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Passive control of a 4-scroll chaotic system

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This paper studies the control of a new chaotic system which can generate 4-scroll attractors. Based on the properties of a passive system, it derives the essential conditions under which this new chaotic system could be equivalent to a passive system and globally asymptotically stabilize at a zero equilibrium point via smooth state feedback. Simulation results and circuit experiment show that the proposed chaos control method is effective.

Keywords: a 4-scroll chaotic system, passive network theory, asymptotically stabilized
PACC: 0545

1. Introduction

In 1963, Lorenz found the first chaotic attractor, which is called Lorenz chaotic system, in a three-dimensional autonomous system when he studied atmospheric convection.[1] Various different chaotic systems are proposed consequently. In 1999, adding a linear feedback controller to the Lorenz chaotic system, Chen found another similar but topologically nonequivalent chaotic attractor,[2] with the most interesting characteristic that it is a dual system to the Lorenz chaotic system, in a well-defined mathematical sense first classified in Ref.[3]. This led to the discovery of a transition system between the Lorenz system and Chen system, which is called the Lü chaotic system.[4] Later, the unified chaotic system[5] which contains the generalized Lorenz, the generalized Chen and Lü chaotic system was discovered by Lü and Chen. Recently, a new simple chaotic system that can generate 4-scroll attractors in a continuous three-dimensional autonomous system is proposed by Liu and Chen.[6] Because this new chaotic system is simple but can display more complex dynamics than the Lorenz, Chen and Lü chaotic systems, the analysis, control, and synchronization of this new chaotic system are studied extensively.[7−10]

Also, the concept of passivity of nonlinear systems has attracted new interest in chaos control recently.[11−13] In this paper, based on the properties of a passive system, the control of this 4-scroll chaotic system is studied. The essential conditions are derived under which this 4-scroll chaotic system could be equivalent to a passive system and globally asymptotically stabilized at a zero equilibrium point via smooth state feedback, and then, an electronic circuit is designed to confirm the effectiveness of this method.

The rest of this paper is organized as follows. In Section 2, based on the properties of a passive system, a rather simple conclusion about controlling dynamical system is drawn for stabilizing at a zero equilibrium point through a feedback control. In Section 3, a brief description of the 4-scroll chaotic system is given and the approach is applied to this system. Section 4 is devoted to numerical simulations for illustration and verification of our results. In Section 5, an electronic circuit is designed and the results of experiment are given for confirmation. Finally, some concluding remarks and comments are given in Section 6.

2. The theory of passive control[13]

Consider the following differential equation:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in X \subset R^n$ is state variable. $f(x)$ and $g(x)$ are the smooth vector fields. $u(t) \in U$ is the input. $f(0) = 0$ and $h(x)$ is a smooth mapping.

Definition 1 The system (1) is a minimum phase system if $L_g h(0)$ is nonsingular and $x = 0$ is one of the asymptotically stabilized equilibrium points of $f(x)$.

Definition 2 The system (1) is passive if the following two conditions are satisfied.

1) $f(x)$ and $g(x)$ exist and are smooth vector fields, $h(x)$ is also a smooth mapping.

2) For any $t \geq 0$, there is a real value β satisfying the inequality

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau \geq \beta, \tag{2}$$

or there are a real value β and $\rho > 0$ satisfying the inequality

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho y^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau. \tag{3}$$

Let $z = \vartheta(x)$, Eq.(1) will be changed into the following generalized form:

$$\begin{aligned} \dot{z} &= f_0(z) + p(z,y)y, \\ \dot{y} &= b(z,y) + a(z,y)u, \end{aligned} \tag{4}$$

where $a(z,y)$ is nonsingular for any (z,y).

Theorem 1 If Eq.(1) is a minimum phase system and has relative degree [1, 1, ...] at $x = 0$, Eq.(4) will be equivalent to a passive system and asymptotically stabilized at an equilibrium point through the local feedback control as follows:

$$\begin{aligned} u = \ & a(z,y)^{-1}\left[-b^{\mathrm{T}}(z,y) - \frac{\partial}{\partial z}W(z)p(z,y)\right. \\ & \left. -\alpha y + v\right], \end{aligned} \tag{5}$$

where $W(z)$ is the Lyapunov function of $f_0(z)$, α is a positive real value and v is an external signal which is connected with the reference input.

