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Suppression of spiral waves using intermittent local electric shock*

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In this paper, an intermittent local electric shock scheme is proposed to suppress stable spiral waves in the Barkley model by a weak electric shock (about 0.4 to 0.7) imposed on a random selected $n \times n$ grids ($n = 1 - 5$, compared with the original 256×256 lattice) and monitored synchronically the evolutions of the activator on the grids as the sampled signal of the activator steps out a given threshold (i.e., the electric shock works on the $n \times n$ grids if the activator $u \leq 0.4$ or $u \geq 0.8$). The numerical simulations show that a breakup of spiral is observed in the media state evolution to finally obtain homogeneous states if the electric shock with appropriate intensity is imposed.

Keywords: excitable media, spiral waves, intermittent control
PACC: 0545, 8700, 8730C

1. Introduction

Outwardly rotating spiral waves and concentric waves are ubiquitous phenomena in a reaction diffusion system. Spiral waves are often observed in a variety of physical, chemical, and biological systems.[1,2] A target pattern is produced by concentric waves which travel away from rhythmic sources while a spiral wave is generated by a rotating source. At present, it is confirmed that spiral wave is related to cardiac arrhythmia, and instability of the spirals will induce the spatiotemporal chaos and ventricular fibrillation.[3−5] Therefore, it is interesting to develop schemes in order to suppress spiral waves and/or prevent a breakup of spiral waves. Relevant topics have been widely investigated.[6−14] Generally, the aim is focused on how to convert spirals to become target waves or other patterns, and how to control the spiral-existing medium to be homogeneous and channelled off the spiral tip and so on. For example, Zhang *et al*[15] proposed a scheme to suppress spiral and chaos via a target wave by inputting periodical signal locally on the system. Then, the drift velocity field describing spiral wave motion in an excitable medium subjected to a two-point feedback control is derived and analysed.[16] The authors of the present paper investigated the dynamics and evolution of spiral waves subjected to an external weak chaotic signal driving, and an external field modulated by a chaotic signal or parameter perturbation.[17,18] The time-delayed method has been also successful to control the chaotic system[19] as well as spiral waves.[20−22] In this paper, an alternate scheme is proposed to suppress the stable spiral waves within a two-step strategy. First, an arbitrary selected grid is sampled to monitor its evolution; Second, a constant weak electric shock is imposed on the square area ($n \times n$ grids compared to the medium with 256 ×256 grids) if the activator variable of sampled grid steps out a threshold. It is found that the scheme under such local and intermittent characteristic will control the stable spiral waves out of the excitable medium to induce a homogeneous state within a short transient period.

2. Model and scheme

The efficiency of the scheme will be testified by the numerical simulations on the Barkley model[7,8] defined by

$$\begin{aligned} \frac{\partial u}{\partial t} &= \varepsilon^{-1} u(1-u)\Big(u - \frac{v+b}{a}\Big) + D\nabla^2 u\,, \\ \frac{\partial v}{\partial t} &= f(u) - v\,, \end{aligned} \tag{1}$$

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where u and v describe the activator and inhibitor variables respectively, D the diffusion coefficient, and the nonlinear function $f(u)$ satisfies $f(u) = 0$ at $u < 1/3$, $f(u) = 1 - 6.75u(u-1)^2$ at $1/3 \leq u \leq 1$ and $f(u) = 1$ at $u > 1$. The decisive parameters are b (positive for the excitable medium) to determine the excitation threshold and ε (the ratio of time scales of the fast activator u and the slow inhibitor variable v). The system owns one stable point ($u = 0$, $v = 0$) and two additional unstable points. Suitable initial conditions and parameters are chosen to generate a stable rotating spiral wave with $a = 0.84$, $b = 0.07$, $\varepsilon = 0.002$ and $D = 1$. The whole system is divided into 256 ×256 grids and the distance between the adjacent grids $\Delta x = \Delta y$=100/256. The Euler forward difference algorithm and the non-flux boundary conditions have been applied for the simulation.

