Vol 16 No 4, April 2007
1009-1963/2007/16(04)/0962-13

Chinese Physics

© 2007 Chin. Phys. Soc.
and IOP Publishing Ltd

Quantum study of Foldy–Wouthuysen–Tani theory on lattice

Liu Da-Qing(刘大庆)†

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, *China*

(Received 23 March 2006; revised manuscript received 13 October 2006)

We study here a quantum version of Foldy–Wouthuysen–Tani (FWT) transformation and compare the similarities and differences between the quantum and the classic FWT theories. Then the improvement of action on lattice is discussed. The result shows that it is not necessary to improve the covariant difference along the time direction on lattice. Finally we discuss briefly the structure of physical vacuum and give a model independent of field condensate.

Keywords: Foldy–Wouthuysen–Tani transformation, glueball
PACC: 0550, 1480D, 1440

1. Introduction

The spectra of quantum chromodynamics (QCD) should in principle contain bound states without constituent quark content, the so-called glueballs, in addition to the mesons and baryons of the quark model. The question, however, arises, what is the difference between a flavour singlet meson and a pure glueball state? Do they mix and, how do they mix?

This paper gives a trial answer to these puzzles. To do this we first study the Foldy–Wouthuysen–Tani theory (FWT theory).[1,2] This theory has been studied in the classical version (in Lagrangian language), but here we study it in the quantum version (in Hamiltonian language) and compare the similarities and differences between them. One can, just as one did in the classical version, study the systematic non-relativistic expansion of the interactions in hadrons in the quantum version of FWT theory. The quantum FWT theory shows strict conservation quantities in bound states, which are referred to as the numbers of modified fermion operators and the modified antifermion operators here. Glueballs and heavy quarkonia are distinguished unambiguously according to this statement. One can calculate the structure of hadrons by using the technique of modified fermion.

The modified quark/antiquark (MQA) is different from the associated (current) quark/antiquark (QA). First, it has a finite size. Second, it also includes components of gluon degree, multi-quark degree and even more, the degree of other type of quark. In fact, the property of number conservation determines not only the form of theory, nonrelativistic quantum chromodynamic (NRQCD) theory,[3−8] or heavy quark effective theory (HQET),[9,10] but also the content of modified quarks in bound states.

Owing to the quantization, there are some differences between quantum FWT theory and classical FWT theory. For instance, there appear additional terms in the quantum version. Besides, we think the definitions here of chromo-electric fields and covariant derivative operator along the time direction, especially on lattice, are better than those in Refs.[4-11].

We first write out briefly the quantum form of chromodynamics in Section 2. Then we study the FWT theory in quantum language and compare the similarities and differences between quantum FWT theory and classic FWT theory in Section 3. We study the improvement of action on lattice in Section 4 and then give a brief summary in Section 5.

2. Quantization

We work here in four-dimensional Euclidean space. For simplicity, we always take the gauge condition $A_0 = 0$ and definition $\not{A} = \sum_i^3 \gamma^i \cdot A^{\rm i}$ in this paper (i = 1, 2, 3). With notations $D_\mu = \partial_\mu + {\rm i}gA_\mu^a T^a$ we have $[D_\mu, D_\nu] = {\rm i}gF_{\mu\nu}$, where $B_k = \dfrac{1}{2}\epsilon_{ijk}F_{ij}$.

First, we quantize gauge fields and fermion fields simultaneously. After the quantization, every classical field, such as spinor ψ, will turn to the corresponding field operator acting on the completed Fock space.

†E-mail: liudq@mail.ihep.ac.cn and liudq@mail.xjtu.edu.cn

Since the gauge fields and the fermion fields are all quantized, the total Fock space is $\mathcal{F} = \mathcal{F}_{\rm F} \times \mathcal{F}_{\rm G}$, where $\mathcal{F}_{\rm F}$ is Fock space for fermions while $\mathcal{F}_G$ is Fock space for bosons. We list the basic properties of these operators as follows. Other properties are discussed in many text books, for example in Ref.[14].

We denote the corresponding conjugate field operators of canonical field operators A^a_x (boson operator) and Ψ^i_x (fermion operator) as $\mathrm{i}\Pi^a_x$ and $\mathrm{i}\Psi^{\dagger i}_x$ respectively. After the quantization it is natural to adopt their commutation or anti-commutation relations as

$$\{\Psi^i_{\mu x}, \Psi^{\dagger j}_{\nu y}\} = \delta^{ij}_{\mu\nu}\delta_{xy}, \quad [A^a_{ix}, \Pi^b_{jy}] = \delta^{ab}_{ij}\delta_{xy}, \tag{1}$$

where the first i, j for Ψ or $\Psi^\dagger$ are colour indices while the second i, j for A or Π are orientation indices. Notice here that As are Hermite operators while Πs are anti-Hermite ones.

We use a functional to expand Hilbert space, for instance, an eigenstate Θ can be written as $\Theta(\boldsymbol{A}(\boldsymbol{x}), \psi^\dagger(\boldsymbol{x}), \psi(\boldsymbol{x}))$. Then, for the conjugate field operator $\Pi^a_{i\boldsymbol{x}}$, we have

$$\Pi^a_{i\boldsymbol{x}}\Theta = -\frac{\partial}{\partial A^a_{i\boldsymbol{x}}}\Theta. \tag{2}$$

And up to a normalized constant

$$\langle \boldsymbol{A}\boldsymbol{x}|\boldsymbol{\Pi}\boldsymbol{x}\rangle = \exp^{-\int \mathrm{d}\boldsymbol{x}\boldsymbol{A}^a_{\boldsymbol{x}}\cdot\boldsymbol{\Pi}^a_{\boldsymbol{x}}}. \tag{3}$$

The requirement for the local gauge invariant of state Θ is

$$(D_i\Pi_{i\boldsymbol{x}})^a\Theta = 0. \tag{4}$$

The complete conditions are

$$\begin{aligned} &\int \mathrm{d}[\boldsymbol{A}(\boldsymbol{x})]|\boldsymbol{A}\rangle\langle\boldsymbol{A}| = 1, \\ &\int \mathrm{d}[\psi^\dagger\psi]|\psi\rangle \exp\left(-\int \mathrm{d}\boldsymbol{x}\psi^\dagger\psi\right)\langle\psi| = 1. \end{aligned} \tag{5}$$

These complete conditions suggest how to expand the state functionals. To expand a state in the gauge field, we use

$$\Theta = \int \mathrm{d}[\boldsymbol{A}(\boldsymbol{x})]|\boldsymbol{A}\rangle\Theta(A), \tag{6}$$

where $\Theta(\boldsymbol{A}) = \langle\boldsymbol{A}|\Theta$, while to expand a state in the fermion field, we use a slightly complicated formula,

$$\begin{aligned} \Theta = &\int \mathrm{d}[\psi^\dagger\psi\psi^{\dagger\prime}\psi']|\psi\rangle \exp\left(-\int \mathrm{d}\boldsymbol{x}\psi^\dagger\psi\right) \\ &\times\Theta(\psi^\dagger, \psi')\exp(-\int \mathrm{d}\boldsymbol{x}\psi^{\dagger\prime}\psi')\langle\psi'|, \end{aligned} \tag{7}$$

where $\Theta(\psi^\dagger, \psi') = \langle\psi|\Theta|\psi'\rangle$.

To describe the dynamics of this Fock space we should write out the Hamiltonian operator, which, in principle, determines all the information about the dynamical behaviour such as spectra of mesons and baryons. The full Hamiltonian operator in QCD is

$$H = H^0_{\rm F} + H_{\rm G}. \tag{8}$$

The Hamiltonian operator for gauge fields is

$$H_G = \frac{1}{2}\{-(\Pi^a_x)^2 + (B^a_x)^2\}, \tag{9}$$

and the Hamiltonian operator for a heavy quark with mass m is

$$H^0_{\rm F} = m\sum_x \Psi^{\dagger i}_\mu(\beta(1 + \not{D}/m))^{ij}_{\mu\nu}\Psi^j_\nu, \tag{10}$$

where the summations over repeated indices are always assumed. We refer to the fermions, of which Hamiltonians are given by the expression (10), as current fermions thereinafter.

We emphasize here that all the above operators, since they act on Fock space, are time independent. Therefore, the Hamiltonian operators, such as H, $H^0_{\rm F}$ and $H_{\rm G}$, are also time independent.

For γ matrices, we set

$$\gamma_i = \begin{pmatrix} 0 & \tau_i \\ \tau_i & 0 \end{pmatrix}, \ \beta = \gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \tag{11}$$

and also

$$\begin{aligned} \sigma_{ij} = &\ \frac{\mathrm{i}}{2}[\gamma_i, \gamma_j] = -\epsilon_{ijk}\Sigma_k \\ = &\ -\epsilon_{ijk}\begin{pmatrix} \tau_k & 0 \\ 0 & \tau_k \end{pmatrix}. \end{aligned} \tag{12}$$

3. The Foldy–Wouthuysen–Tani transformation

Suppose that there exists a unitary transformation $\mathcal{U}$ that does not depend on time explicitly, then for a vector Θ in Fock space $\mathcal{F}$ we will have the following transformation:

$$\Theta' = \mathcal{U}\Theta, \tag{13}$$

while for a given operator, such as Hamiltonian H, the transformation is

$$\mathcal{H}' = \mathcal{U}H\mathcal{U}^\dagger. \tag{14}$$

It should be emphasized that, unlike what we did in the classical version of FWT transformation,[16] here

it is not necessary to consider the correction $-\mathrm{i}\mathcal{U}\partial_t\mathcal{U}^\dagger$ in Eq.(14), since $\mathcal{U}$ and H are both time independent.

