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Intensity modulation in single-mode microchip Nd:YAG lasers with asymmetric external cavity*

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Intensity modulation induced by the asymmetric external cavity in single-mode microchip Nd:YAG lasers is presented. Two kinds of experimental results are discussed based on multiple feedback effects. In one case, the intensity modulation curve is a normal sine wave, whose fringe frequency is four times higher than that of a conventional optical feedback system, caused by multiple feedback effects. In the other case, the intensity modulation curve is the overlapping of the above quadruple-frequency signal and conventional optical feedback signal, which is determined by the additional phase difference induced by the asymmetric external cavity. The theoretical analyses are in good agreement with the experimental results. The quadruple-frequency modulation of the laser output intensity can greatly increase the resolution of displacement measurement of an optical feedback system.

Keywords: optical feedback, asymmetric external cavity, multiple feedbacks
PACC: 4210J, 4255R, 8170G

1. Introduction

It is well known that the light reflected into a laser cavity by external mirror causes intensity variations. The laser intensity modulation is similar to that produced by a conventional optical interferometer, i.e. a fringe shift corresponding to every half wavelength displacement of the movable external mirror.[1] This phenomenon, also called self-mixing interference, has been widely used for imaging,[2] displacement,[3−5] velocity and vibration[6−8] in laser diodes. In recent years, diode-pumped microchip lasers have attracted much attention for their ultra-high sensitivity to optical feedback. Their photon lifetime is much shorter than the population lifetime, resulting in a high magnification of feedback signal.[6] The ultra-high-sensitivity self-mixing microchip lasers are widely used in imaging,[6,7] laser Doppler velocimetry,[8−10] or vibrometry.[11]

The previous work[12] has described the effects of asymmetric external cavity in semiconductor lasers. It was found that the fringe frequency of an optical feedback system can be doubled. Addy *et al*[13] tried to use this double-frequency fringe to increase the resolution of optical feedback sensing system. But the measurement range of this optical feedback system cannot be greater than 1mm. A method of doubling the fringe frequency in He–Ne laser induced by birefringence of an external cavity is investigated by Liu *et al.*[14] But the size of He–Ne Laser is too large to be propitious to miniaturization of the optical sensing system. Cheng and Zhang[15] presented the triple fringe frequency in vertical-cavity surface-emitting lasers (VCSELs) caused by the misalignment of the focusing lenses and external reflector. But the focusing lenses are needed for the divergence of the light in VCSELs.

In this paper, we study the intensity modulation in single-mode Nd:YAG lasers induced by the asymmetric external cavity. Two different cases are observed. One is that the fringe frequency of intensity modulation is four times higher than what the conventional optical feedback can achieve, which is called quadruple-frequency fringe. Every fringe corresponds to a $\lambda/8$ displacement in the single-mode Nd:YAG laser. The other is that the intensity modulation curve is the superposition of the quadruple-frequency signal and the conventional optical feedback signal. In this case, we find that the additional phase difference due to the asymmetric external cavity determines the shape of intensity modulation curves. Experimental results, as well as theoretical analyses, are presented

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in this paper. Our results present a potential method to greatly increase the measurement resolution of an optical feedback system.

2. Experimental setup

Experiments are carried out on a diode-pumped half-intracavity Nd:YAG laser with the experimental setup shown in Fig.1.

The laser chip used consists of parallel-plane plate, with a thickness $d = 2$ mm and a diameter $\Phi 5$ mm, made of YAG crystal containing 1.1 at % Nd^{3+} ions. Its pumping surface is dielectric-coated to have a reflectivity of $r_1 = 100\%$ at the lasing wavelength of $\lambda = 1.064$ μm, but an antireflectivity at the pumping wavelength of λ =808 nm. The other surface is coated to have a high transmission of 99.9% at the lasing wavelength. The laser cavity with a cavity length l about 50mm is composed of pumping surface of YAG and a concave mirror M which has a radius of 200mm and a reflectivity of $r_2 = 98.8\%$ at $\lambda = 1.064$ μm. The pump source is a fibre-coupled laser diode, whose output is focused on the laser crystal. We keep the pumping light at about 220 mW when the laser outputs single longitudinal mode observed by F-P. Throughout the whole process, the laser works in the fundamental transverse mode state. M_{E} is an external reflective mirror with a radius of 1500mm and a reflectivity of $r_3 = 60\%$, used to reflect beams back into the laser. Both M_{E} and M form the feedback external cavity, whose length L is about 300mm. PZT is used to modify the external cavity length. The laser modes are observed by F-P, while the detector D can be used to detect the output laser intensity; and θ is the tilted angle between the surface normal to M_{E} and the laser axis, which represents the misalignment degree of the external cavity.