Compared with a continuous electric shock on the whole medium,[9] we introduce an intermittent weak shock on a local region. The external electric shock shown in Eq.(2) is included in the right side of the first formula in Eq.(1),

$$I = (I_0 H(u - u_{01}) + I_0 H(u_{02} - u))\delta_{i,\alpha}\delta_{j,\beta}. \quad (2)$$

Here, i, j are the integers corresponding to the discretized x, y variables as $x_i = (i-1)\Delta x$, $y_j = (j-1)\Delta y$, α and β are integers. The monitored grid is selected as a square lattice in the centre with $n \times n$ sites (For example, for $n = 2$, α, $\beta = 127$, 128, and for $n = 5$, α, $\beta = 126$, 127, 128, 129, 130 etc.). The standard Heaviside function H in Eq.(2) is defined by $H(x) = 1$ if $x \geq 0$ and $H(x) = 0$ if $x < 0$. I_0 is a constant, u is the same activator variable as in Eq.(1), and u_{01} and u_{02} are the selected threshold to be specifically chosen.

3. Simulations and results

In our numerical simulations, different I_0 values, as well as the u_{01} and u_{02} data will be chosen to testify whether the stable rotating spiral waves are suppressed, or its breakup and drift are deduced successfully. The numerical results confirm that the stable spiral wave can be killed due to the generated spiral drift, and the whole medium reaches a homogeneous state within 490 time units.

Next, the effect of intensity is considered by taking a sequence of external electric shock with $I_0 = 0.4$, 0.5, 0.7, 0.8, 0.9, 0.95, 1.0. The results of the simulation are summarized in the following table where T presents the transient period of killing spirals in the time units. It takes a much longer time period to break up the spirals for the cases of $I_0 = 0.8$ and 0.9 compared with all the other cases.

It requires a shorter transient period to make a homogeneous state at $I_0 = 0.6$ when the thresholds are set as $u_{01} = 0.8$, $u_{02} = 0.4$; to our surprise, it fails to be that stronger intensity of external perturbation and/or shock makes a shorter transient period as expected. Generally, when an external constant and/or periodical signal with the eigenfrequency of the system is imposed on the medium continuously, the stronger the intensity of the signal, the shorter the transient period. In our comprehension, the scheme based on Eq.(2) induces an intermittent perturbation of the activator of Eq.(1) in a local area as the amplitude of the monitored grid steps out of the selected threshold. Perhaps it may produce a quasi-periodical signal in the local area, and therefore the tip of the spiral wave is driven to the border of the medium and the spiral wave is killed after a collision with the border. It is a quasi-periodical driving in a local area for which the stronger intensity of driving fails to make a shorter transient period as we expected. Then the intensity I_0 was further set as 1.1, 1.2, 1.3, and the breakup of spiral wave is also observed within a relatively short period. Figure 2 shows an especial case for $I_0 = 1.1$ as an illustration.

Table 1. Evolution of spiral wave under an intermittent electric shock with different intensity I_0 on the local area of α, $\beta = 126$, 127, 128, 129, 130 by Eq.(2).

I_0	0.4	0.5	0.6	0.7	0.8	0.9	0.95	1.0
T/time units	1600	1200	490	600	~ 3200	~ 3200	360	180

It is found that, in general, a stronger intensity of the electric shock I_0 will break up the spiral wave within a shorter period to induce new patterns. The cases of $I_0 = 1.2$, 1.3 were investigated, to find the break-up of spirals. It may be explained that the medium is perturbed greatly and so a new defect appeared, thus the dynamics of Eq.(1) changed rapidly.