For exemplification, we first let $\mathcal{U} = \exp\left\{\frac{1}{2m}\Psi^\dagger \not{D}\Psi\right\} \equiv \mathrm{e}^V$, where summations over colour, spinor and position indices have been suppressed here and thereinafter. We have

$$H_{\mathrm{F}}^{0\prime} = \mathcal{U}H_{\mathrm{F}}^0\mathcal{U}^\dagger = m\Psi^\dagger\beta\left[1-\frac{1}{2}\left(\frac{\not{D}}{m}\right)^2 +\frac{1}{3}\left(\frac{\not{D}}{m}\right)^3-\frac{1}{8}\left(\frac{\not{D}}{m}\right)^4+\cdots\right]\Psi. \quad (15)$$

$\mathcal{U}$ is a unitary transformation, therefore, the commute relations in Eq.(1) still hold after this transformation.

Under this unitary transformation, the operator H_{G} turns to

$$\begin{aligned} H'_{\mathrm{G}} &= \mathcal{U}H_{\mathrm{G}}\mathcal{U}^\dagger \\ &= H_{\mathrm{G}} + [V, H_{\mathrm{G}}] + \frac{1}{2}[V,[V,H_{\mathrm{G}}]] + \cdots \\ &= H_{\mathrm{G}} - \frac{\mathrm{i}g}{2m}\Psi^\dagger \not{\Pi}\Psi \\ &\quad -\frac{\mathrm{i}g}{8m^2}\sum_{xy}[\Psi^\dagger \not{D}\Psi(x), \Psi^\dagger \not{\Pi}\Psi(y)] + \cdots . \end{aligned} \quad (16)$$

Because of the quantization of the gauge fields, there exist additional terms imposed on the initial H_{G}. These terms should be absorbed into the Hamiltonian for fermion fields. Therefore, under the transformation, H_{G} is invariant while Hamiltonian for fermions turns to

$$\begin{aligned} H'_{\mathrm{F}} = m\Psi^\dagger\beta&\left[1-\frac{1}{2}\left(\frac{\not{D}}{m}\right)^2+\frac{1}{3}\left(\frac{\not{D}}{m}\right)^3 -\frac{1}{8}\left(\frac{\not{D}}{m}\right)^4\right]\Psi - \frac{\mathrm{i}g}{2m^2}\Psi^\dagger\not{\Pi}\Psi \\ &-\frac{\mathrm{i}g}{8m^2}\sum_{xy}[\Psi^\dagger \not{D}\Psi(x), \Psi^\dagger \not{D}\Psi(y)] + \cdots . \end{aligned} \quad (17)$$

We refer to the operators that keep the conservation of fermion number and antifermion number as even operators, such as operator $\Psi^\dagger\beta\Psi$. Otherwise, we call them odd operators, such as operator $\Psi^\dagger\beta\not{D}\Psi$. The odd operators can be eliminated by a series of unitary transformations according to the powers of v step by step. After performing a series of time independent transformations, such as $\exp\left[-\frac{\mathrm{i}g}{4m^2}\Psi^\dagger\beta\not{\Pi}\Psi\right]$, $\exp\left[\frac{1}{6}\Psi^\dagger(\not{D}/m)^3\Psi\right]$ and so on, H_{G} becomes invariant, but the Hamiltonian for fermion changes to

$$\begin{aligned} H_{\mathrm{F}} &= m\Psi^\dagger\beta\left[1-\frac{D^2}{2m^2}+\frac{g\boldsymbol{\Sigma}\cdot\boldsymbol{B}}{2m^2}-\frac{D^4}{8m^4}\right]\Psi \\ &\quad -\frac{\mathrm{i}g}{8m^2}\sum_{xy}[\Psi^\dagger \not{D}\Psi(x), \Psi^\dagger \not{\Pi}\Psi(y)] + \cdots , \end{aligned} \quad (18)$$

where we have used the identity $\not{D}^2 = D^2 - \boldsymbol{\Sigma}\cdot\boldsymbol{B}$.

We then decompose a four-component Dirac spinor into two two-component Pauli spinors as follows:

$$\Psi = \begin{pmatrix}\varphi\\ \chi\end{pmatrix}, \quad (19)$$

where φ annihilates a fermion while χ produces an anti-fermion.

Notice that the Hamiltonian is in fact in the normal order form, we can also rewrite Eq.(18) in the two-component Pauli form:

$$\begin{aligned} H_{\mathrm{F}} &= \varphi^\dagger\left[m-\frac{D^2}{2m}+\frac{g\boldsymbol{\tau}\cdot\boldsymbol{B}}{2m}-\frac{D^4}{8m^3}\right]\varphi \\ &\quad -\chi^\dagger\left[m-\frac{D^2}{2m}+\frac{g\boldsymbol{\tau}\cdot\boldsymbol{B}}{2m}-\frac{D^4}{8m^3}\right]\chi \\ &\quad +\frac{g}{8m^2}\{\varphi^\dagger\boldsymbol{\tau}\cdot(\boldsymbol{D}\times\boldsymbol{\Pi}-\boldsymbol{\Pi}\times\boldsymbol{D})\varphi \\ &\quad +\chi^\dagger\boldsymbol{\tau}\cdot(\boldsymbol{D}\times\boldsymbol{\Pi}-\boldsymbol{\Pi}\times\boldsymbol{D})\chi\} \\ &\quad -\frac{\mathrm{i}g}{8m^2}\{\varphi^\dagger(\boldsymbol{D}\cdot\boldsymbol{\Pi}-\boldsymbol{\Pi}\cdot\boldsymbol{D})\varphi \\ &\quad +\chi^\dagger(\boldsymbol{D}\cdot\boldsymbol{\Pi}-\boldsymbol{\Pi}\cdot\boldsymbol{D})\chi\} \\ &\quad +H_4+H_6+\cdots, \end{aligned} \quad (20)$$

where H_4 includes the four-fermion interaction:

$$\begin{aligned} H_4 &= -\frac{\alpha_s\pi}{6m^2}\Big\{4\varphi^\dagger\varphi\chi^\dagger\chi-\frac{4}{3}\varphi^\dagger\tau^i\varphi\chi^\dagger\tau^i\chi \\ &\quad -3\varphi^\dagger T^a\varphi\chi^\dagger T^a\chi+\varphi^\dagger T^a\tau^i\varphi\chi^\dagger T^a\tau^i\chi\Big\} \\ &= \frac{\alpha_s\pi}{m^2}\varphi^\dagger T^a\tau^i\chi\chi^\dagger T^a\tau^i\varphi, \end{aligned} \quad (21)$$

and H_6 includes

$$\begin{aligned} H_6 &= -\frac{1}{16}\varphi^\dagger\frac{D^6}{m^5}\varphi+\frac{1}{16}\chi^\dagger\frac{D^6}{m^5}\chi \\ &\quad +\frac{g}{8m^3}(\varphi^\dagger\{D^2,\boldsymbol{\tau}\cdot\boldsymbol{B}\}\varphi-\chi^\dagger\{D^2,\boldsymbol{\tau}\cdot\boldsymbol{B}\}\chi) \\ &\quad +\frac{\alpha_s\pi}{2m^3}\{-\varphi^\dagger(\boldsymbol{\Pi}\cdot\boldsymbol{\Pi}+\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{\Pi}\times\boldsymbol{\Pi})\varphi \\ &\quad +\chi^\dagger(\boldsymbol{\Pi}\cdot\boldsymbol{\Pi}+\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{\Pi}\times\boldsymbol{\Pi})\chi\}. \end{aligned} \quad (22)$$

In Eq.(21) i, j are colour indices and $\alpha_s \equiv \frac{g^2}{4\pi}$ while in Eq.(22) we have ignored the terms with very small coefficients, for instance, smaller than 1/20. For simplicity, H_6 will be ignored thereinafter. We note here

that terms in H_4 are due to the correction of the quantization for gauge fields

$$\begin{aligned}&\sum_{xy}[\Psi^\dagger \boldsymbol{D}\Psi(x),\Psi^\dagger \boldsymbol{\Pi}\Psi(y)]\\ =&\sum_x \Psi^\dagger[\boldsymbol{D},\boldsymbol{\Pi}]\Psi - \mathrm{i}g\,\Psi_\rho^{\dagger k}\Psi_\mu^{\dagger i}\Psi_\lambda^l\Psi_v^j\gamma_{\mu\nu}^m\gamma_{\rho\lambda}^m T_{kl}^a T_{ij}^a,\end{aligned} \tag{23}$$

which includes odd operators. To eliminate them we perform unitary transformations $\exp\left\{\frac{3\alpha_s\pi}{32m^3}((\chi^\dagger\varphi)^2-(\varphi^\dagger\chi)^2)\right\}$ and $\exp\left\{-\frac{\alpha_s\pi}{96m^3}((\chi^\dagger\tau\varphi)^2-(\varphi^\dagger\tau\chi)^2)\right\}$ successively.