3. The control of 4-scroll chaotic system

Recently, Liu *et al* introduced the new chaotic system that can generate 4-scroll attractors. The nonlinear differential equations that describes this chaotic system are

$$\begin{aligned} \dot{x}_1 &= ax_1 + d_1x_2x_3, \\ \dot{x}_2 &= bx_2 + d_2x_1x_3, \\ \dot{x}_3 &= cx_3 + d_3x_1x_2, \end{aligned} \tag{6}$$

where a, b, c, d_1, d_2 and d_3 are system parameters and x_1, x_2 and x_3 are state variables. Taking $a = 0.5$, $b = -4$, $c = -10$, $d_1 = -1$, $d_2 = 1$ and $d_3 = 1$, we obtain an example of the 4-scroll chaotic attractors depicted in Fig.1. When we add the controller to the first formula of Eq.(6), the Eq.(6) can be changed into the following form:

$$\begin{aligned} \dot{x}_1 &= ax_1 + d_1x_2x_3 + u, \\ \dot{x}_2 &= bx_2 + d_2x_1x_3, \\ \dot{x}_3 &= cx_3 + d_3x_1x_2. \end{aligned} \tag{7}$$

The aim is to determine the controller u for stabilizing this chaotic system at a zero equilibrium point.

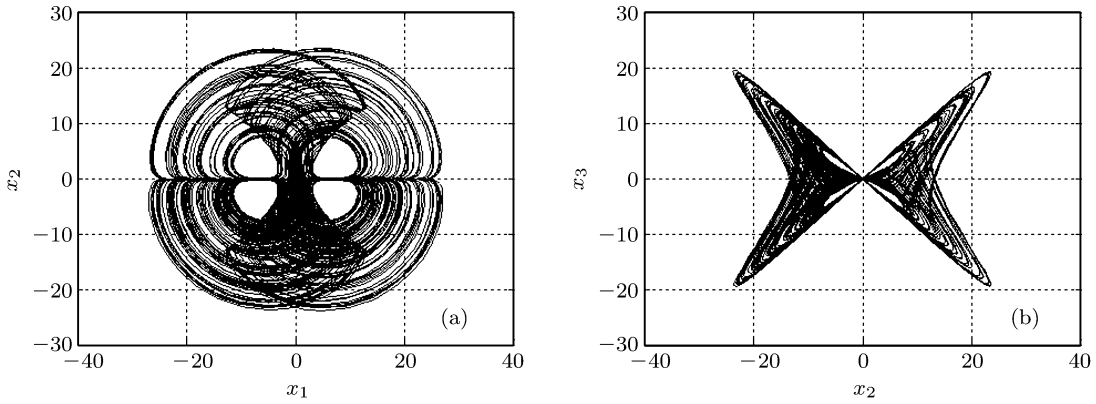


Fig.1. The phase portrait of 4-scroll chaotic system (a) $x_1 - x_2$, (b) $x_2 - x_3$.

Theorem 2 If we choose the controller as follows

$$u = -(\alpha + a)x_1, \tag{8}$$

where α is a positive real constant, then the 4-scroll chaotic system (6) will be asymptotically stabilized at a zero equilibrium point.

Proof

Let $z_1 = x_2, z_2 = x_3, y = x_1$, Eq.(7) can be rewritten as

$$\begin{aligned}\dot{z}_1 &= bz_1 + d_2yz_2,\\ \dot{z}_2 &= cz_2 + d_3yz_1,\\ \dot{y} &= ay + d_1z_1z_2 + u.\end{aligned} \tag{9}$$

So

$$\begin{aligned}\dot{z} &= f_0(z) + p(z,y)y,\\ \dot{y} &= b(z,y) + a(z,y)u,\end{aligned} \tag{10}$$

where

$$\begin{aligned}f_0(z) &= [bz_1 \quad cz_2]^{\mathrm{T}},\\ p(z,y) &= [d_2z_2 \quad d_3z_1]^{\mathrm{T}},\\ b(z,y) &= ay + d_1z_1z_2,\\ a(z,y) &= 1.\end{aligned} \tag{11}$$