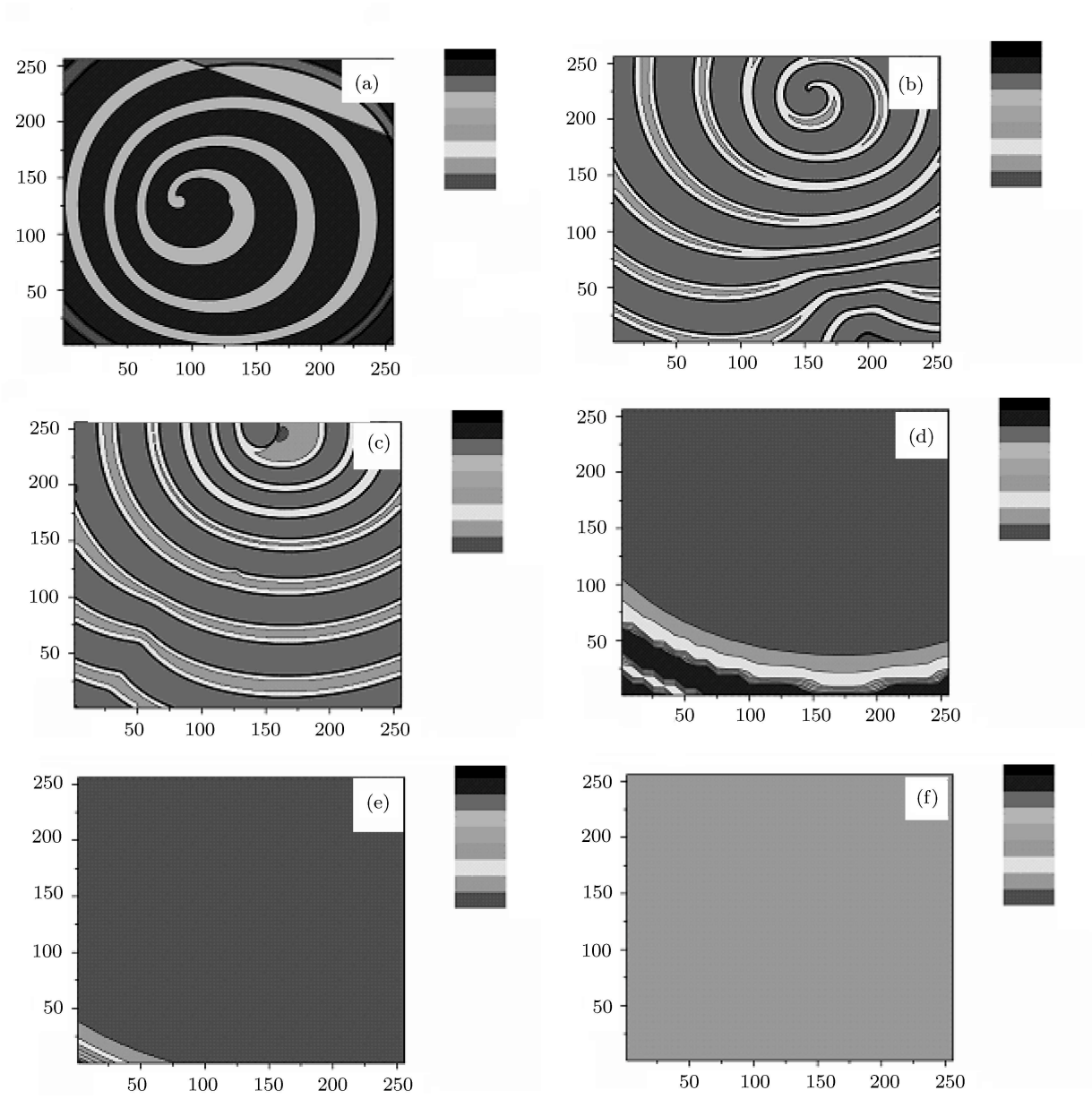


Fig.1. Evolution of stable spiral wave under the control of Eq.(2) for $I_0 = 0.6$ and $u_{01} = 0.8$, $u_{02} = 0.4$, α , $\beta = 126$, 127, 128, 129, 130(5×5 grids), versus the time at $t = 100$(a), 400(b), 440(c), 470(d), 480(e), 490(f) units, respectively.

How does this scheme behave as it is applied to a smaller control area, for example the intermittent weak electric shock is imposed on a 4×4 square in the space centre? The numerical results are shown in Fig.3.