To apply Eqs.(20)–(22) to quarkonia, one should know the orders in the velocity expansion of each term in these equations. To do this one should return to the Coulomb gauge and use power counting.[4] From the discussion in obtaining Eq.(28), it is equivalent in power counting between $\boldsymbol{\Pi}$ and E. Suppressing the integral factor $\int \mathrm{d}x^3 \sim (mv)^{-3}$ in each term, we have respectively $\varphi^\dagger m\varphi \sim m$, $\varphi^\dagger\frac{D^2}{m}\varphi \sim mv^2$, $\varphi^\dagger\frac{D^4}{m^3}\varphi \sim mv^4$, $\varphi^\dagger\frac{g\boldsymbol{\tau}\cdot\boldsymbol{B}}{m}\varphi \sim mv^4$, $\frac{1}{m^2}\{\varphi^\dagger\boldsymbol{\tau}\cdot(\boldsymbol{D}\times g\boldsymbol{\Pi}-g\boldsymbol{\Pi}\times\boldsymbol{D})\varphi \sim \frac{1}{m^2}\{\varphi^\dagger(\boldsymbol{D}\cdot g\boldsymbol{\Pi}-g\boldsymbol{\Pi}\cdot\boldsymbol{D})\varphi \sim mv^4$ in Eq.(20). We also have $H_6 \sim mv^6$ and $H_4 \sim mv^4$ if we set $\alpha_s(mv) \sim v$.

We have thus reproduced the results of classical FWT theory,[1,2,10] except for the quantum corrections in H_4, which is on the order of v^4. These four-fermion interactions cannot occur at a tree level in classical version. As argued in Ref.[4], such terms can only exhibit themselves by a loop integral in the classical version, therefore, the four-fermion interaction in H_4 is on the order of α_s^2 or beyond,[1,4,16] while in the quantum version, these terms appear at a tree level, just due to the uncommutation of field operators. They stem from the uncommutation between the gauge fields and the associated conjugate fields. This is the significant difference between the quantum version and classical version of FWT theory. These four-fermion interactions cause spin–spin interactions of modified fermions in bound states, which have been studied by other NRQCD approaches,[5,7] with the utilizing of matching conditions.

In some theory using classical FWT transformation there is no four-fermion interaction or multi-fermion interaction. An example is lattice Fermilab action.[16] The $J/\psi-\eta_c$ mass splitting is mis-estimated in some lattice work[17] based on this action. Since the four-fermion interactions are spin–spin ones, we suggest here that one should add such interactions to the action in lattice simulations.

Notice that the translation fermion operators in Eq.(18) and Eq.(20) are different from the initial current fermion operators in Eq.(10). They are determined by a unitary transformation. Correspondingly, we refer to the translation fermion operators as modified fermion/antifermion (MFA) operators. Define the number operator of modified fermion as

$$N(\varphi) \equiv \int \mathrm{d}\boldsymbol{x}\varphi^\dagger(\boldsymbol{x})\varphi(\boldsymbol{x}), \tag{24}$$

then $[N(\varphi),H]=0$ will be obtained, which implies that the modified fermion is the fermion with conservational particle number, although it is very complicated to represent the modified fermion in the form of (current) fermion. We have a similar result for modified antifermion operators. From this statement, one can therefore unambiguously define bound states with $N(\varphi)\Theta = N(\chi)\Theta = \Theta$ as quarkonia and bound states with $N(\varphi)\Theta = N(\chi)\Theta = 0$ as glueballs respectively. Although in the language of current quarks there does exist the admixture, this statement is not an accurate one because the number of current quark is not a conservational quantity in bound states. One can calculate this 'admixture' by utilizing the conservation of modified fermion in bound states.

Since we refer to the operators that break the conservation of MQA number as odd ones, the conservation is equivalent to the statement that there is no odd operator in the ultimate translation Hamiltonian. Thus, equivalently, as long as it can be demonstrated that one can always perform a unitary transformation to eliminate all the odd operators, the conservation of MFA in QCD is proven. The exact form of the unitary transformation is unknown; correspondingly, the exact analytical forms of the modified fermions and antifermions are unknown also. As the product of unitary transformations is still a unitary transformation, one can use the product of series of unitary transformations to approximate the ultimate transformation. Therefore, what is necessary is only to prove that one can perform unitary transformations (FWT transformations) to eliminate the odd terms step by step. In heavy quark systems one can do it according to dimensions of operators. What are required are only the two constraints: 1) the Hamiltonian being Hermite; 2) the relative quark numbers in Hamiltonian being conservational.

Before the proof we make some comments here. As is well known, in classical FWT transformation one can decouple fermion from antifermion. However, in quantum FWT theory one cannot do that. There is an obvious counter example in Eq.(21). In fact, there are differences between conservation of MFA and decouples of MFA. For instance, four-fermion interaction in Eq.(21) has conservation property but no decoupling, while an interaction such as $\int \mathrm{d}\boldsymbol{x}\epsilon_{ijk}\varphi^{i\dagger}\varphi^{j\dagger}\varphi^{k\dagger}$ has the decoupling property but breaks the conservation property. As we shall see, all the arguments below are for the conservation property. Our discussion in the following is very general, although we shall meet a rather more complicated situation than that in the classical theory.

We prove it by using induction. The initial Hamiltonian satisfies these two constraints, of course. Suppose that through a series of unitary transformations the two constraints still hold, then the Hamiltonian can be written as $H = H_{\mathrm{G}} + H_{\mathrm{F}} = H_{\mathrm{G}} + H_{\mathrm{F}}^{\mathrm{even}} + H_{\mathrm{F}}^{\mathrm{odd}}$, where $H_{\mathrm{F}}^{\mathrm{even}}$ maintains the modified quark conservation, $H_{\mathrm{F}}^{\mathrm{even}} = \int \mathrm{d}\boldsymbol{x}m(\varphi^{\dagger}\varphi - \chi^{\dagger}\chi) - \frac{1}{2m}(\varphi^{\dagger}D^2\varphi - \chi^{\dagger}D^2\chi) + \cdots$, while $H_{\mathrm{F}}^{\mathrm{odd}}$ breaks modified quark conservation. Notice that all the dimensions of operators in Hamiltonian are positive, except $\int \mathrm{d}\boldsymbol{x}(\varphi^{\dagger}\varphi - \chi^{\dagger}\chi)$, of which the dimension is zero. For instance, $\int \mathrm{d}\boldsymbol{x}(\varphi^{\dagger}D^2\varphi - \chi^{\dagger}D^2\chi)$ and $\frac{1}{2}\int \mathrm{d}\boldsymbol{x}B_i^aB_i^a$ have dimensions 2 and 1 respectively. Because of the two constraints, in $H_{\mathrm{F}}^{\mathrm{odd}}$, the odd operator, in particular, the odd operator with the lowest dimension can always be written as $O^p = \frac{c}{m^{p-1}}\int \mathrm{d}\boldsymbol{x}(\varphi^{\dagger i_1}\ldots\varphi^{\dagger i_k}O^{i_1\ldots i_k}_{j_1\ldots j_k}\chi^{j_1}\ldots\chi^{j_k} + \chi^{\dagger j_1}\ldots\chi^{\dagger j_k}(O^{i_1\ldots i_k}_{j_1\ldots j_k})^{\dagger}\varphi^{i_1}\ldots\varphi^{i_k})$, where $p = 3(k-1)+l$, and l is the dimension of gauge covariant operator O, which itself is an even operator with colour charge. Here the operator has dimension p, c is a dimensionless constant and m^{p-1} in factor is to make the total dimension be unitary.

Under a unitary transformation $\mathcal{U} = \mathrm{e}^V$, where

$$V = \frac{c}{2km^p}(\varphi^{\dagger i_1}\ldots\varphi^{\dagger i_k}O^{i_1\ldots i_k}_{j_1\ldots j_k}\chi^{j_1}\ldots\chi^{j_k} - \chi^{\dagger j_1}\ldots\chi^{\dagger j_k}(O^{i_1\ldots i_k}_{j_1\ldots j_k})^{\dagger}\varphi^{i_1}\ldots\varphi^{i_k}), \quad (25)$$

Hamiltonian H turns into $H' = \mathcal{U}H\mathcal{U}^{\dagger} = H + H^{\mathrm{add}}$, where $H^{\mathrm{add}} = [V,H] + \frac{1}{2}[V,[V,H]] + \cdots$. In H^{add}, the only operator with dimension p is the term $[V, m\int \mathrm{d}\boldsymbol{x}(\varphi^{\dagger}\varphi - \chi^{\dagger}\chi)] = -O^p$, which just eliminates O^p in $H_{\mathrm{F}}^{\mathrm{odd}}$. Since $p > 0$ and other operators in H have positive dimensions, all the dimensions of the other operators in H^{add} are larger than p.

V still maintains the relative quark conservation and $V^{\dagger} = -V$; therefore, the translation Hamiltonian still satisfies the two constraints. Notice that the coefficients before V are convergent rather quickly, which determines the order of coefficients in the ultimate Hamiltonian also to converge quickly. The convergence is as rapid as being of the order of $\frac{1}{2^k k!}$. Thus, one can always perform a series of transformations to eliminate all the odd terms in Hamiltonian.

The conservation character of MFA is beyond the tree level. In the initial Hamiltonian, there exist interactions such as $\bar{\Psi}\boldsymbol{p}\Psi$ where $|\boldsymbol{p}| > m$. To apply renormalization theory we should integrate out the degree of interactions with $|\boldsymbol{p}| > \mu$ where $mv < \mu < m$. As argued in Ref.[7], this integration leads to additional local interactions imposed on the initial Hamiltonian H. But since we think that the final Hamiltonian of QCD also satisfies the two constraints, the conservations of modified quarks and antiquarks still hold true in the renormalization theory, although the unitary transformations and therefore the ultimate Hamiltonian should be modified at this time. For the renormalization theory in Ref.[10], most of its results still hold here although it is working on classical FWT theory.

