts resonantly with the cavity field. Then, in the interaction picture, the atom–cavity interaction is described by the Jaynes–Cummings Hamiltonian $H_{\rm I} = g(a^+S^- + a^-S^+)$, where a^+ and a^- are the creation and annihilation operators for the cavity mode respectively, g is the atom–cavity coupling strength and $S^+ = |e\rangle\langle g^-|$ and $S^- = |g^-\rangle\langle e|$.

Assuming that atom 1 initially prepared in the excited state $|e\rangle_1$, we send this atom through the cavity 1, initially in the vacuum state $|0\rangle_{\rm c1}$. Atom 1 is made resonant with the cavity and undergoes a $\pi/2$ pulse. The state of atom 1 and cavity is given by

$$|\Psi\rangle_{1,c1} = \frac{1}{\sqrt{2}}(|e\rangle_1|0\rangle_{\rm c1} + |g^-\rangle_1|1\rangle_{\rm c1}). \tag{1}$$

Atom 1 and cavity 1 are in the entangled state $|\Psi\rangle_{1,\rm c1}$. At the same time, let atom 2 with the excited state $|e\rangle_2$ cross the cavity 2. Undergoing the same evolution as atom 1, cavity 2 plus atom 2 end up in the entangled state

$$|\Psi\rangle_{2,c2} = \frac{1}{\sqrt{2}}(|e\rangle_2|0\rangle_{\rm c2} + |g^-\rangle_2|1\rangle_{\rm c2}). \tag{2}$$

The state of two atomic system and two cavity fields can be expressed as

$$|\Psi\rangle_{1,2,c1,c2} = \frac{1}{2}(|e\rangle_1|0\rangle_{\rm c1} + |g^-\rangle_1|1\rangle_{\rm c1}) \times (|e\rangle_2|0\rangle_{\rm c2} + |g^-\rangle_2|1\rangle_{\rm c2}). \tag{3}$$

Rewrite the state of the whole system $|\Phi\rangle_{1,2,c1,c2}$

$$\begin{aligned}|\Psi\rangle_{1,2,c1,c2} = {}& \frac{1}{2}|\Psi^+\rangle_{2,c1}\Big(\frac{1}{\sqrt{2}}|e\rangle_1|0\rangle_{\rm c2}+\frac{1}{\sqrt{2}}|g^-\rangle_1|1\rangle_{\rm c2}\Big)\\ &+\frac{1}{2}|\Psi^-\rangle_{2,c1}\Big(\frac{1}{\sqrt{2}}|e\rangle_1|0\rangle_{\rm c2}-\frac{1}{\sqrt{2}}|g^-\rangle_1|1\rangle_{\rm c2}\Big)\\ &+\frac{1}{2}|\Phi^+\rangle_{2,c1}\Big(\frac{1}{\sqrt{2}}|e\rangle_1|1\rangle_{\rm c2}+\frac{1}{\sqrt{2}}|g^-\rangle_1|0\rangle_{\rm c2}\Big)\\ &+\frac{1}{2}|\Phi^-\rangle_{2c1}\Big(\frac{1}{\sqrt{2}}|e\rangle_1|1\rangle_{\rm c2}-\frac{1}{\sqrt{2}}|g^-\rangle_1|0\rangle_{\rm c2}\Big),\end{aligned} \tag{4}$$

where $|\Psi^\pm\rangle_{2,\rm c1}$ and $|\Phi^\pm\rangle_{2,\rm c1}$ are the Bell states

$$\begin{aligned}|\Phi^\pm\rangle_{2,c1} &= \frac{1}{\sqrt{2}}(|e\rangle_2|1\rangle_{\rm c1} \pm |g^-\rangle_2|0\rangle_{\rm c1})\,,\\ |\Psi^\pm\rangle_{2,c1} &= \frac{1}{\sqrt{2}}(|e\rangle_2|0\rangle_{\rm c1} \pm |g^-\rangle_2|1\rangle_{\rm c1})\,.\end{aligned} \tag{5}$$

From Eq.(4), taking joint measurement on atom 2 and cavity 1 by a Bell state, we find that atom 1 and cavity 2 are automatically projected into a maximally entangled state. Entanglement swapping is successfully realized. The next work is how to detect atom 2 and cavity 1. To simplify joint measurement, we let atom 2 undergo the transformations listed in Table 1.