Compared with the results in Fig.2, a smaller control area costs a longer transient period under the same intensity and threshold. Then we confirm the character to investigate the scheme by taking even smaller 3×3 grids (α, β = 127, 128, 129) and 2×2 grids (α, β = 88, 89), and the corresponding results are plotted in Figs.4 and 5 respectively.

Up to now, the square control area has been located on the centre of the medium, and the whole medium can become homogeneous after the intermittent electric shocks were imposed on the local area (3×3 grids among all 256×256 grids in the medium). Furthermore we have to confirm whether the approach is also effective when the control area is away from the central area of the medium.

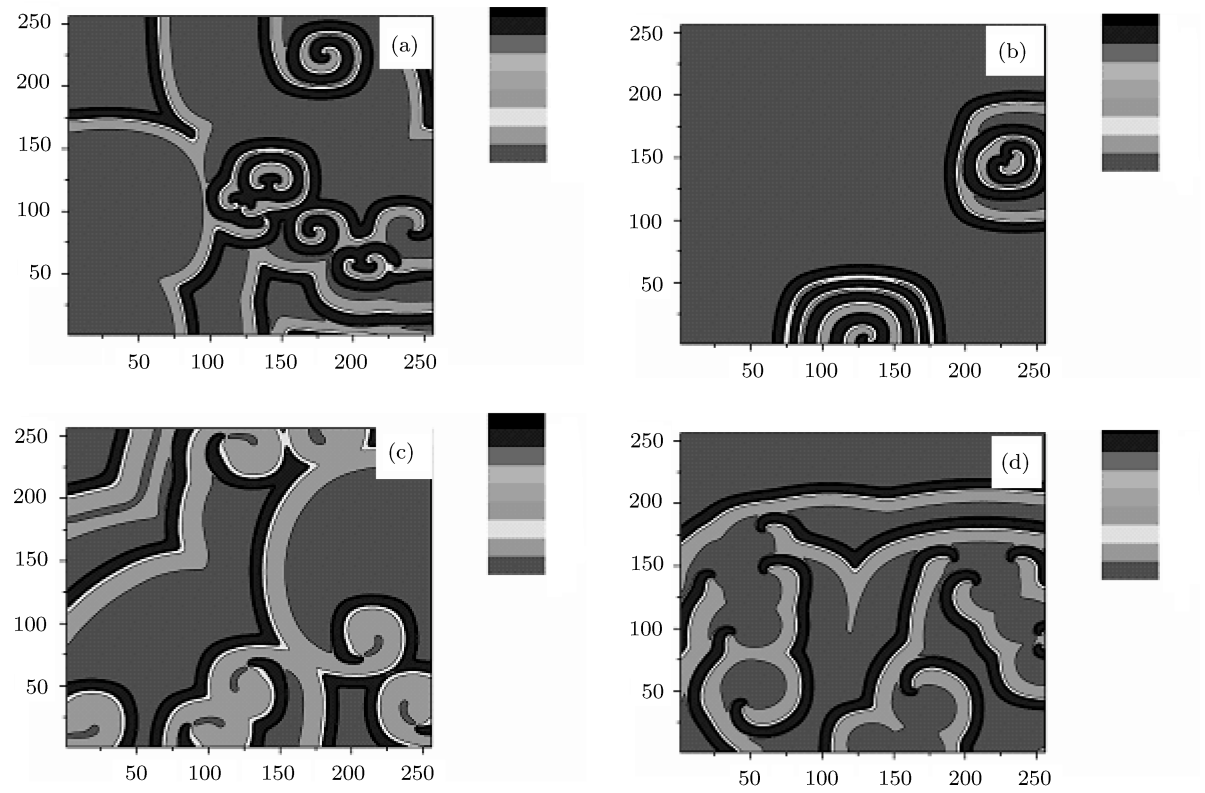


Fig.2. Evolution of stable spiral wave under the control of Eq.(2) for $I_0 = 1.1$ with $u_{01} = 0.8$, $u_{02} = 0.4$, α, $\beta = 126$, 127, 128, 129, 130 (5×5 grids) versus the time at $t = 100$(a), 140(b), 170(c), 200(d) time units respectively.

