Vol 16 No 4, April 2007
1009-1963/2007/16(04)/0942-04

Chinese Physics

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A new multi-scroll chaotic generator

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(Received 16 August 2006; revised manuscript received 23 August 2006)

In this paper a new simple multi-scroll chaotic generator is studied. The characteristic of this new multi-scroll chaotic generator is that it is easy to generate different number of scroll chaotic attractors through modifying the nature number n after fixing the suitable system parameters and it does not need complex mathematical derivation. Various number of scroll chaotic attractors are illustrated not only by computer simulation but also by the realization of an electronic circuit experiment on Electronic Workbench (EWB).

Keywords: multi-scroll chaotic attractor, circuit, scroll
PACC: 0545

1. Introduction

Recently, the generation, control, synchronization and application of multi-scroll chaotic attractors have been studied with increasing interest and have become a central topic in research due to its great potential in chaos communication technology.[1−10] A large number of various multi-scroll chaotic generators have been presented for a few decades by some researchers. For instance, Suykens and Vandewalle proposed the quasi-linear function approach to generate a family of n-double scroll chaotic attractors.[1] Tang *et al* introduced a sine function to generate n-scroll chaotic attractors in a simple circuit.[2] Lü *et al* presented a switching manifold approach for generating chaotic attractors with multi-merged basins of attractions.[3] Very recently, Yu *et al* introduced a novel nonlinear modulating function approach for generating n-scroll chaotic attractors based on a general jerk circuit.[4] Yu *et al* also constructed a family of multi-folded torus chaotic attractors, which are quite different from the scroll chaotic attractors,[5] etc. All the above works develop this research direction greatly and take this research direction further and deeper.

In this paper, we present a new simple multi-scroll chaotic generator. If the nature number n of this system is modified, the different number of scroll attractors will be obtained easily. And then, an electronic circuit is designed and simulated on Electronic Workbench (EWB).

2. A new multi-scroll chaotic system

The autonomy differential equations of this new multi-scroll chaotic system can be described as

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x + ay + f(x+z), \\ \dot{z} &= -z + f(x+z), \end{aligned} \tag{1}$$

where

$$f(x+z) = b \sum_{i=-n}^{n} \mathrm{sgn}(x+z+2i), \tag{2}$$

a, b are constants and x, y, z are variables of system (1). Let $u = x + z$, then Eq.(2) will be changed into the following form:

$$f(u) = b \sum_{i=-n}^{n} \mathrm{sgn}(u+2i). \tag{3}$$

Taking $a = 0.1$, $b = 0.5$, $n = 0$, and letting the right-hand side of Eq.(1) equal zero, we can obtain

$$\begin{aligned} y &= 0, \\ -x + ay + f(x+z) &= 0, \\ -z + f(x+z) &= 0. \end{aligned} \tag{4}$$

So, the equilibrium points of Eq.(1) can be solved from Eq.(4): $(-b, 0, -b)$, (0,0,0), $(b, 0, b)$. Linearizing Eq.(1) at each equilibrium point, one can observe that the Jacobians are the same as

$$J = \begin{bmatrix} 0 & 1 & 0 \\ -1 & a & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{5}$$

So, the characteristic roots of Eq.(1) from Jacobian are the same as $0.05+0.9987i$, $0.05-0.9987i$, -1. We can conclude that the trajectories of Eq.(1) will not stay in any equilibrium point because each equilibrium is unstable. In fact, for a given initial value, as time elapses, the trajectory of Eq.(1) spirally diverges around its equilibrium point or changes around to another. After a long time, the trajectories will overflow the whole chaotic region and 2-scroll chaotic attractors can be formed, which are shown in Fig.1(a). Likewise, the number of equilibrium points is varied with the variation of nature number n of Eq.(1), each equilibrium point is also unstable and the $2n$+2-scroll chaotic attractors can be obtained. For example, $n = 1$, 2, 3, 4, 5, the 4-scroll, 6-scroll, 8-scroll, 10-scroll and 12-scroll attractors are obtained as shown in Figs.1(b), (c), (d), (e), (f) respectively.