Table 1. The process of evolution of atom 2 from left to right, where FT denotes field type, TR denotes transformation, and CF means classical field.

FT	σ^- CF	σ^+ CF	σ^- cavity 1	σ^+ CF	σ^- CF
TR	$\|e\rangle \to \frac{1}{\sqrt{2}}(\|e\rangle + \|g^-\rangle)$ $\|g^-\rangle \to \frac{1}{\sqrt{2}}(\|g^-\rangle - \|e\rangle)$	$\|e\rangle \to \|g^+\rangle$	$\|g^-\rangle\|1\rangle \to$ $-\|g^-\rangle\|1\rangle$	$\|g^+\rangle \to$ $\|e\rangle$	$\|e\rangle \to \frac{1}{\sqrt{2}}(\|e\rangle + \|g^-\rangle)$ $\|g^-\rangle \to \frac{1}{\sqrt{2}}(\|g^-\rangle - \|e\rangle)$

Atom 2 and cavity 1 become

$$|\Psi^\pm\rangle_{2,c1} \to |g^-\rangle_2\Big(\frac{1}{\sqrt{2}}|0\rangle_{\rm c1} \mp \frac{1}{\sqrt{2}}|1\rangle_{\rm c1}\Big), \qquad |\Phi^\pm\rangle_{2,c1} \to |e\rangle_2\Big(\frac{1}{\sqrt{2}}|1\rangle_{\rm c1} \pm \frac{1}{\sqrt{2}}|0\rangle_{\rm c1}\Big). \tag{6}$$

Now, atom 3 with the atomic state $|g^-\rangle_3$ is introduced. Let atom 3 undergo the following evolution in turn:

$$\text{atom 3} \xrightarrow{\sigma^-\ \text{c1}} \begin{matrix}|g^-\rangle|0\rangle \to |g^-\rangle|0\rangle\\ |g^-\rangle|1\rangle \to -|e\rangle|0\rangle\end{matrix} \xrightarrow{\sigma^-\ \text{CF}} \begin{matrix}|e\rangle \to \frac{1}{\sqrt{2}}(|e\rangle + |g^-\rangle),\\ |g^-\rangle \to \frac{1}{\sqrt{2}}(|g^-\rangle - |e\rangle).\end{matrix}$$

In the end, the state of the cavity 1 and atoms 2 and 3 becomes

$$\begin{aligned}&|\Psi^+\rangle_{2,c1}|g^-\rangle_3 \to |g^-\rangle_2|g^-\rangle_3|0\rangle_{\rm c1}, \quad |\Psi^-\rangle_{2,c1}|g^-\rangle_3 \to -|g^-\rangle_2|e\rangle_3|0\rangle_{\rm c1},\\ &|\Phi^+\rangle_{2,c1}|g^-\rangle_3 \to -|e\rangle_2|e\rangle_3|0\rangle_{\rm c1}, \qquad |\Phi^-\rangle_{2,c1}|g^-\rangle_3 \to -|e\rangle_2|g^-\rangle_3|0\rangle_{\rm c1}.\end{aligned} \tag{7}$$

Hence, detecting atoms 2 and 3 through state selective field ionization detectors, we can finish the joint measurement. With the help of measurement outcomes, we can get an entangled state of cavity 2 and atom 1, because there is not strong correlation between atom 1 and cavity 2 before measurement. So we realize entanglement swapping between atom and cavity field. The successful probability of this process is one.