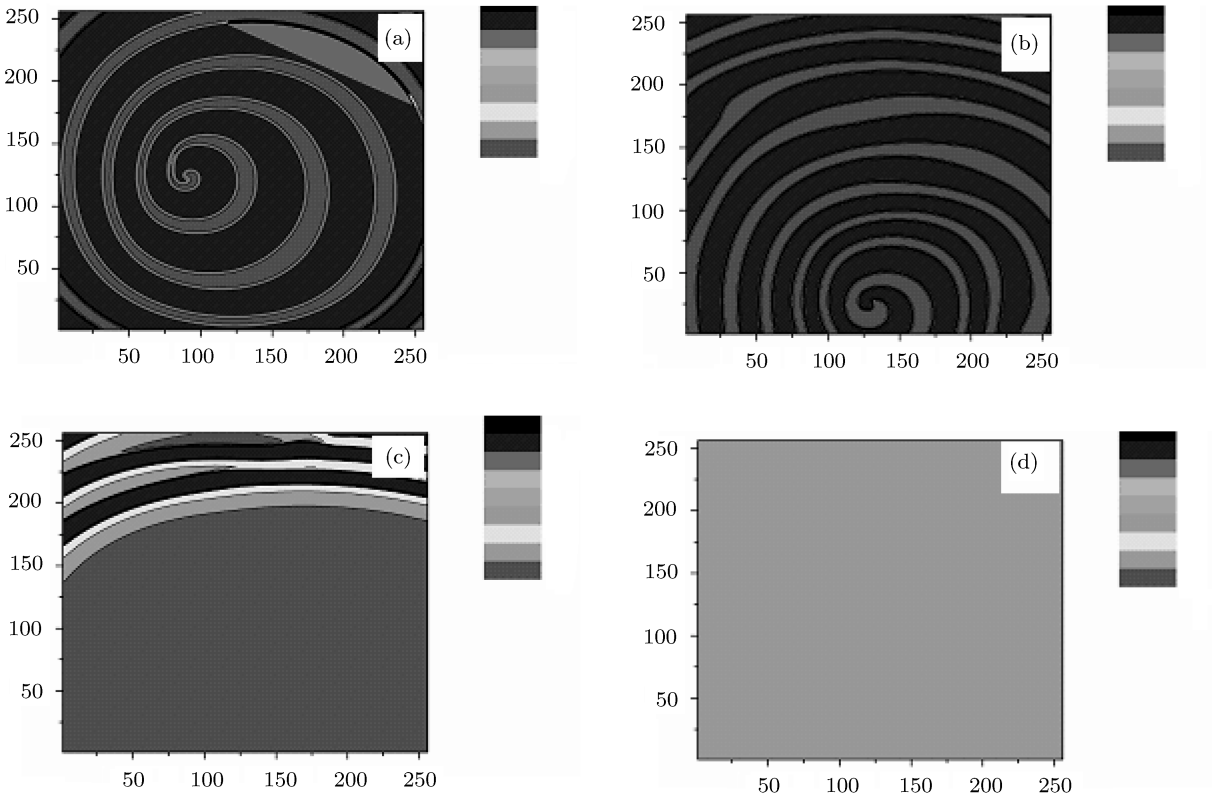


Fig.3. Evolution of stable spiral wave under the control of Eq.(2) for $I_0 = 0.6$, and $u_{01} = 0.8$, $u_{02} = 0.4$, α, $\beta = 127$, 128, 129, 130(4×4 grids) versus the time at $t = 100$(a), 600(b), 700(c), 800(d) time units.