Let

$$W(z) = \frac{1}{4}z_1^2 + \frac{1}{4}z_2^2, \tag{12}$$

Then according to Eq.(12), we have

$$\dot{W}(z) = \frac{1}{2}\dot{z}_1z_1 + \frac{1}{2}\dot{z}_2z_2 = \frac{1}{2}(bz_1^2 + cz_2^2). \tag{13}$$

For $b = -4$, $c = -10$

$$\dot{W}(z) \le 0. \tag{14}$$

So, $W(z)$ is the Lyapunov function of $f_0(z)$ and that $f_0(z)$ is globally asymptotically stable. Meanwhile, $L_gh(0)$ is nonsingular. According to the definition 1, Eq.(9) is a minimum system. On the other hand, Eq.(9) has relative degree [1, 1, ...] at $x = 0$. According to Theorem 1, the feedback controller can be derived

$$u = -(\alpha + a)y + v, \tag{15}$$

namely

$$u = -(\alpha + a)x_1 + v. \tag{16}$$

When we let $v = 0$, Eq.(7) will be asymptotically stabilized at a zero equilibrium point.

4. Simulation results

In this section, the fourth-order Runge–Kutta method is used to solve the system of differential equations (7), with step size equal 0.001 in numerical simulations. Choose the value of α equal 1.5 and the initial conditions of Eq.(7): $x_1(0) = -0.9$, $x_2(0) = 0.1$, $x_3(0) = 4.1$. Let $v = 0$ and add the controller u after 130 s. As expected, one can observe that the trajectories of the 4-scroll chaotic system asymptotically stabilized at a zero equilibrium point as illustrated in Fig.2.

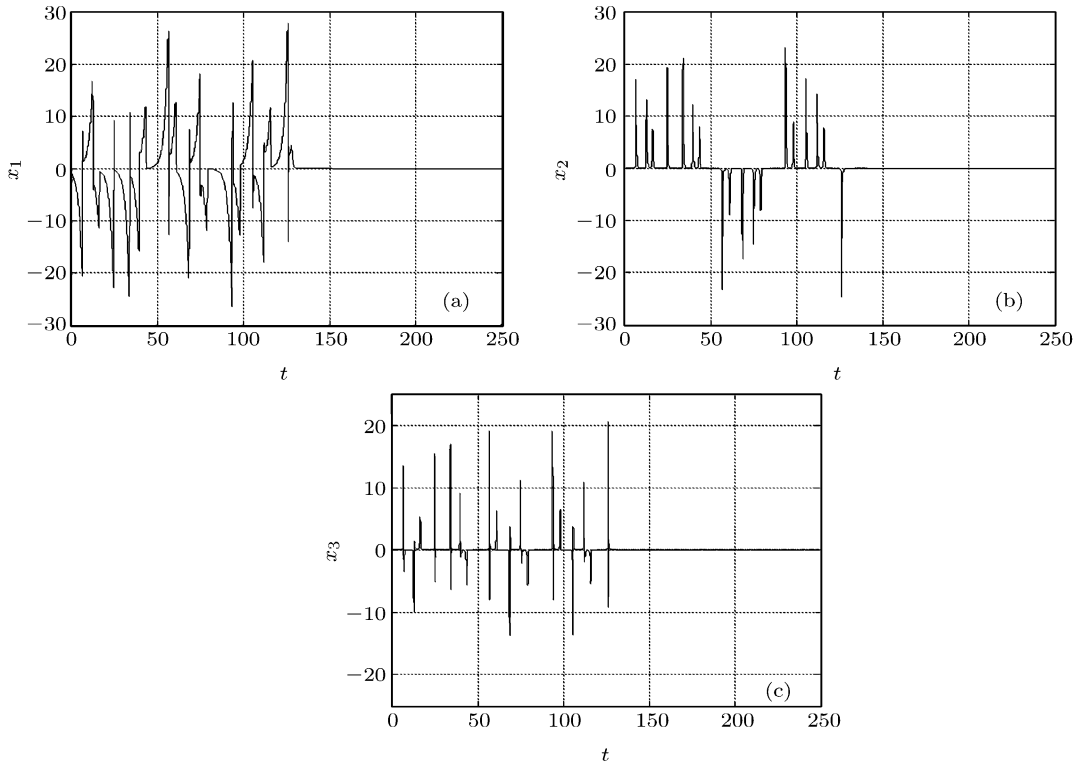


Fig.2. The time series of variables under control after 130s: (a) x_1, (b) x_2, (c) x_3.