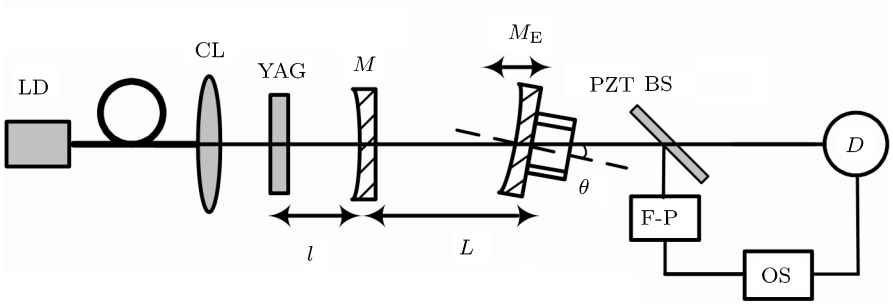


Fig.1. Experimental setup for optical feedback system. M, M_{E}: mirrors; BS: beam splitter; PZT: piezoelectric transducer; YAG: YAG crystal; CL: collimating lens; LD: laser diode; F-P: Fabry–Perot scanning interferometer; OS: oscilloscope. D: photoelectronic detector.

3. Experimental results

When θ=0, the external cavity is perfectly aligned. A fringe of an optical feedback system corresponds to $\lambda/2$ change of the external cavity length as shown in Fig.2(a), which we call the conventional optical feedback signal, with λ being the wavelength of the laser. However, when $\theta = 10$ mrad, the fringe frequency of the optical feedback system is quadrupled, compared with that of conventional optical feedback as shown in Fig.2 (b). When $\theta = 8$ mrad, there are two components in the intensity modulation curve. The envelope signal is the conventional optical feedback signal, while the quadruple-frequency modulation part is caused by the multiple feedback effects. The experimental results are shown in Fig.2.

Figures 2(a)–2(c) show three kinds of intensity modulation curves: (a) The conventional optical feedback signal, i.e. a fringe shift corresponds to $\lambda/2$ change of the external cavity length. (b) The fringe frequency of intensity modulation curve is four times higher than that of conventional optical feedback, i.e. a fringe shift corresponds to $\lambda/8$ change of the external cavity length. This result can be used to greatly improve the resolution of optical feedback sensing system. (c) The intensity modulation curve is the combination of the above quadruple-frequency signal and the conventional optical feedback signal.