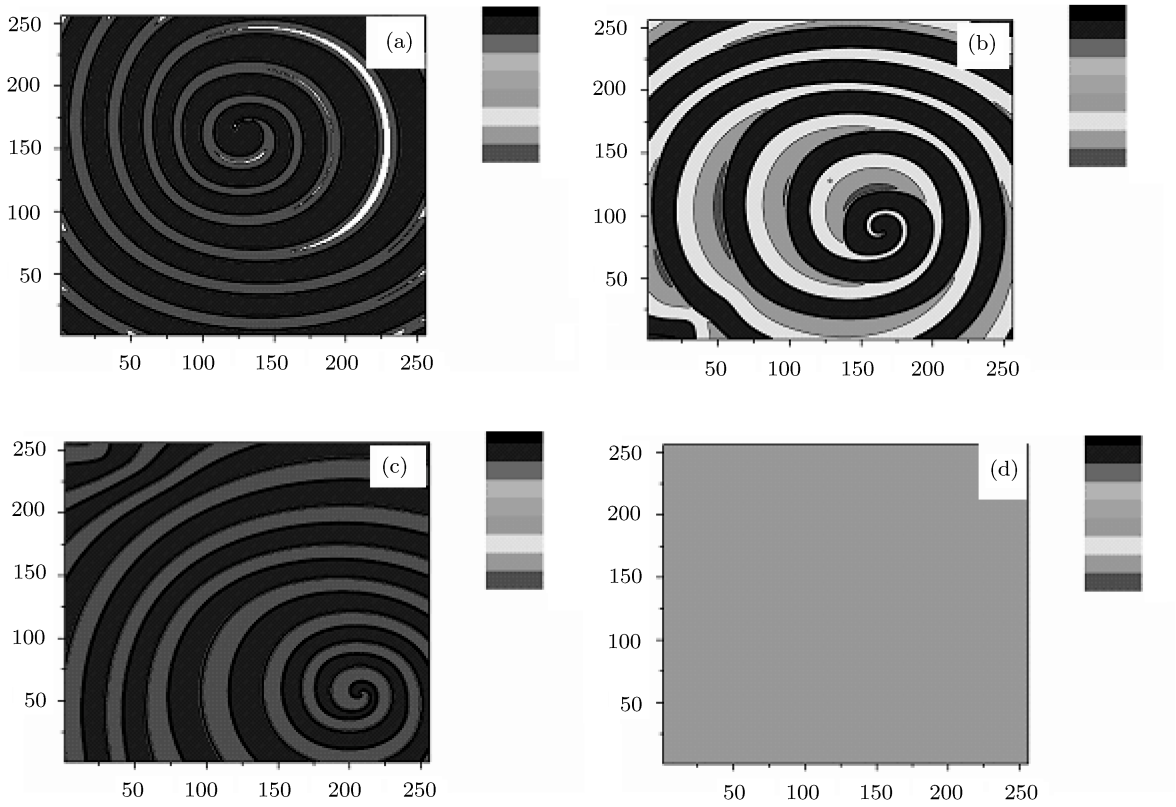


Fig.4. Evolution of stable spiral wave under the control of Eq.(2) for $I_0 = 0.6$ and $u_{01} = 0.8$, $u_{02} = 0.4$, α, $\beta = 127$, 128, 129 (3×3 grids) versus the time at $t = 400$(a), 800(b), 1000(c), 1150(d) time units.