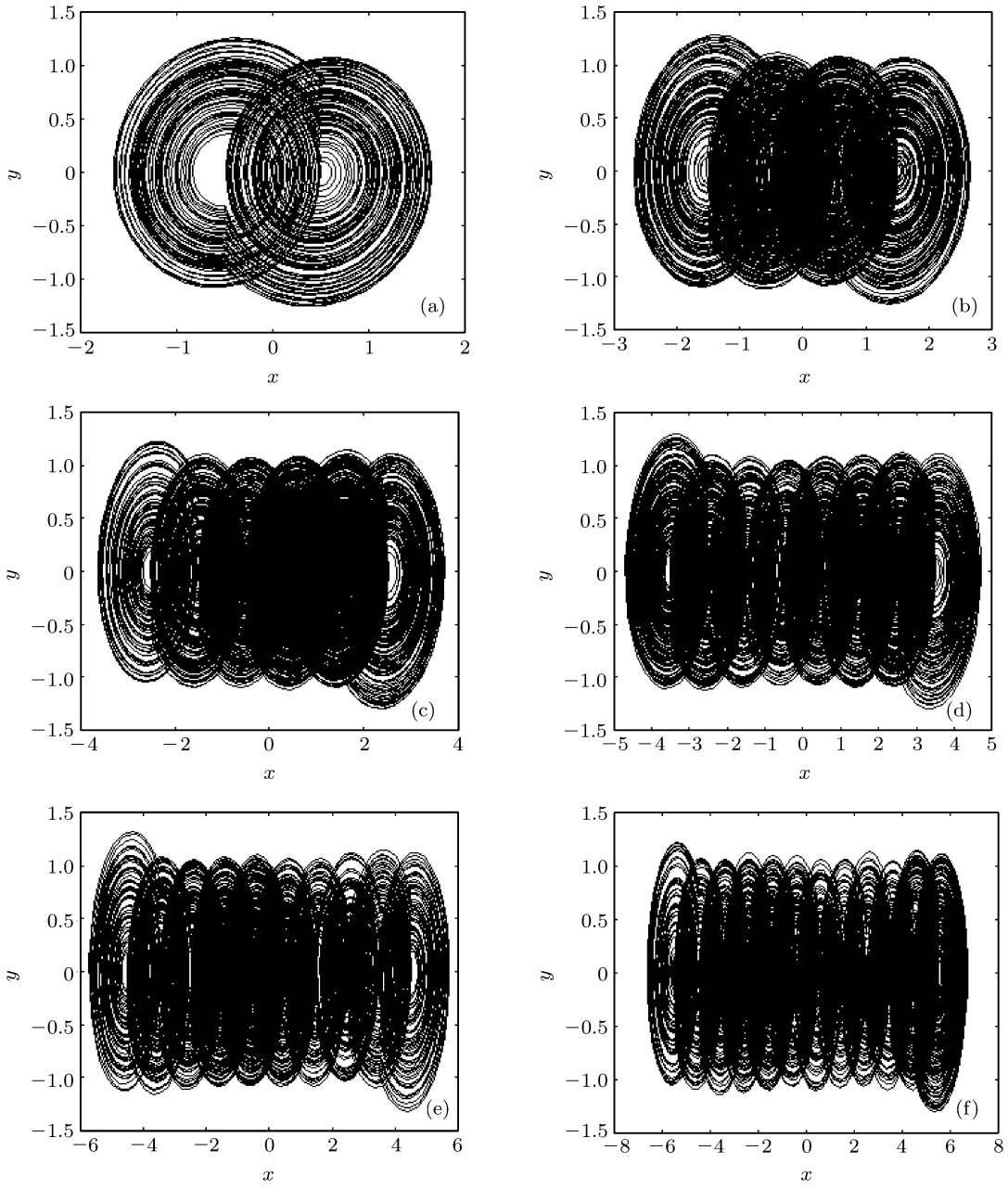


Fig.1. The chaotic attractors (a) 2-scroll, (b) 4-scroll, (c) 6-scroll, (d) 8-scroll, (e) 10-scroll, (f) 12-scroll.

3. Circuit experiment on EWB

In this section, an electronic circuit is designed on EWB to confirm this new multi-scroll chaotic generator. LF347 is used to realize addition and subtraction of signal and nonlinear function. The whole circuit is designed and shown in Fig.2. The power supply is $\pm 15\,\mathrm{V}$. According to the theory of circuit and the characteristic of each circuit component, we can obtain the following equations:

$$\begin{aligned}\dot{x} &= \frac{1}{R_1C_0}y,\\ \dot{y} &= -\frac{1}{R_1C_0}x + \frac{1}{R_2C_0}y\\ &\quad + \frac{15}{R_3C_0}\sum_{i=-n}^{n}\operatorname{sgn}(x+z+2i),\\ \dot{z} &= -\frac{1}{R_1C_0}z + \frac{15}{R_3C_0}\sum_{i=-n}^{n}\operatorname{sgn}(x+z+2i).\end{aligned} \tag{6}$$