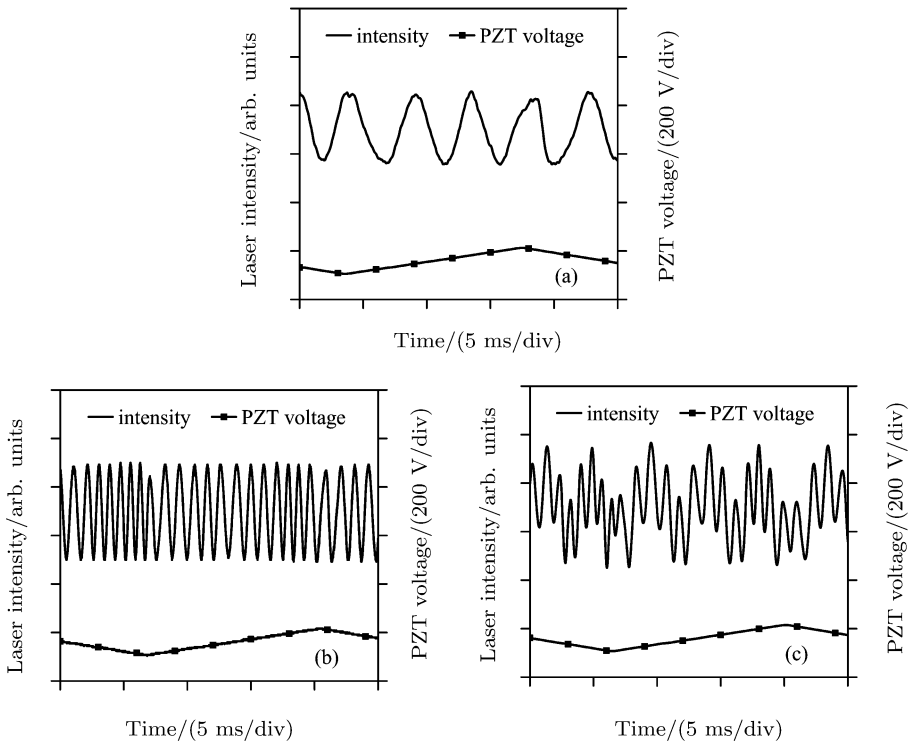


Fig.2. Intensity modulation curves in oscilloscope waveforms for different optical feedbacks: (a) Conventional optical feedback signal. (b) Quadruple-frequency signal of laser output intensity. (c) The combination of quadruple-frequency modulation signal and conventional optical feedback signal.

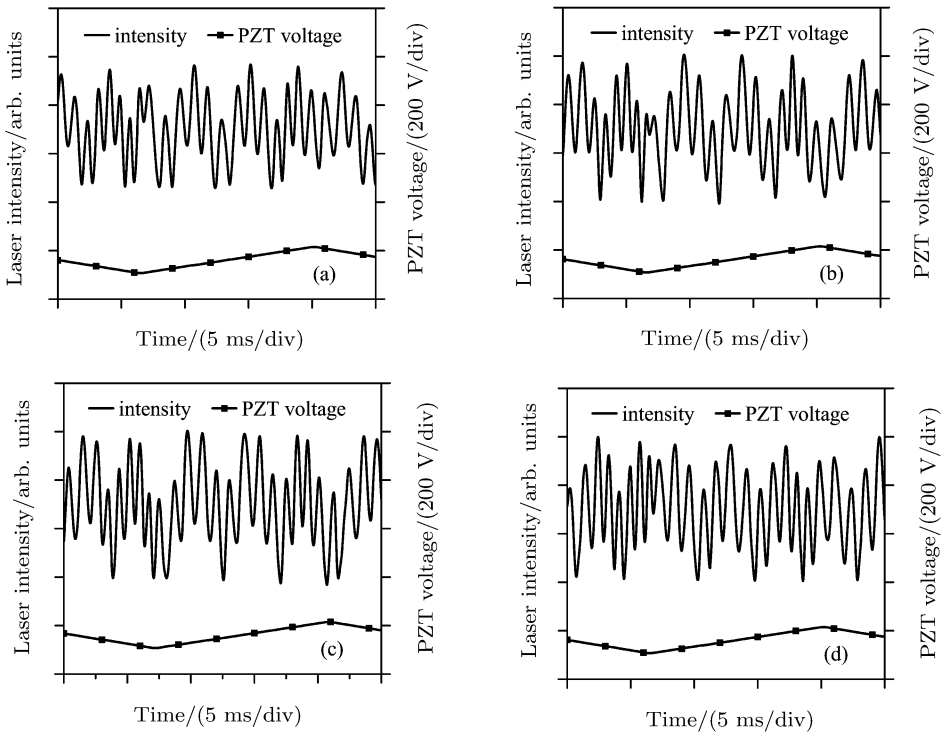


Fig.3. Intensity modulation curves in oscilloscope waveforms for different $\Delta\theta$: (a) $\Delta\theta = 0$, (b) $\Delta\theta = 2$ μrad, (c) $\Delta\theta = 4$ μrad, (d) $\Delta\theta = 6$ μrad.

We keep the tilted angle $\theta = 8$ mrad, i.e. the intensity modulation curve is the overlapping of the quadruple-frequency signal and the conventional optical feedback signal. Then, we change the tilted angle θ by a small amount of $\Delta\theta$ as follows: (a) $\Delta\theta = 0$; (b) $\Delta\theta = 2$ μrad; (c) $\Delta\theta = 4$ μrad; (d) $\Delta\theta = 6$ μ rad. The results for different $\Delta\theta$ are shown in Fig.3.

When $\theta = 8$ mrad, the intensity modulation curves are slowly varying envelope signals of the mono-frequency (i.e. the conventional optical feedback signal) superposed by quadruple-frequency signals as shown in Figs.3(a)–3(d). With the small variation $\Delta\theta$ of the tilted angle θ, the quadruple-frequency signal shifts relatively to the mono-frequency signal. Thus, the positions and heights of the four peaks in the envelope curves are changed with the different $\Delta\theta$, as shown in Fig.3. This phenomenon may be used in precision angle measurement.