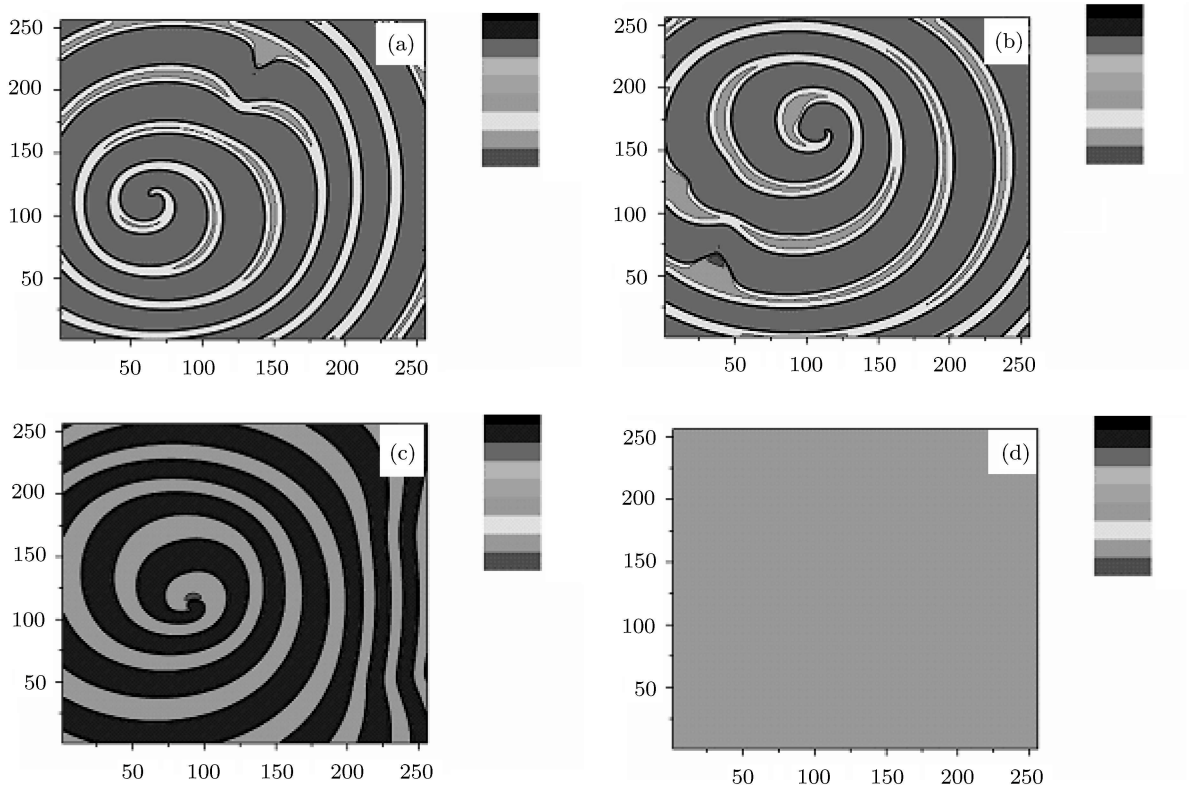


Fig.5. Evolution of stable spiral wave under the control of Eq.(2) for $I_0 = 0.6$ and $u_{01} = 0.8$, $u_{02} = 0.4$, α, $\beta = 88$, 89 (2×2 grids) versus the time at $t = 400$(a), 600(b), 800(c), 1000(d) time units.

The numerical simulation results confirm that, as expected, the whole medium can become homogeneous, though the scheme may cost a longer transient period. Now it is interesting to discuss its characters and advantages. The control algorithm of the local intermittent electric shock works effectively with small intensity (e.g. $I_0 = 0.6$) to suppress the stable spiral waves and the whole medium can become homogeneous state whether the control area is located on the centre or not. As we know, the spiral wave is linked to a topological defect, which is a local zero of the order parameter.[23] Here, it relates to the unstable point in the Eq.(1) and requires the following equation of $f(u_0) - v_0 = 0$ at $1/3 < u_0 = (v_0 + b)/a < 1$.[8] The defect may be eliminated as the appropriate intermittent electric shock was imposed locally. It makes no much difference whether the control area is generated in the centre (spiral tip) or not of the medium. The spiral drift and breakup could be all the times observed for the cases studied by our simulations.