The other modification of Hamiltonian originates from the truncation of Hamiltonian according to v in practice, since the ultimate Hamiltonian has infinite terms. To include the effects of interactions of the higher order, the coefficients of the truncated Hamiltonian should also be modified. For instance, the six-fermion interactions can modify the four-fermion interactions through a quark propagator, if we do not write these six-fermion interactions in the practice Hamiltonian. The detailed structures of modified quarks are shown in Section 4.3.

We clarify the conservation of MFA from two aspects. Firstly, these results cannot be generalized to the cases of light quark freedom and gluon freedom. That is, it is hard to define modified light quark or gluon with conservational properties. Therefore, for example, for a meson η_{c}, there exist relative amplitudes of other operators with higher dimensions or with higher powers of typical velocity v of heavy quark in the meson, which act on the physical vac-

uum states, besides the dominant operator that belongs to representation colour-singlet ${}^1S_0$, $\varphi_c^\dagger\gamma_5\bar{\chi}_c$.[17] These operators include $c\bar{c}g$, four-fermion interaction, etc, provided that the total $J^{\rm PC}$ of the operators is equal to 0^{-+}. This phenomenon has been studied through quite a lot of NRQCD approaches (c.f. Ref.[12]). Secondly, the conservation of MFA is attributed to the unitary transformation between MFA and FA (fermion/antifermion). To describe mesons bounded by strong interactions, we always use the notation of MFA in an effective model, such as potential model, NRQCD or potential NRQCD (pNRQCD). However, it is more convenient to use the notation of FA in many other high energy processes, such as decay by electric-weak interaction. The difference between FA and MFA implies that one cannot take results in the basis of MFA as inputs in the basis of FA. For instance, one can extract different condensates using lattice NRQCD, while applying these results in the decay process one should perform corrections that are due to the FWT transformation. On the other hand, there are some relationships between MFA and FA. Some of them are shown in Section 4.3. The relationships between them imply that these theories are in fact different sides of QCD.

Combining these two aspects, the conservation property of MFA is not in contradiction with the statement of meson–gluon mixing,[13] for these two statements are in different bases.

Although at leading order the Hamiltonian of FWT theory $H^{\rm FWT}$ and that of NRQCD theory $H^{\rm NRQCD}$ are equal, $H^{\rm FWT} = H^{\rm NRQCD}$, it is not a general conclusion beyond leading order. As is well known, the discrepancy between the ultimate Hamiltonian in FWT theory with notation of modified quark and the initial Hamiltonian with notation of asymptotic (free) quark lies in a unitary transformation. Therefore, their evolution matrices $S^{\rm FWT}$ and $S^{\rm QCD}$ also have a distinction in unitary transformation. Since the evolution matrix $S^{\rm NRQCD} = S^{\rm QCD}$ at any order by matching conditions, one has $S^{\rm FWT} = \mathcal{U}S^{\rm NRQCD}\mathcal{U}^\dagger$. This unitary transformation can be expanded according to v, which is considered as a small quantity. $S^{\rm FWT} = S^{\rm NRQCD} + o(v)$, or, we have a relation $H^{\rm FWT} = H^{\rm NRQCD} + o(v)$. In other words, only at leading order, can we have $H^{\rm FWT} = H^{\rm NRQCD}$.

A more crucial difference between quantum FWT and NRQCD is that here we are working in functional space while in NRQCD we are working in function space. To see it more clearly, we consider a vacuum state. Then $H_{\rm F}\Theta_0 = E_0\Theta_0$. As seen in the appendix, in the language of modified quark and antiquark, we obtain functional $\Theta_0(\varphi^\dagger, \varphi, \chi, \chi^\dagger) = 1$ and $E_0 = 0$. It is obvious that there is neither modified quark nor modified antiquark in the vacuum. For quenched QED, we obtain functional $\Theta_0(A_i(\boldsymbol{p}))$ of Eq.(31).

There are also two differences between the quantum version and the classical version of FWT theory, which are shown in Refs.[1,2] and especially a lattice version of FWT theory in Ref.[16]. Firstly, in the quantum version, we use time-independent operators to construct the Hamiltonian. Therefore, we obtain a time-independent Hamiltonian, even with the consideration of chromo-electric interactions. Thus, for instance, the time evolution of a free meson with a definite mass M is expressed as $\exp(-Et)$ where E is time-independent energy. But in the classical FWT theory we have to take a time-dependent Hamiltonian when we consider the effect of chromo-electric fields. Secondly, we obtain multi-fermion interactions which do not exist in the classical version at a tree level.

4. Improvement of action on lattice

In this section we first study the action on lattice by utilizing a distribution function $\mathcal{Z}$. A similar case in classical version was studied in Ref.[16], in which however, only the case of fermion fields was investigated. We study here the cases of both the fermion fields and the gauge fields. Surely we are now working in the quantum version. In Section 4.3 we show some results of the structure of MFA and physical vacuum.

4.1. Improvement on gauge fields

On lattice, we often use the chromo-electric field $\boldsymbol{E}$ instead of $\boldsymbol{\Pi}$. Here we show the relationships between $\boldsymbol{\Pi}$ and $\boldsymbol{E}$ on lattice. Then we show that it is unnecessary to improve the covariant difference along the time direction on lattice. The distribution function on lattice is

$$\begin{aligned}\mathcal{Z} = & \int [{\rm d}\boldsymbol{A}_t][{\rm d}\psi_t^\dagger d\psi_t]\ldots\langle\boldsymbol{A}_{t+1}(\boldsymbol{x})\\ & \times\langle|a_{t+1}(\boldsymbol{x})|{\rm e}^{-Ha}|a_t(\boldsymbol{x})\rangle|\boldsymbol{A}_t(\boldsymbol{x})\rangle\\ & \times\exp\left(-\int {\rm d}\boldsymbol{x}\psi_t^\dagger(\boldsymbol{x})\psi_t(\boldsymbol{x})\right)\langle\boldsymbol{A}_t(\boldsymbol{x})|\langle a_t(\boldsymbol{x})|\ldots,\end{aligned} \tag{26}$$

where subscripts are time indices, lattice spacing a will always be set as being unitary thereinafter and $|a_t\rangle$ ($\langle a_t|$) is the right (left) eigenstate for annihilator $\hat{a}$ (producer $\hat{a}^\dagger$).

We write out only the distribution related to the gauge fields:

$$\begin{aligned}\mathcal{Z}_{\rm G} = & \int [{\rm d}\boldsymbol{A}_t(\boldsymbol{x})]\ldots\langle \boldsymbol{A}_{t+1}(\boldsymbol{x})| \\ & \cdot\exp\left\{-\sum_{\boldsymbol{x}}\frac{1}{2}(-\boldsymbol{\Pi}^2(\boldsymbol{x})+\boldsymbol{B}_t^2(\boldsymbol{x})) \right. \\ & +\frac{g}{8m^2}\psi^\dagger\boldsymbol{\Sigma}\cdot(\boldsymbol{D}\times\boldsymbol{\Pi}-\boldsymbol{\Pi}\times\boldsymbol{D})\psi \\ & \left.-\frac{{\rm i}g}{8m^2}\psi^\dagger(\boldsymbol{D}\cdot\boldsymbol{\Pi}-\boldsymbol{\Pi}\cdot\boldsymbol{D})\psi\right\} \\ & \cdot|\boldsymbol{A}_t(\boldsymbol{x})\rangle\ldots. \end{aligned} \tag{27}$$

Insert $\int[{\rm d}\boldsymbol{\Pi}]|\boldsymbol{\Pi}\rangle\langle\boldsymbol{\Pi}|$ into the above equation and integrate out conjugate field $\boldsymbol{\Pi}$ by utilizing Eq.(3) and the stationary technique since Π is anti-hermitian. After rewriting the final result in gauge invariant form, one obtains

$$\begin{aligned}\mathcal{Z}_{\rm G} = & \int [{\rm d}A_\mu(t,\boldsymbol{x})] \\ & \cdot\exp\Big\{-\sum_{\boldsymbol{x},t}\frac{1}{2}{\rm tr}(\boldsymbol{E}^2(t,\boldsymbol{x})+\boldsymbol{B}^2(t,\boldsymbol{x})) \\ & +\frac{g}{8m^2}\psi^\dagger\boldsymbol{\Sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}-\boldsymbol{E}\times\boldsymbol{D})\psi \\ & -\frac{{\rm i}g}{8m^2}\psi^\dagger(\boldsymbol{D}\cdot\boldsymbol{E}-\boldsymbol{E}\cdot\boldsymbol{D})\psi\Big\}, \end{aligned} \tag{28}$$

up to an irrelevant constant and operators with higher order in spatial lattice. In the above equation we have defined the chromo-electric field in a two-leaf-clover form as

$$\begin{aligned}E_i(t,\boldsymbol{x}) = & \frac{1}{4}\{(U_0(t,\boldsymbol{x})U_i(t+1,\boldsymbol{x})U_0^\dagger(t,\boldsymbol{x}+\hat{\boldsymbol{i}}) \\ & \cdot U_i^\dagger(t,\boldsymbol{x})-U_0(t,\boldsymbol{x})U_i^\dagger(t+1,\boldsymbol{x}-\hat{\boldsymbol{i}}) \\ & \cdot U_0^\dagger(t,\boldsymbol{x}-\hat{\boldsymbol{i}})U_i(t,\boldsymbol{x}-\hat{\boldsymbol{i}}))-{\rm h.c.}\}. \end{aligned} \tag{29}$$

This equation is the same as that for lattice NRQCD and Fermilab action, except for the expression of $\boldsymbol{E}$. This means that one can improve the accuracy of the discrete form of chromo-magnetic field by next-leading terms or next-next-leading terms, with the discrete form of chromo-electric field just kept invariant. The correct form of chromo-electric field $\boldsymbol{E}$ is just the covariant forward difference, at least up to v^6. This statement is generally strict if one studies only pure gauge fields.