4. Theoretical analysis

In the presence of optical feedback, a light beam can be split into two separate beams. One travels within the internal cavity, and the other travels in the external cavity, and then they couple together to get into the internal cavity. These two beams are superposed in the internal cavity and construct the self-mixing interference. After two round-trips, the initial electric field $E_0(t)$ in the laser cavity turns into the final electric field $E(t)$ and these round-trips can be expressed as[3,14]

$$E(t) = r_1 r_2 \exp\Big(\mathrm{j}4\pi\nu\frac{nl}{c} + gd\Big)E_0(t) + r_1 T_2 r_3 C_1 \exp\Big(\mathrm{j}4\pi\nu\frac{nl+L}{c} + gd\Big)E_0(t), \quad (1)$$

where r_1, r_2 and r_3 are the reflectivities of the pump entrance surface, M and M_{E}, respectively; $T_2 = 1 - r_2^2$ is the transmission coefficient of M; C_1 is the feedback coupling coefficient of M when the beam travels from the external cavity to the internal cavity, which can be calculated by means of the 3×3 matrix formalism[15] for optical misaligned elements; L and l are the external cavity length and the laser cavity length, respectively; c is the speed of light in vacuum; g is the linear gain per unit length due to the stimulated emission inside the laser cavity in the presence of feedback; n is the effective refractive index of the laser cavity material, and d is the length of gain medium.

Equation (1) only takes into account the light intensity of first roundtrip. However, the asymmetric external mirror can cause multiple feedbacks. Dual or even triple feedback has been reported.[13−15] For He-Ne laser or laser diodes, the higher-order feedback can hardly be observed, because the light intensity of higher-order feedback is much less than the total output of the laser. Thus, the fluctuation induced by higher-order feedback can hardly be observed. For YAG lasers, there is the high magnification (10^6) of feedback signal because their photon lifetime is much shorter than the population lifetime.[6] Hence, the higher-order feedback can be realized in YAG lasers.

For small tilted angles, to a reasonable degree of accuracy, the electric field $E_{\mathrm{in}}(t)$, which is reflected into the laser cavity from the asymmetric external cavity due to multiple feedbacks, can be expressed as

$$E_{\mathrm{in}}(t) = r_1 T_2 r_3 \sum_{i=1}^{n} (r_2 r_3)^{i-1} C_i \times \exp\Big(\mathrm{j}4\pi\nu\frac{nl+iL}{c} + gd + \delta_i\Big)E_0(t), \quad (2)$$

where n represents the largest round-trip times in external feedback cavity of the light, which can reenter into the laser; i represents the light's ith round-trip in the external cavity; C_i is the coupling coefficient of the ith feedback into the laser cavity, and δ_i is the additional phase caused by the asymmetric external mirror M_{E}.

Using the ray-trace method, we find that only the light which travels four round-trips between M_2 and M_{E} can feed back into the laser cavity, besides a portion of the laser beam which travels only one round-trip in the external feedback cavity when $\theta = 8$ mrad. Because the diameter of the YAG crystal is only 5mm, most of the beams reflected back by the tilted mirror M_{E} will not then be fed back into the laser cavity beyond the first round-trip. However, they may be reflected from the laser cavity mirror, and after four round-trips they can be fed back into the laser cavity, because M_{E} used as a feedback mirror is a concave mirror, not a plane mirror. Thus, we can obtain

$$E_{\mathrm{in}}(t) = r_1 T_2 r_3 \Big[C_1 \exp\Big(\mathrm{j}4\pi\nu\frac{nl+L}{c} + gd + \delta_1\Big) + (r_2 r_3)^3 C_4 \exp\Big(\mathrm{j}4\pi\nu\frac{nl+4L}{c} + gd + \delta_4\Big)\Big]E_0(t). \quad (3)$$

According to the experimental results, supposing $C_4 \approx C_1$, we can modify Eq.(1) into the following equation:

$$E(t) = r_1 r_2 \exp\Big(\mathrm{j}4\pi\nu\frac{nl}{c} + gd\Big)E_0(t) + E_{\mathrm{in}}(t). \quad (4)$$