5. Circuit experiment

In this section, an electronic circuit is designed to confirm the proposed method. Because of the restriction in the acceptable voltage of operational amplifiers and analogue multipliers, we must decrease the amplitude of every variable of Eq.(7) by 0.5. The following form can be obtained:

$$\begin{aligned}\dot{x}_1 &= a_1 x_1 + 2d_1 x_2 x_3 + u,\\ \dot{x}_2 &= b_1 x_2 + 2d_2 x_1 x_3,\\ \dot{x}_3 &= c_1 x_3 + 2d_3 x_1 x_2.\end{aligned} \tag{17}$$

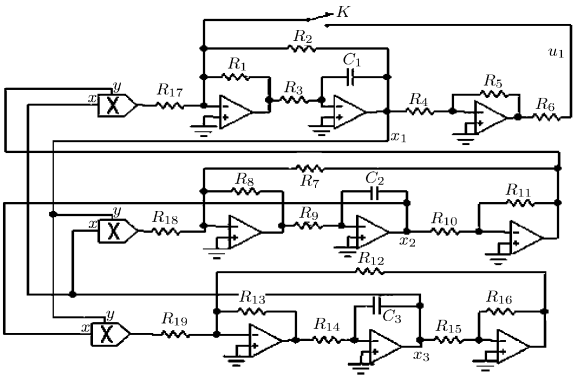


Fig.3. Circuitry realization of system (17).

The circuit which can be realized for Eq.(17) is designed as shown in Fig.3. With the theory of circuit, we can obtain

$$\begin{aligned}\dot{x}_1 &= \frac{R_1}{R_2R_3C_1}x_1 - \frac{R_1R_{11}}{10R_3R_{10}R_{17}C_1}x_2x_3 - \frac{R_1R_5}{R_3R_4R_6C_1}x_1,\\ \dot{x}_2 &= -\frac{R_8R_{11}}{R_7R_9R_{10}C_2}x_2 + \frac{R_8}{10R_9R_{18}C_2}x_1x_3,\\ \dot{x}_3 &= -\frac{R_{13}R_{16}}{R_{12}R_{14}R_{15}C_3}x_3 + \frac{R_{13}}{10R_{14}R_{19}C_3}x_1x_2.\end{aligned} \tag{18}$$

Let $R_1, R_4, R_5, R_8, R_{10}, R_{11}, R_{13}, R_{15}, R_{16} = 10\,\mathrm{k\Omega}$, $R_3, R_9, R_{14} = 100\,\mathrm{k\Omega}$, $R_6 = 50\,\mathrm{k\Omega}$, $R_{17}, R_{18}, R_{19} = 5\,\mathrm{k\Omega}$, $R_2 = 200\,\mathrm{k\Omega}$, $R_7 = 25\,\mathrm{k\Omega}$, $R_{12} = 10\,\mathrm{k\Omega}$, $C_1, C_2, C_3 = 1\,\mu\mathrm{F}$, Eq.(18) will be equal to Eq.(17). When the switch K is opened, the phase portrait of circuit experiment is shown in Fig.4 and when the switch K is closed, as expected, one can observe that the trajectories of each variable will be asymptotically stabilized at a zero equilibrium point, and shown in Fig.5. By comparing the Fig.1 with Fig.2, one can observe that there is a good qualitative agreement between the numerical simulation and the experimental measurement on circuit.