This is an advantage of the lattice theory in the Hamilton form. In the Lagrange form the chromo-electric fields have to be defined as a covariant derivative along the time direction, which implies that it seems we should use the covariant difference along the time direction to represent the chromo-electric fields on lattice. As is well known,[16] this representation has many disadvantages on lattice, so we always use an 'approximate form': covariant forward or backward difference on lattice as a substitute for the covariant difference. In contrast, in the Hamilton form, this approximate form is just the exact expression of the 'chromo-electric fields'.

For the lattice gauge theory, there seems to be little difference in action between starting from Lagrangian and starting from Hamiltonian. But, this is not true. Deep-going study shows that our action will deviate from the action written directly from Lagrangian at a higher order, for instance, at $o(a^8)$.

Since there is no admixture between heavy quarkonia and heavy hybrid quarkonia with pure glueballs, we can use the gauge invariant operators containing only gauge degree to extract glueballs. For instance, we use an operator ${\rm tr}B_i^aB_i^a$[19] to extract 0^{++} glueball. One question arises, then, just as raised by some persons, whether it is also necessary to include chromo-electric fields to extract the glueball state. The answer is negative.

To illuminate this statement we recall that in the quantum dynamic a harmonic system can be depicted by using the Hamiltonian as $\frac{1}{2}(-\partial_x^2+x^2)\psi(x)=E\psi(x)$. The wavefunction corresponding to a state with energy E_i is $\psi_i(x)=H_i\psi_0(x)$ where $\psi_0={\rm e}^{-x^2}$ is the vacuum wavefunction and H_i is Hermite polynomial, for instance, $H_0=1$, $H_1=2x$, $H_2=4x^2-2$, and so on. When working in path integral quantization, the operator x acting on a vacuum state extracts the $\frac{1}{2}\psi_1$ state, while the operator x^2 extracts the state $\frac{1}{4}\psi_2+\frac{1}{2}\psi_0$, which means that when using x^2 to extract state ψ_2, one should perform a vacuum subtraction, and so on. But, the operator $-{\rm i}p=\partial_x$ acting on vacuum state does not extract new state: $\partial_x\psi_0=-\frac{1}{2}\psi_1$.

It is similar in the field theory. It is enough to discuss this by using only gauge fields. An arbitrary state, including vacuum, can be written as a functional with respect to canonical fields and the covariant (spatial) derivative of the canonical fields: $\Theta=\Theta(\boldsymbol{A})$, at

a definite time. If we require the state to be a gauge invariant, the functional will be functional of chromomagnetic fields and their covariant (spatial) derivative, $\Theta = \Theta(\boldsymbol{B}, \boldsymbol{D})$. In Fock space, it is not obvious on lattice to define a chromo-electric field at a definite time slice, for the definition of the chromo-electric field should come down to the functional at two time slices (in quantum FWT theory) or more time slice (in classical FWT theory). But, from the argument in obtaining Eq.(28) one may propose an equivalent scheme: constructing glueball by utilizing conjugate fields, $\boldsymbol{\Pi}$. From Eq.(2), it is equivalent to using an operator $-\mathrm{i}\dfrac{\partial}{\partial A_i^a}$ acting on a state functional, which will produce another state functional with respect to the canonical fields and the covariant derivative of the canonical fields. Since the generated state functional should be a gauge invariant, it can also be written as the form of $\Theta'(\boldsymbol{B}^a, \boldsymbol{D})$. Then, running over all the independent colour-singlet functionals, of which the independent variables consist of chromomagnetic fields and their covariant derivative, implies completing all the eigenstates of Hamiltonian. These functional can be categorized into different J^{PC} according to the representation of group $SO(3)^{\mathrm{PC}}$. In the basis of canonical field A_i^a (and therefore, chromomagnetic field B_i^a), these functionals can be considered as being generated by corresponding operators acting on a vacuum state, which include the operator of chromomagnetic fields and their covariant derivative operator. Therefore, performing action on a vacuum state by using all the independent colour-singlet operators consisting of chromomagnetic fields and their covariant (spatial) derivative in the vacuum will generate all the independent state wavefunctionals.

We exemplify this statement by the case of quenched (free) QED. The equation of wave functional is

$$\begin{aligned} &\frac{1}{2}\int \mathrm{d}\boldsymbol{x}\left(-\frac{\partial^2}{A_i(\boldsymbol{x})^2}+B_i(\boldsymbol{x})^2\right)\Theta(A_i(\boldsymbol{x})) \\ =\;& E\Theta(A_i(\boldsymbol{x})). \end{aligned} \tag{30}$$

We obtain its vacuum wavefunctional as follows:

$$\begin{aligned} &\Theta_0(A_i(\boldsymbol{p})) \\ \propto\;& \exp\left\{-\int \mathrm{d}\boldsymbol{p}\frac{1}{2p}A_i(-\boldsymbol{p})(p^2\delta_{ij}-p_ip_j)A_j(\boldsymbol{p})\right\} \\ =\;& \exp\left\{-\int \mathrm{d}\mathbf{p}\frac{1}{2p}B_i(-\boldsymbol{p})B_i(\boldsymbol{p})\right\}, \end{aligned} \tag{31}$$

which is gauge-independent. To extract a definite J^{PC} state, one can use the gauge invariant operators acting on a vacuum state, which determine the property of the state. For instance, up to a phase factor, suppose $\boldsymbol{p} = (0, 0, p_3)$, then the functional generated by operator $A_1(\boldsymbol{p}) \propto \dfrac{p_3}{\boldsymbol{p}^2}B_2(\boldsymbol{p})$ will generate a single photon, $A_1(\boldsymbol{p})\Theta_0 \propto \dfrac{p_3}{\boldsymbol{p}^2}B_2(\boldsymbol{p})\Theta_0$, of which the polarization direction is along the x-axis, i.e. perpendicular to the momentum $\boldsymbol{p}$. However, it is not difficult to verify that $\mathit{\Pi}_1(\boldsymbol{p})$, the conjugate field of canonical field $A_1(\boldsymbol{p})$, will also generate such a state, up to a normalization factor. (It is worth paying special attention to it and we shall make comment on it elsewhere.)

In summary, as long as we have run over all the independent colour-singlet operators, which are made up of chromomagnetic fields and can be categorized into different J^{PC}, we shall obtain all the independent wave functionals of glueballs. Therefore, it is unnecessary to use chromo-electric fields in the calculation of glueball spectrum, of which the result will be used to argue whether there exist three 2^{++} glueballs in the $2-3$GeV range in the future. Analogously, we do not use operators such as $\bar{\psi}\boldsymbol{E}\psi$ to extract hybrid mesons. Using chromo-electric field operators to construct glueballs may be considered as being their checking.

4.2. Improvement on Fermion fields

Here we study the improvement on fermion fields, which can be generalized into Fermi laboratory form[16] or NRQCD form.[4,7,8] We rewrite the Hamiltonian of fermion fields here

$$\begin{aligned} H_{\mathrm{F}} =\;& \varphi^\dagger\left(m-\frac{D^2}{2m}+\frac{g\boldsymbol{\tau}\cdot\boldsymbol{B}}{2m}-\frac{D^4}{8m^3}\right)\varphi \\ &+\chi\left(m-\frac{D^2}{2m}+\frac{g\boldsymbol{\tau}\cdot\boldsymbol{B}}{2m}-\frac{D^4}{8m^3}\right)^{\mathrm{T}}\chi^\dagger \\ &-\frac{\mathrm{i}g}{8m^2}\{\varphi^\dagger[\boldsymbol{\tau}\cdot\boldsymbol{D},\boldsymbol{\tau}\cdot\boldsymbol{E}]\varphi-\chi[\boldsymbol{\tau}\cdot\boldsymbol{D},\boldsymbol{\tau}\cdot\boldsymbol{E}]^{\mathrm{T}}\chi^\dagger\} \\ &+\frac{\alpha_{\mathrm{s}}\pi}{m^2}\varphi^\dagger T^a\tau^i\chi\chi^\dagger T^a\tau^i\varphi. \end{aligned} \tag{32}$$

Then the distribution function for the fermion part is

$$\begin{aligned} \mathcal{Z}_F &= \mathrm{tr}(\mathrm{e}^{-H_{\mathrm{F}}}) \\ &= \int[\mathrm{d}\varphi\mathrm{d}\varphi\mathrm{d}\chi\mathrm{d}\chi^\dagger]\ldots|\varphi_{t+1}\rangle I_t\langle\chi_{t-1}|\ldots, \end{aligned} \tag{33}$$

where

$$I_t = \mathrm{e}^{-\varphi_t^\dagger\varphi_{t+1}}\langle\chi_t|\langle\varphi_t^\dagger|\mathrm{e}^{-H_{\mathrm{F}}}|\varphi_t\rangle|\chi_t^\dagger\rangle\mathrm{e}^{-\chi_{t-1}\chi_t^\dagger}. \tag{34}$$