So, $1/(R_1C_0) = 1$, $1/(R_2C_0) = a$, $15/(R_3C_0) = b$. Substituting $R_0 = 10\,\mathrm{k\Omega}$, $R_1 = 100\,\mathrm{k\Omega}$, $R_2 = 1\,\mathrm{M\Omega}$, $R_3 = 3\,\mathrm{M\Omega}$, $C_0 = 10\,\mu\mathrm{F}$ into Eq.(6), we will find that Eq.(6) equals to Eq.(1). When the switches K_0, K_1, K_2, K_3, …, K_n are closed, the $2n+2$-scroll chaotic attractors can be generated. For example, when $n = 3$, only K_0, K_1, K_2, K_3 are closed, the 8-scroll chaotic attractors can be obtained on EWB, which are shown in Fig.3(a), when $n = 5$, only K_0, K_1, K_2, K_3, K_4, K_5 are closed, the 12-scroll chaotic attractors can be obtained on EWB, which are shown in Fig.3(b). Comparing Fig.1(d) with Fig.1(f), one can observe that there is a good qualitative agreement between the numerical simulation and the experimental measurement on EWB.

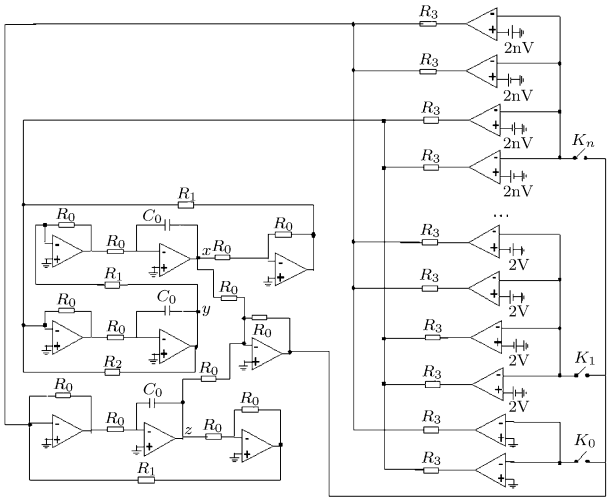


Fig.2. The circuit of realizing system (1).

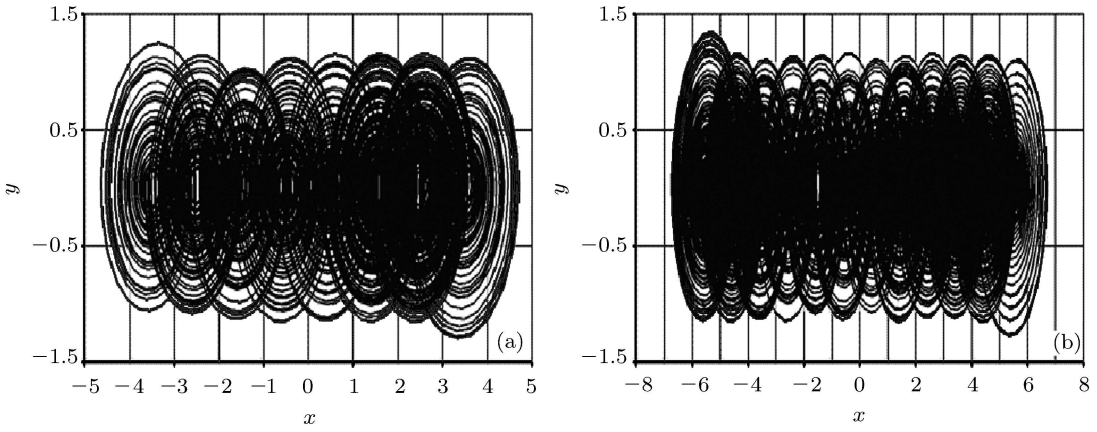


Fig.3. The chaotic attractors on EWB (a) 8-scroll, (b) 12-scroll.

4. Conclusion

In summary, this new multi-scroll chaotic generator is very simple and $2n + 2$ number of scroll attractors can be obtained through modifying the nature number n. The results of simulation and the circuit experiment on EWB are uniform. It is hoped that the applications of chaos to engineering problems would benefit from the results of this paper.

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