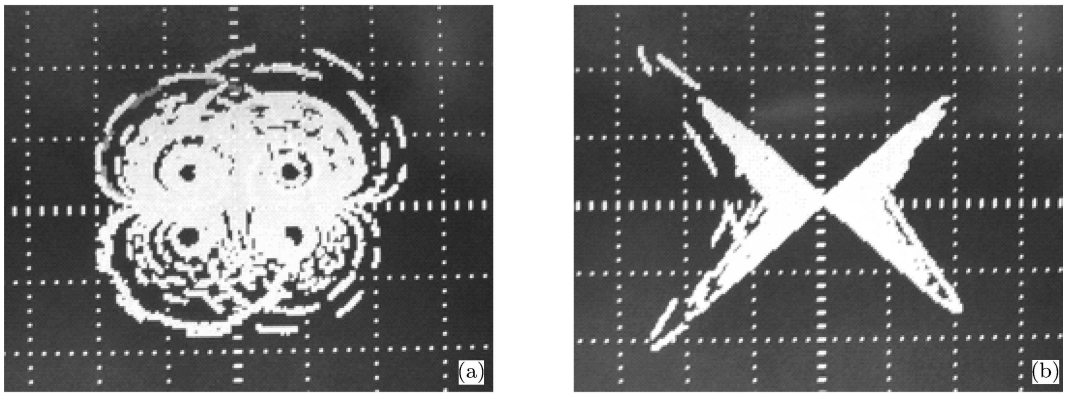


Fig.4. The phase portrait of circuit experiment (5V/div, 5V/div)(a) $x_1 - x_2$, (b) $x_2 - x_3$.

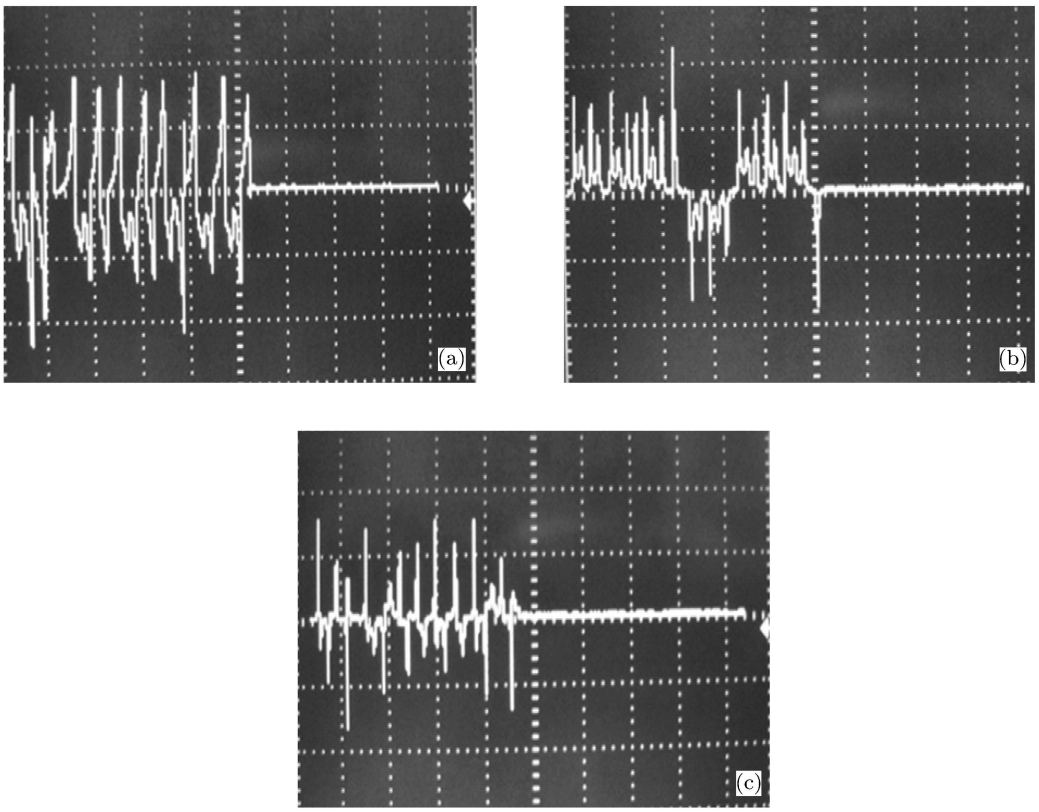


Fig.5. The time series of variables of circuit experiment under control (a) x_1, (b) x_2, (c) x_3.

6. Conclusion

Based on the properties of a passive system, the feedback controller for controlling 4-scroll chaotic system to stabilize at a zero equilibrium point is derived. The effectiveness of this proposed control method has been validated not only by numerical simulations results, but also by electronic circuit experiment. In addition, this method can be applied to control other chaotic or hyperchaotic systems.

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