Here subscript t denotes time. After some direct calculations by using Berezin integral, we obtain the results under anti-periodic condition for fermion fields

as

$$\mathcal{Z}_{\rm F} = \int [{\rm d}\bar{\psi}{\rm d}\psi] \exp\{-\sum_{t,\boldsymbol{x}} S\}, \tag{35}$$

where

$$\begin{aligned} S = \ & \bar{\psi} D_t^w \psi + \bar{\psi} \left[(1 - {\rm e}^{-m}) - \frac{{\rm e}^{-m} D^2}{2m} \right. \\ & \left. + \frac{{\rm e}^{-m} g \Sigma \cdot \boldsymbol{B}}{2m} - \frac{{\rm e}^{-m} D^4}{8m^3}(1+m) \right] \psi \\ & - \frac{{\rm i}g{\rm e}^{-m}}{8m^2} \psi^\dagger [\boldsymbol{\tau} \cdot \boldsymbol{D}, \boldsymbol{\tau} \cdot \boldsymbol{E}] \psi \\ & - \frac{\alpha_s \pi {\rm e}^{-2m}}{4m^2} \bar{\psi}(1+\gamma_0)\gamma_i T^a \psi \bar{\psi}(1-\gamma_0)\gamma_i T^a \psi. \end{aligned} \tag{36}$$

In the above equation $D_t^w = \gamma_0 D_t + \frac{1}{2} D_t^2$ where $D_t = \frac{1}{2}(D_t^+ + D_t^-)$, $D_t^2 = D_t^+ - D_t^-$, and D_t^+ (D_t^-) is the covariant forward (backward) difference along the time direction. Just like Eq.(28), we have ignored the higher-order operators in spatial directions here.

Here we encounter a similar situation to that we have in the lattice gauge field theory. It seems that one should perform higher covariant time difference on lattice to improve the accuracy of the discrete form of the covariant time derivation of fermion fields when working in the Lagrange frame. In contrast, in the Hamilton form, the time derivation should be expressed just as the time difference with the Wilson type on lattice and the improvement on it is not necessary.

Notice here that S is not equal to the Lagrangian $\mathcal{L}$. This is because of a finite lattice spacing. In fact, $S = \mathcal{L}$ is only a special case even in the continuum. Here we see that, on the discrete lattice, $S \neq \mathcal{L}$ generally. Surely, these correction terms should vanish in the continuum: $m \to 0$. But on the lattice with a finite spacing, we cannot ignore them.

We substitute the static mass of quark m by $1 - {\rm e}^{-m}$ in the path integrand, which possesses a property that the pole position of quark on lattice just equals the current mass of the associated quark. To see this one can neglect all the higher terms and write out the propagator of static quarks: $G(p_0, \mathbf{0}) = ({\rm e}^{{\rm i}p_0} - 1 + (1 - {\rm e}^{-m}))^{-1}$. The propagator has pole at $E(= -{\rm i}p_0) = m$. Just as expected, the decay behaviour of correlation of fermion with mass m is like ${\rm e}^{-mt}$.

On lattice one should use the covariant difference $\sum_i \Delta_i \left(1 - \frac{1}{12}\Delta_i\right)$, where $\Delta_i = D_i^+ D_i^-$ and D_i^+ (D_i^-) is covariant forward (backward) difference operator in direction i ($i = 1, 2, 3$), to substitute covariant differential operator $\sum_i D_i^2$. The evolution of propagator, if it should be stable[18] ($|1 - H| < 1$), requires that $m > 1.7$. Therefore, $D \sim mv < 1 < \frac{m}{1.7}$, or, $1.7 < m < \frac{1}{v}$. This procedure has a narrow window to perform calculations for ψ family but it has enough room to perform calculations for Υ family. This window can be widened by introducing the gauge field or, by setting $1 - {\rm e}^{-m}$ as being unitary, which decouples the static energy from the dynamic energy of fermion completely.

In summary, Sections 4.1 and 4.2 are devoted to the quantum version of theory introduced in Ref.[16]. But, as we are working in quantum theory on lattice while Ref.[16] is in the classical version on lattice, there occur some differences between them. Some of them are shown in the last paragraph in Section 3. However, there are some other differences. A significant one is, for example, the treatment of chromo-electric fields. In Ref.[16], a common discretization for chromo-electric field is the use of the four-leaf clover terms, which will cause some trouble. As stated there and in Ref.[22], the transfer matrix in theory, which contains derivatives of gauge fields over more than two time slices, is not positive. However, there is no chromo-electric field in the Hamiltonian in the quantum version; instead, and we have the conjugate operators, Π_is. After the path quantization and integrating out the conjugate operators, we meet covariant forward difference operators along the time direction, $D_t^+ \boldsymbol{A}$s, which are the lattice definitions of chromo-electric fields. Therefore, here we have got rid of the trouble we met in Ref.[16].

4.3. The structure of modified fermion and vacuum

Although current fermions and associated modified fermions in quarkonia are both fermion operators and their many quantum quantities, such as charge, spin and so on, are the same, there exist differences between them.

Firstly, if the free quark is point-like as we consider, the modified quark will spread over a finite region, of which the size is about Compton wavelength.[1] As $\boldsymbol{D} \sim {\rm i}\boldsymbol{p}$ stands in superscript and it plays a role of phase factor, the larger the momentum of fermion, the smaller the region that the modified fermion occupies.

Secondly, the modified quark has not only the component of the associated current quark, but also components of gauge degree, multi-quark degree, and even more, the degrees of other types of free quarks. For instance, a modified quark should have degree of multi-quark through the transformations such as $\exp\left\{\frac{3\alpha_s\pi}{32m^3}((\chi^\dagger\varphi)^2-(\varphi^\dagger\chi)^2)\right\}$ in Section 3. In bound states, one can calculate these components, such as two couples of current quarks in meson by using the notation of modified quark.

To exemplify it, we consider how a modified quark also includes the degree of other types of current quarks. We suppose that there exist two types of fermions, then the Hamiltonian of fermion can be written as

$$H_{\rm F}^0 = m_1\sum_x \Psi_1^\dagger\left(\beta\left(1+\frac{\not{D}}{m_1}\right)\right)\Psi_1 + m_2\sum_x \Psi_2^\dagger\left(\beta\left(1+\frac{\not{D}}{m_2}\right)\right)\Psi_2. \quad (37)$$

After performing unitary transformations $\exp\left\{\frac{1}{2m_1}\Psi_1^\dagger\not{D}\Psi_1\right\}$ and $\exp\left\{\frac{1}{2m_2}\Psi_2^\dagger\not{D}\Psi_2\right\}$, we have four-fermion interaction

$$H_4 = \frac{-g^2}{8m_1^2}(\Psi_{1\rho}^{\dagger k}\Psi_{1\mu}^{\dagger i}\Psi_{1\lambda}^{l}\Psi_{1\nu}^{j}+\Psi_{2\rho}^{\dagger k}\Psi_{2\mu}^{\dagger i}\Psi_{2\lambda}^{l}\Psi_{2\nu}^{j}) \times\gamma_{\mu\nu}^m\gamma_{\rho\lambda}^m T_{kl}^a T_{ij}^a + \frac{g^2}{4m_1m_2}\Psi_2^\dagger\gamma_i T^a\Psi_2\Psi_1^\dagger\gamma_i T^a\Psi_1. \quad (38)$$

Both the two types of fermion number conservations are broken by the last term in the above expression. To eliminate this term we should perform a unitary transformation that takes the admixture between these two fermions. This means that a modified fermion, such as a modified charm, has a small component of other types of fermion pairs, such as bottom quarks. This phenomenon will contaminate our measurements of Cabibbo–Kobayashi–Maskawa (CKM) matrix, although this contamination is vanishing when $V(\mathrm{CKM})=1$. Unfortunately, this contamination is just a unitary transformation and therefore it is unmeasurable.

The particle numbers of MFA are conservational quantities in mesons, such as in quarkonia. This implies that the modified fermion is a physical notation while the current fermion is not, at least in quarkonium. A quarkonium is therefore made up of two constituents with a finite size. In contrast, if the current fermion were physical notation, there would be non-vanishing possibility to measure four point-like constituents in a quarkonium.

There exist such Feynman diagrams in perturbation QCD that one fermion and its associated antifermion are annihilated into one or more gluons, which, successively, decay into other type of FA pair or multi-pairs of fermion/antifermion. For instance, in $c\bar{c}$ quarkonium, one couple of $c\bar{c}$ produces one couple of $b\bar{b}$ in Ψ bound states. Because of time inverse invariant, the inverse process is also able to occur. If there is no wavefunctional collapse, the relative phases of $c\bar{c}$ and $b\bar{b}$ are correlative and one can therefore define a modified charm operator, to make the number of the corresponding fermion conservational. But when we have measured a decay or collision event, for example, a decay by weak interaction, this statement will be invalid in meaning by the wavefunctional collapse. Therefore, the conservation of modified quark number is valid only in bound states.

In the following, we discuss the structure of QCD vacuum, especially the content of heavy quark freedom. The conservation property shows that there is no MFA in vacuum. However, in many high energy processes, such as collision process or decay process, it is more convenient to use the notation of FA. The number of current fermion is not a conservational quantity in QCD bound states, there exist zero, one, two, ..., pairs of FA at any position in QCD vacuum. The QCD vacuum is therefore a complex superposition state of the perturbation vacuum, one pair of FAs, two pairs of FAs and so on.