Next, we use this kind of technique of entanglement swapping to generate entangled state of cavity fields. Two identical cascade-type three-level atoms whose states are denoted by $|g\rangle$, $|e\rangle$ and $|i\rangle$ and two single-mode cavity fields are considered. At the initial time, atom 1, prepared in the state $|e\rangle_1$, is sent across cavity 1, in the vacuum state $|0\rangle_{c1}$. The transition between the states $|g\rangle_1$ and $|e\rangle_1$ is resonant with the cavity field, and the transition frequency between the states $|i\rangle_1$ and $|e\rangle_1$ is highly detuned from cavity frequency and thus the state $|i\rangle_1$ is not affected when atom 1 interacts with the cavity. Here, atom 1 undergoes a $\pi/2$ pulse. The state of atom 1 and cavity 1 can be expressed as

$$|\Psi\rangle_{1,c1} = \frac{1}{\sqrt{2}}(|e\rangle_1|0\rangle_{c1} + |g\rangle_1|1\rangle_{c1}). \tag{8}$$

Using the similar method to atom 1, we find that atom 2 and cavity 2 are in the state

$$|\Psi\rangle_{2,c2} = \frac{1}{\sqrt{2}}(|e\rangle_2|0\rangle_{c2} + |g\rangle_2|1\rangle_{c2}). \tag{9}$$

The state of the whole system can be expressed as

$$\begin{aligned}|\Psi\rangle_{1,2,c1,c2} = {} & \frac{1}{2}(|e\rangle_1|0\rangle_{c1} + |g\rangle_1|1\rangle_{c1})(|e\rangle_2|0\rangle_{c2} + |g\rangle_2|1\rangle_{c2}) = \frac{1}{2}|\Psi^+\rangle_{1,2}\Big(\frac{1}{\sqrt{2}}|0\rangle_{c1}|0\rangle_{c2} + \frac{1}{\sqrt{2}}|1\rangle_{c1}|1\rangle_{c2}\Big) \\ & + \frac{1}{2}|\Psi^-\rangle_{1,2}(\frac{1}{\sqrt{2}}|0\rangle_{c1}|0\rangle_{c2} - \frac{1}{\sqrt{2}}|1\rangle_{c1}|1\rangle_{c2}) + \frac{1}{2}|\Phi^+\rangle_{1,2}\Big(\frac{1}{\sqrt{2}}|1\rangle_{c1}|0\rangle_{c2} + \frac{1}{\sqrt{2}}|0\rangle_{c1}|1\rangle_{c2}\Big) \\ & + \frac{1}{2}|\Phi^-\rangle_{1,2}\Big(\frac{1}{\sqrt{2}}|1\rangle_{c1}|0\rangle_{c2} - \frac{1}{\sqrt{2}}|0\rangle_{c1}|1\rangle_{c2}\Big),\end{aligned} \tag{10}$$

where

$$|\Psi^\pm\rangle_{1,2} = \frac{1}{\sqrt{2}}(|e\rangle_1|e\rangle_2 \pm |g\rangle_1|g\rangle_2), \quad |\Phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}(|g\rangle_1|e\rangle_2 \pm |e\rangle_1|g\rangle_2). \tag{11}$$

From Eq.(10), taking joint measurement on atoms 1 and 2 by a Bell state, we find that cavity fields 1 and 2 are automatically projected into an entangled state of Fock state. In order to perform joint measurement, we take atom 1 as the control qubit and atom 2 as the target qubit, and perform a controlled-not (CNOT) operation,[22] subsequently perform a Hadamard (H) transformation on atom 1. Now, we show the process of the performance of CNOT and H gates. Let atoms 1 and 2 undergo some transformations listed in Table 2.

Table 2. The process of performing CNOT operation on atoms 1 and 2, where FT denotes field type, TR denotes transformation, and CF means classical field. In cavity, two atoms simultaneously enter the cavity.