For $E(t) = E_0(t)$ when the laser comes to equilibrium, Eq.(4) can be rewritten as

$$r_1 r_2 \exp\Big(\mathrm{j}4\pi\nu\frac{nl}{c} + gd\Big)\Big[1 + \frac{T_2 r_3 C_1}{r_2}\exp\Big(\mathrm{j}4\pi\nu\frac{L}{c} + \delta_1\Big) + \frac{T_2 r_3 (r_2 r_3)^3 C_4}{r_2}\exp\Big(\mathrm{j}4\pi\nu\frac{4L}{c} + \delta_4\Big)\Big] = 1. \quad (5)$$

Define

$$\phi = 4\pi\upsilon L/c,$$
$$\alpha = T_2 r_3 C_1/r_2,$$
$$\beta = T_2 r_3 (r_2 r_3)^3 C_4/r_2.$$

Solving Eq.(5), we obtain

$$g = -\frac{1}{d}[\ln(r_1 r_2) + \alpha\cos(\varphi + \delta_1) + \beta\cos(4\varphi + \delta_4)]. \quad (6)$$

In the absence of M_{E}, there is no feedback and $\alpha = \beta = 0$, so the linear gain per unit length is

$$g_0 = -\frac{1}{d}\ln(r_1 r_2). \quad (7)$$

The excess required gain Δg can be expressed as

$$\Delta g = g - g_0 = -\frac{\alpha}{d}\cos(\varphi + \delta_1) - \frac{\beta}{d}\cos(4\varphi + \delta_4). \quad (8)$$

Since the laser intensity can be expressed as[16,17]

$$I = I_0(1 - K\Delta g), \quad (9)$$

where K is a constant and I_0 is the initial laser intensity without optical feedback, substituting Eq.(8) into Eq.(9), the laser output intensity with asymmetric optical feedback can be written as

$$I_1 = I_0[1 + \alpha K/d\cdot\cos(\varphi + \delta_1) + \beta K/d\cdot\cos(4\varphi + \delta_4)]. \quad (10)$$

Equation (10) indicates that the laser intensity with optical feedback consists of three parts. One is the constant part I_0, the others are the mono-frequency part, i.e. conventional optical feedback signal, and the quadruple-frequency signal. When only the first round-trip of light is considered, i.e. $C_4 = 0$, the intensity modulation curve is a sine wave, and its fringe shift corresponds to a $\lambda/2$ change of the external cavity length. When there is only the fourth round-trip of light, i.e. $C_1 = 0$, the intensity modulation curve is also a sine wave, but the frequency is the quadruple of that of the conventional feedback signal, i.e., its fringe shift corresponds to $\lambda/8$ change of the external cavity length. When both the first and fourth round-trips of light coexist, the intensity modulation curve is an envelope signal of the mono-frequency signal superposed by a quadruple-frequency signal. Thus, there are four peaks in every envelope signal. The simulation curves based on the above analysis are shown in Fig.4.

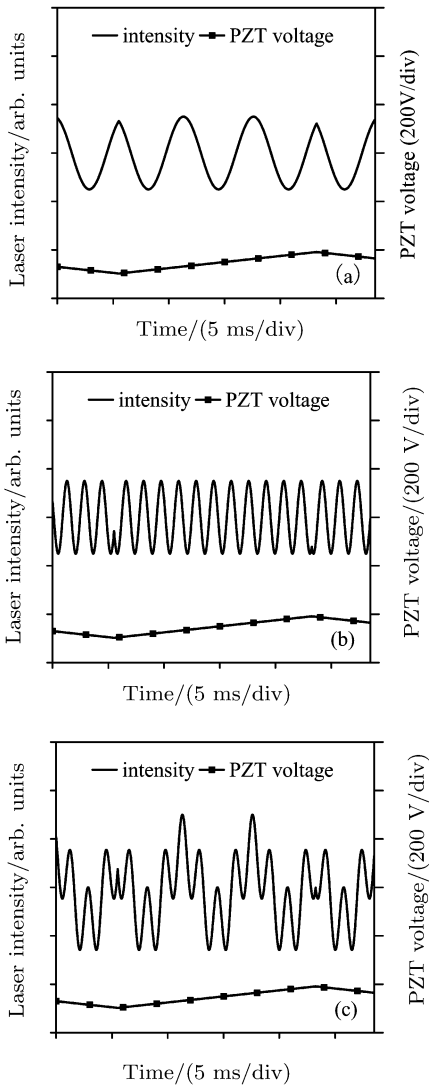


Fig.4. Simulation of intensity modulation curves for different optical feedbacks: (a) Only the first round-trip of the light can be reflected into laser cavity. (b) Only the fourth round-trip of the light can be reflected into laser cavity. (c) Both the first and fourth round-trips of the light can be reflected into laser cavity.