To clarify this we consider a toy model. Suppose that the ultimate of FWT transformation is a very simple unitary transformation

$$\mathcal{U}=\exp\left\{\frac{1}{2m}\int \mathrm{d}\boldsymbol{x}\Psi^\dagger\not{D}\Psi\right\}\approx 1+\frac{1}{2m}\int \mathrm{d}\boldsymbol{x}\Psi^\dagger\not{D}\Psi,$$

in other words, the difference between MFA and FA is a known unitary transformation $\mathcal{U}$. Under such a transformation, if a state, particularly vacuum state, can be written as Θ_0 in the basis of MFA, or Θ_0' in the basis of FA, we have the relation $\Theta_0'=\mathcal{U}^\dagger\Theta_0$, from Eq.(13). We know that the number of MFA in vacuum is vanishing, that is $\Theta_0(\varphi,\,\varphi^\dagger,\,\chi,\,\chi^\dagger)=1$ in the basis of MFA. Therefore, in the basis of FA, we have $\Theta_0'(\varphi^\dagger,\varphi,\chi,\chi^\dagger)=\mathcal{U}^\dagger\Theta_0=1-\frac{1}{2m}\int \mathrm{d}\boldsymbol{x}\varphi^\dagger\boldsymbol{\tau}\cdot D\chi+\cdots$, which is a superposition state of trivial vacuum (or perturbation vacuum, in which there are no FA or FA pairs.) states, one pair of FA, etc.

This superposition stems from the quantum

mechanism, especially from the conservational property of MFA. Since there are pairs of FA in the vacuum, we should obtain the non-vanishing quark condensates in QCD vacuum, although the vacuum energy related to the fermion part is vanishing.

Here we show how to compute it in the first order. To do it we turn it into the Fourier space. We write

$$\begin{aligned}\Psi &= \frac{1}{\sqrt{\Omega}}\sum_{\boldsymbol{p},s}\left(a^i_{\boldsymbol{p},s}u^i_{\boldsymbol{p},s}+b^{\dagger i}_{-\boldsymbol{p},s}v^i_{-\boldsymbol{p},s}\right)\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}},\\ \Psi^\dagger &= \frac{1}{\sqrt{\Omega}}\sum_{\boldsymbol{p},s}\left(a^{\dagger i}_{\boldsymbol{p},s}u^{\dagger i}_{\boldsymbol{p},s}+b^i_{-\boldsymbol{p},s}v^{\dagger i}_{-\boldsymbol{p},s}\right)\mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}},\end{aligned} \tag{39}$$

where a ($a^\dagger$) and b ($b^\dagger$) annihilate (produce) fermion and antifermion respectively, and

$$\begin{aligned}u^i_{\boldsymbol{p},s} &= \frac{1}{\sqrt{2E_p(m+E_p)}}\begin{pmatrix}(m+E_p)\varphi^i_s\\ -\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{p}\varphi^i_s\end{pmatrix},\\ v^i_{-\boldsymbol{p},s} &= \frac{1}{\sqrt{2E_p(m+E_p)}}\begin{pmatrix}-\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{p}\chi^i_s\\ (m+E_p)\chi^i_s\end{pmatrix}.\end{aligned} \tag{40}$$

In the above equations, Ω is vacuum volume, subscript s denotes helicity, superscript i is colour while φ and χ are spinors of rest fermion and rest antifermion respectively.

Also in the toy model, suppose that we can write Θ_0 and Θ_0' as $|\rangle_0$ and $|\rangle$ respectively, then $|\rangle=\mathcal{U}^\dagger|\rangle_0$. Since $|\rangle_0$, the vacuum wavefunctional in the basis of MFA is known, this relation reveals how to write $|\rangle$, the vacuum functional in the basis of FA. To study the condensate we do not work in the basis of MFA but in the basis of FA. Then for a condensate of current $\bar\Psi^i\Psi^i$ (here $\bar\Psi$ and Ψ are both in the basis of FA), we have

$$\langle|:\bar\Psi^i\Psi^i:|\rangle=\langle|\mathcal{U}:\bar\Psi^i\Psi^i:\mathcal{U}^\dagger|\rangle_0. \tag{41}$$

Let $\mathcal{U}=\mathrm{e}^V$, then

$$\begin{aligned}V=\sum_{\boldsymbol{p}\boldsymbol{p}'ss'ij}&\frac{-\mathrm{i}}{2m\sqrt{2E_p(m+E_p)}\sqrt{2E_{\boldsymbol{p}'}(m+E_{\boldsymbol{p}'})}}\\ &\times\{a^{\dagger i}_{\boldsymbol{p},s}a^j_{\boldsymbol{p}',s'}\varphi^{\dagger i}_sC^{ij}(\boldsymbol{p},\boldsymbol{p}')\varphi^j_{s'}\\ &+b^i_{-\boldsymbol{p},s}b^{\dagger j}_{-\boldsymbol{p}',s'}\chi^{\dagger i}_sC^{ij}(\boldsymbol{p},\boldsymbol{p}')\chi^j_{s'}\\ &+a^{\dagger i}_{\boldsymbol{p},s}b^{\dagger j}_{-\boldsymbol{p}',s'}\varphi^{\dagger i}_sB^{ij}(\boldsymbol{p},\boldsymbol{p}')\chi^j_{s'}\\ &+b^i_{-\boldsymbol{p},s}a^j_{\boldsymbol{p}',s'}\chi^{\dagger i}_sB^{ij}(\boldsymbol{p},\boldsymbol{p}')\varphi^j_{s'}\}.\end{aligned} \tag{42}$$

Here,

$$\begin{aligned}C^{ij} &= \mathrm{i}(\not{p}\not{q}'^{ij}(m+E_{\boldsymbol{p}'})-\not{q}'^{ij}\not{p}'(m+E_p)),\\ B^{ij} &= (m+E_p)(m+E_{p'})\not{q}'^{ij}+\not{p}\not{q}'^{ij}\not{p}',\end{aligned} \tag{43}$$

where $\not{p}'=\tau_ip_i$, energy $E_p=\sqrt{m^2+p^2}$ and $q^{ij}_k(p,p')=p_k\delta_{ij}\delta_{\boldsymbol{p},\boldsymbol{p}'}+A^a_k(\boldsymbol{p}-\boldsymbol{p}')T^a_{ij}$.

One also has

$$\begin{aligned}\sum_{\boldsymbol{x}}:\bar\Psi^i\Psi^i:=&\sum_{\boldsymbol{p},s}\frac{m}{E_p}\left(a^{\dagger i}_{\boldsymbol{p},s}a^i_{\boldsymbol{p},s}+b^{\dagger i}_{-\boldsymbol{p},s}b^i_{-\boldsymbol{p},s}\right)\\ &-\sum_{p,s,s'}\frac{\mathrm{i}}{E_p}\left(a^{\dagger i}_{\boldsymbol{p},s}b^{\dagger i}_{-\boldsymbol{p},s'}\varphi^{\dagger i}_s\not{p}'\chi^i_{s'}\right.\\ &\left.-b^i_{-\boldsymbol{p},s}a^i_{\boldsymbol{p},s'}\chi^{\dagger i}_s\not{p}'\varphi^i_{s'}\right)\\ \equiv\ &G+T.\end{aligned} \tag{44}$$

Then from Eq.(44), in the first order, the condensate for G is

$$\begin{aligned}\langle G\rangle=&2\times\frac{1}{(2m)^2}\\ &\sum_{\boldsymbol{p}\boldsymbol{p}_1si i_1}\left\langle\frac{mb^i_{-\boldsymbol{p},s}b^{\dagger i}_{-\boldsymbol{p},s}\chi^{\dagger i}_sB^{ii_1}(\boldsymbol{p},\boldsymbol{p}_1)B^{i_1i}(\boldsymbol{p}_1,\boldsymbol{p})\chi^i_s}{E_{p_1}2E_p(m+E_p)2E_{p_1}(m+E_{p_1})}\right\rangle_0.\end{aligned} \tag{45}$$

Similarly,

$$\langle T\rangle=-2\left\langle\sum_{\boldsymbol{p}si}\frac{b^i_{-\boldsymbol{p},s}b^{\dagger i}_{-\boldsymbol{p},s}\chi^{\dagger i}_sB^{ii}(\boldsymbol{p},\boldsymbol{p})\chi^i_s}{2m2E_p^2(m+E_p)}\right\rangle_0. \tag{46}$$

When going to infinite volume $\Omega\to\infty$, one should replace $\sum\limits_{\boldsymbol{p}}$ by $\displaystyle\int\frac{\mathrm{d}\boldsymbol{p}}{8\pi^3}$. The integration of the above two equations are divergent and we take a cutoff $p<\Lambda$.

Concerning the factor 6 from colour space and spinor space in the above two expressions and noticing $B^{ij}(\boldsymbol{p},\boldsymbol{p}')\simeq(m+E_p)(m+E_{p'})\not{p}'\delta_{ij}\delta_{\boldsymbol{p},\boldsymbol{p}'}$, we obtain

$$\begin{aligned}\langle T+G\rangle=&-\frac{3\Lambda^4}{2\pi^2m}\int_0^1\mathrm{d}p\,\frac{p^4\left(x+\sqrt{p^2+x^2}\right)}{p^2+x^2}\\ &\times\left(1-\frac{x+\sqrt{x^2+p^2}}{4\sqrt{x^2+p^2}}\right),\end{aligned} \tag{47}$$

where $x=\dfrac{m}{\Lambda}$. When $x\gg1$, that is, in the case of heavy quarks such as bottom quarks and top quarks, the condensate will decrease as m^{-2} for in this case the integration in Eq.(47) can be written as $\displaystyle\int_0^1\mathrm{d}p\,\frac{3p^4}{4x}$. If we naively extract this result in the region $x\lesssim1$, this condensate will decreases like $1/m$ for in the case of $x<1$ the integration in Eq.(47) can be written as $\displaystyle\int_0^1\mathrm{d}p\,\frac{3}{4}p^3$.