	FT	CF1	CF2	Cavity	CF3	CF4
atom1	TR			$\|g\rangle_1\|i\rangle_2 \to \|g\rangle_1\|i\rangle_2$ $\|g\rangle_1\|g\rangle_2 \to \|g\rangle_1\|g\rangle_2$		
atom 2	TR	$\|e\rangle \to \frac{1}{\sqrt{2}}(\|e\rangle + \|g\rangle)$ $\|g\rangle \to \frac{1}{\sqrt{2}}(\|g\rangle - \|e\rangle)$	$\|e\rangle \to \|i\rangle$	$\|e\rangle_1\|i\rangle_2 \to -\|e\rangle_1\|i\rangle_2$ $\|e\rangle_1\|g\rangle_2 \to \|e\rangle_1\|e\rangle_2$	$\|i\rangle \to \|e\rangle$ $\|g\rangle \to \|g\rangle$	$\|e\rangle \to \frac{1}{\sqrt{2}}(\|e\rangle + \|g\rangle)$ $\|g\rangle \to \frac{1}{\sqrt{2}}(\|g\rangle - \|e\rangle)$

After performing CNOT operation, $|\Psi^\pm\rangle_{1,2}$ and $|\Phi^\pm\rangle_{1,2}$ evolve to

$$|\Psi^\pm\rangle_{1,2} \to \frac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_2 \pm |g\rangle_1|g\rangle_2), \quad |\Phi^\pm\rangle_{1,2} \to \frac{1}{\sqrt{2}}(|g\rangle_1|e\rangle_2 \pm |e\rangle_1|e\rangle_2). \tag{12}$$

Next we need to carry out H operation on atom 1. In order to do so, let atom 1 cross a classical fields and undergo the transition

$$|e\rangle_1 \rightarrow \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1)\,.$$
$$|g\rangle_1 \rightarrow \frac{1}{\sqrt{2}}(|g\rangle_1 - |e\rangle_1)\,. \qquad (13)$$

Eq.(12) becomes

$$|\Psi^+\rangle_{1,2} \rightarrow |g\rangle_1|g\rangle_2\,,$$
$$|\Psi^-\rangle_{1,2} \rightarrow |e\rangle_1|g\rangle_2\,,$$
$$|\Phi^+\rangle_{1,2} \rightarrow |g\rangle_1|e\rangle_2\,,$$
$$|\Phi^-\rangle_{1,2} \rightarrow -|e\rangle_1|e\rangle_2\,. \qquad (14)$$

Hence, the joint measurement can be achieved by detecting atoms 1 and 2 separately. Depending on the measurement outcome, we can assure that cavity fields 1 and 2 are in entangled state of Fock states. That is, there are strong correlations between photons of two cavities. At the same time, with the help of different measurement outcomes, we can assure the concrete form of the state of cavity fields.

In summary, we have proposed a scheme to realize entanglement swapping between atom and cavity and generate the entangled state of two separate cavity fields. By the cavity QED techniques, our studies show that we can realize the information swapping of interatom and intercavity or between atom and cavity. This kind of technique of information swapping can be used in quantum information processing, such as quantum teleportation,[23] remote state preparation,[24] and quantum communication.[25] Entangled state of cavity fields which we generated can be used as quantum channels to teleport an unknown atomic state. Supposing the unknown atomic state can be expressed as $|\psi\rangle = \alpha|e\rangle + \beta|g^-\rangle$, the sender performs joint measurement on atom and cavity 1. According to measurement outcomes from sender, the receiver can recover the original state from the cavity 2. As a final remark we briefly discuss the experimental feasibility of this scheme. In our scheme, there is never more than one photon in the cavity. The preparation of this type of cavity was reported.[26] For the Rydberg atoms, recent experiments showed that the decay time of the cavity was longer than the interaction time of the atom with the cavity field.[27] Therefore, based on the present or future cavity QED techniques, our scheme might become accessible to the experiments.

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