From Eq.(10), there is an additional phase difference $\delta = \delta_1 - \delta_4$, which is induced by the asymmetric external mirror M_{E}. The different δ will result in different relative positions and heights of the four peaks in every envelope signal. The simulation curves of different δ are shown in Fig.5.

The additional phase difference δ can be changed by the change of tilted angle θ of M_{E} in the order of a few μrad. Under our experimental conditions, the relationship between the change of tilted angle $\Delta\theta$ and the additional phase difference δ can be expressed as

$$3D\Delta\theta = \frac{\delta\lambda}{4\pi}, \tag{11}$$

where $D = 20$ mm is the distance between the light spot on M_{E} and the rotation axis of M_{E}.

So, $\delta = \pi/2$ corresponds to $\Delta\theta$=2.08 μrad. This $\Delta\theta$ is so small that the coefficients C_1 and C_4 can be considered constant during the changes of tilted angle in the order of a few μrad.

Theoretical analyses agree well with the experimental results.

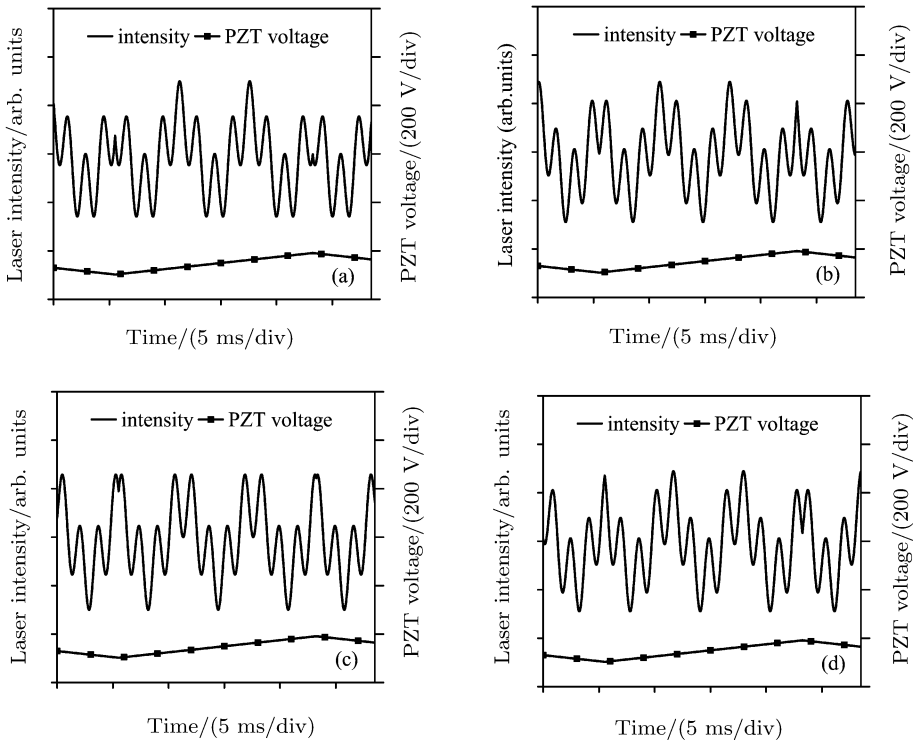


Fig.5. Simulation of intensity modulation curves for different additional phase difference δ: (a) $\delta = 0$; (b) $\delta = \pi/2$; (c) $\delta = \pi$; (d) $\delta = 3\pi/2$.

5. Conclusions

Intensity modulation induced by multiple feedbacks of asymmetric external cavity in single-mode microchip Nd:YAG laser has been studied and demonstrated. Two different optical feedback cases are discussed. One is that only the fourth round-trip of the feedback light can be reflected into laser cavity. In this case, the intensity modulation curve is a quadruple-frequency signal relative to the case of conventional optical feedback. Hence, a fringe shift of this case corresponds to a $\lambda/8$ change of the external cavity length. The other is that both the first and fourth round-trips of feedback lights can be reflected into the laser cavity. In this case, the additional phase difference δ determines the shapes of the intensity modulation curves, i.e., resulting in different relative positions and heights of the four peaks in the envelope curve. The results presented here can be used to realize the $\lambda/8$ resolution of displacement measurement based on optical feedback, i.e. 133nm for YAG lasers. It also suggests a new way of angle measurement with a high resolution.

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