Unlike Ref.[21], where the condensates originate from topological nontrivial and/or vacuum fluctuation, the condensate introduced here is a pure quantum effect. This is an interesting result that we here

have a non-vanishing condensate of fermion fields and a vanishing vacuum energy related to fermion part simultaneously.

5. Summary

To construct the Hamiltonian or Lagrangian of HQET, one may follow two approaches: one is Harvard theory and the other is (classical) FWT theory. The normalization of fields in the former is not correct while the latter cannot produce multi-quark interactions, for instance four-quark interactions. These two disadvantages possessed by the two theories have been remedied in the quantum FWT theory as shown in this paper.

There exist higher order time-derivative terms in Harvard theory. This is another flaw of the Harvard theory. When one goes into the lattice theory, the situation becomes worse for one should consider interactions beyond two neighbouring time slices. This flaw does not occur in quantum and classical FWT theories. But if the accuracy of the discretization is to be improved, there appear such bad interactions in theory.[10] There is no such embarrassment here. Furthermore, when the gauge fields are included, one suffers a similar trouble in the classical FWT theory. To construct the chromo-electric fields one should always perform interactions beyond two neighbouring time slices in the classical FWT theory. Here we overcome this by defining the chromo-electric fields in the two-leaf-clover form. This amounts to the statement that it is not necessary to improve the discrete form of chromo-electric fields and covariant time difference on lattice.

Just like the classic version, there exist some conservational quantities, the numbers of modified quarks and antiquarks, in bound states, although a more complicated situation appears in the quantum version. One can use such conservational quantities to distinguish between quarkonia and glueball states unambiguously. Furthermore, we have studied the contents of modified quarks in bound states, and by utilizing the technique of modified quark, we have also studied the structure of the vacuum, for instance, we have obtained a non-vanishing condensate of fermion fields and a vanishing vacuum energy related to the fermion part simultaneously.

Appendix: The Berezin integral

We first consider a system of fermion operators $\{\hat{a}_i, \hat{a}_i^\dagger\}$ $(i = 1, 2, \ldots)$: (Here $\hat{a}$ and $\hat{a}^\dagger$ denote producing and annihilating operators for fermion or antifermion respectively.)

$$\{\hat{a}_i, \hat{a}_j^\dagger\} = \delta_{ij}; \quad \{\hat{a}_i, \hat{a}_j\} = \{\hat{a}_i^\dagger, \hat{a}_j^\dagger\} = 0. \tag{A1}$$

The corresponding Grassmann variables satisfy

$$\{a_i, a_j^\dagger\} = \{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} = 0. \tag{A2}$$

One defines the right eigenstate of operator $\hat{a}_i$ as

$$|a\rangle = \mathrm{e}^{\hat{a}_i^\dagger a_i}|0\rangle, \tag{A3}$$

where $|0\rangle$ is the vacuum state. Similarly, the left eigenstate of operator $\hat{a}_i^\dagger$ may be defined as

$$\langle a| = \langle 0|\mathrm{e}^{a_i^\dagger \hat{a}_i}. \tag{A4}$$

One obtains

$$\begin{aligned} &\langle a|a\rangle = \mathrm{e}^{a_i^\dagger a_i}, \\ &\int [\mathrm{d}a^\dagger \mathrm{d}a]|a\rangle \mathrm{e}^{-a_i^\dagger a_i}\langle a| = 1. \end{aligned} \tag{A5}$$

The representation in terms of Grassmann variables for an arbitrary $\hat{A}$ in Fock space is

$$A(a^\dagger, a) \equiv \langle a|\hat{A}(\hat{a}^\dagger, \hat{a})|a\rangle, \tag{A6}$$

where A is the associated Grassmann quantity of operator $\hat{A}$. Here are some examples:

$$\begin{aligned} &\mathbf{1}(a^\dagger, a) = \exp(a_j^\dagger a_j); \\ &\hat{a}_k^\dagger(a^\dagger, a) = a_k^\dagger \exp(a_j^\dagger a_j); \\ &\exp(\hat{a}_k^\dagger \Lambda_{kl}\hat{a}_l(a^\dagger, a) = \exp(a_k^\dagger(\mathrm{e}^\Lambda)_{kl}a_l); \\ &\exp(\hat{a}_k^\dagger \Lambda_{kl}\hat{a}_l(a^\dagger, a))\exp(\hat{a}_k^\dagger \Gamma_{kl}\hat{a}_l(a^\dagger, a)) \\ = \;& \exp(a_k^\dagger(\mathrm{e}^\Lambda \mathrm{e}^\Gamma)_{kl}a_l). \end{aligned} \tag{A7}$$

Simply, we assume here that there is only one spinor component on each lattice grid $\hat{a}^\dagger$ $(\hat{a})$. We set the Hamiltonian of fermion as $H_{\mathrm{F}} = \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{y}\hat{a}^\dagger(\boldsymbol{x})A_{\boldsymbol{x},\boldsymbol{y}}\hat{a}(\boldsymbol{y})$. Suppose the vacuum state of fermion is $\Theta_0(a^\dagger, a) = \exp \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{y}a^\dagger(\boldsymbol{x})B_{\boldsymbol{x},\boldsymbol{y}}a(\boldsymbol{y})$, then one obtains

$$\begin{aligned} &\int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{y}\mathrm{d}\boldsymbol{z}a^\dagger(\boldsymbol{x})A_{\boldsymbol{x},\boldsymbol{y}}B_{\boldsymbol{y},\boldsymbol{z}}a(\boldsymbol{z})\Theta_0(a^\dagger, a) \\ = \;& E_0\Theta_0(a^\dagger, a). \end{aligned} \tag{A8}$$

Therefore, the vacuum state for fermion can be written as $\Theta_0(a^\dagger, a) = 1$ and the vacuum energy E_0 is zero. Define a number operator on position $\boldsymbol{x}$ as $N(\boldsymbol{x}) = a^\dagger(\boldsymbol{x})a(\boldsymbol{x})$, then we will have $N(\boldsymbol{x})\Theta_0 = 0$ for an arbitrary $\boldsymbol{x}$. Thus, the vacuum state corresponds

to the zero eigenvector of the modified fermion number operator, which is, in fact, naturally inferred from the modified fermion number conservation. This can be confirmed further by the fact that $\hat{a}(\boldsymbol{x}_0)\Theta_0 = 0$. Incidentally, it is easy to see that the state with only one fermion at position $\boldsymbol{x_0}$ can be written as $a^\dagger(\boldsymbol{x_0})$.

References

[1] Foldy L L and Wouthuysen S A 1950 *Phys. Rev.* **78** 29

[2] Tani S 1951 *Prog. Theor. Phys.* **6** 267

[3] Feng X C, Li D M and Feng J L 2005 *Acta Phys. Sin.* **54** 4084

[4] Lepage G P, Magnea L and Nakhleh C 1992 *Phys. Rev.* D **46** 4052

[5] Pineda A and Soto J 1998 *Phys. Rev.* D **58** 114011

[6] Chen Y Q, Kuang Y P and Oakes R J 1995 *Phys. Rev.* D **52** 264

[7] Bodwin G T, Braaten E and Lepage G P 1995 *Phys. Rev.* D **51** 1125

[8] Manohar A V 1997 *Phys. Rev.* D **56** 230

[9] Körnor J G and Thompson G 1992 *Phys. Lett.* B **264** 204

[10] Balk S, Ilakovac A, Korner J G and Pirjol D hep-ph/9311348;
Balk S, Korner J G and Pirjol D 1994 *Nucl. Phys.* B **429** 449

[11] Wilson K G and Kogut J 1974 *Phys. Reports* C **12** 75

[12] Bratten E 1997 hep-ph/9702225

[13] Chen Y Q and Bratten E 1998 *Phys. Rev. Lett.* **80** 5060

[14] Bali G S 2000 hep-ph/0001312

[15] Coleman S and Mandula J 1967 *Phys. Rev.* **159** 1251
Weinberg S 2000 *The Quantum Theory of Fields* **(**Cambridge: Cambridge University Press)

[16] El-khadra A X, Kronfeld A S and Mackenzie P B 1997 *Phys. Rev.* D **55** 3933

[17] CP-PACS Collaboration 2002 *Phys. Rev.* D **65** 094508

[18] Thacker B A and Lepage G P 1991 *Phys. Rev.* D **43** 196

[19] Liu D Q and Wu J M 2002 *High Energy Phys. Nucl. Phys.* **26** 222
Liu D Q and Wu J M 2002 *Mod. Phys. Lett.* A **17** 1419

[20] Chen Y, Dong S J and Liu K F 2003 the XXI International Symposium on Lattice Field Theory (Lattice 2003), Tsukuba, Japan. p285

[21] Langfeld K and Schaden M 1991 *Phys. Lett.* B **272** 100
Langfeld K, Alkofer R and Reinhardt H 1992 *Phys. Lett.* B **277** 163

[22] Luscher M and Weisz P 1984 *Nucl. Phys.* B **240** 349