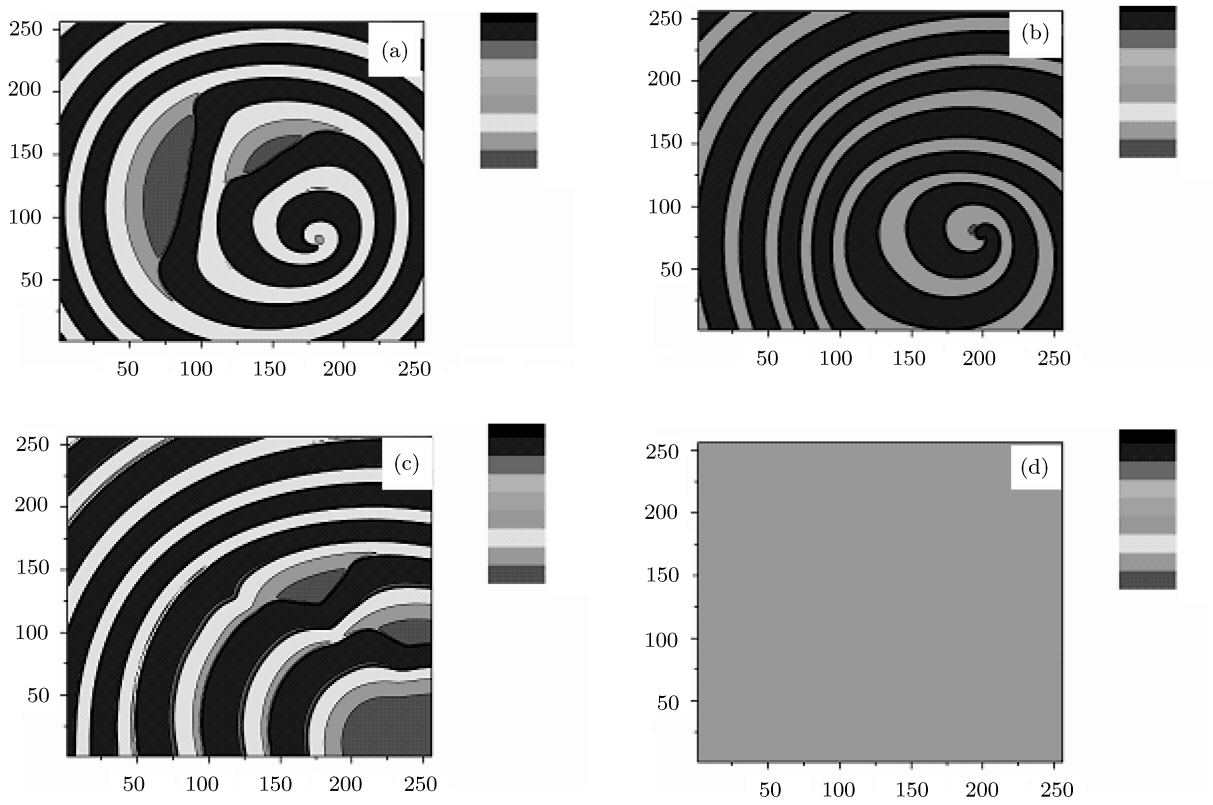


Fig.6. Evolution of stable spiral wave under the control of Eq.(2) for $I_0 = 0.6$ and $u_{01} = 0.8$, $u_{02} = 0.4$, α, $\beta = 100$ (1×1 grid) versus the time at $t = 400$(a), 500(b), 800(c), 1000(d) time units.

4. Conclusions

In this paper, a pattern formation in a two-dimensional excitable medium is investigated. We propose a new scheme to suppress the stable spiral waves in the Barkley excitable medium by introducing locally a weak electric shock on a few grids when the activator of an arbitrary sampled grid steps out of a selected threshold. The control area and intensity are adjusted to study its efficiency for several cases. It is found that the stable spiral wave states will be killed to induce the whole medium to be homogeneous steadily as expected. The simulations confirm that the scheme behaves with some advantages, such as its shorter transient period, smaller control area (a few grids perturbed) and weaker intensity controlled by the external inputting signal, compared with the previous studies.[15] It shows another advantage compared with some present schemes based on the external periodical driving[10,15] and/or parameter perturbation. It confirms that the external periodical driving calls for an frequency which equals to the eigenfrequency of the system so that a shorter transient period will induce based on the stochastic resonance, and the eigenfrequency could be found based on the

linear stability analysis at the bifurcation point. As a result, all the parameters of the medium must be known beforehand. In our scheme, all the parameters could be unknown, it just needs to monitor the few arbitrary selected grids (or one grid only), the controller will work immediately as the amplitude of the sampled and monitored grids or grid step out the required threshold. The work will be carried on in other reaction–diffusion systems to check its effectiveness and independence on models, and search for new proofs and evidence in experiments.

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