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Study on bifurcation and stability of the closed-loop current-programmed boost converters

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In order to predict bifurcation point of the closed-loop current-programmed boost converter and enable this converter to operate at stable parameter space, this paper firstly establishes stroboscopic maps for this converter in the continuous conduction mode according to operating characteristics and topple of this converter. Parameter space at the steady state and bifurcation types are analysed together with stability theory of nonlinear equation. In the solving course, the duty cycle is avoided because of inversive solution, and accuracy is increased. Finally, correction is proved by numerical calculation.

Keywords: boost converter, discrete map, bifurcation
PACC: 0545

1. Introduction

DC–DC converters are widely used in industry, communication, aerospace and civil applications. Their main function is to stabilize a DC voltage to a desired DC value. DC–DC converters are nonlinear systems composed of a piecewise linear system. Various bifurcations, subharmonics and chaos have been found in them.[1−8] When bifurcation occurs in this system, the stability of periodic state will disappear, and a new stable or unstable state arises.[8,9] Meanwhile some paradoxical and irregular phenomena appear, such as high amplitude electromagnetic noise, interval instability, sudden collapse in critical performance and feedback oscillation, etc.[1,2] Bifurcation of DC–DC converters must be deeply studied so as to find the proper parameter space to be assured of stability of their performance. Proper model and analytical method are critical to study bifurcation and stability in the DC–DC converters. There are three types of mostly used methods to model the nonlinear dynamics of DC–DC converters to date. The state space averaging method[10,11] is a frequently adopted one. However, this approach is restricted to analyse the characteristics of the low-frequency domain of converters. The piecewise numerical method completely depends on numerical calculation by computer, so that the operating physical meaning of converters is difficult to understand. It has been demonstrated by many previous researches that the discrete map method with higher degree of accuracy is the optimum to model and analyse the nonlinear dynamic nature of converters.[4,12−14]

In recent years, the open-loop current-programmed boost converter (stabilizing a DC voltage to a higher value) has been deeply studied, while the closed-loop mode is seldom analysed mostly because of the difficulty to determine the border condition.

The main goal of this paper is to obtain a steady parameter space for closed-loop current-programmed boost converter by discrete map model. The paper is arranged as follows. The circuit's model and operating principle are presented in Section 2. In Section 3, a discrete map model for DC–DC converter is derived, and asynchronous switching function is defined. In Section 4, stability for the DC–DC converter is studied based on an inversive method. Finally, we draw some conclusions in Section 5.

2. The circuit model and its principle of operation

The schematic diagram of closed-loop current-programmed boost converter is shown in Fig.1. Provided that the circuit operates in continuous mode (namely the inductor current $i_L > 0$ in a switching cycle), the circuit has two modes of operation based on turning *on* or off of switch T and diode D.

Mode 1: Switch T turns *on* and diode D turns off;

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Mode 2: Switch T turns off and diode D turns on.

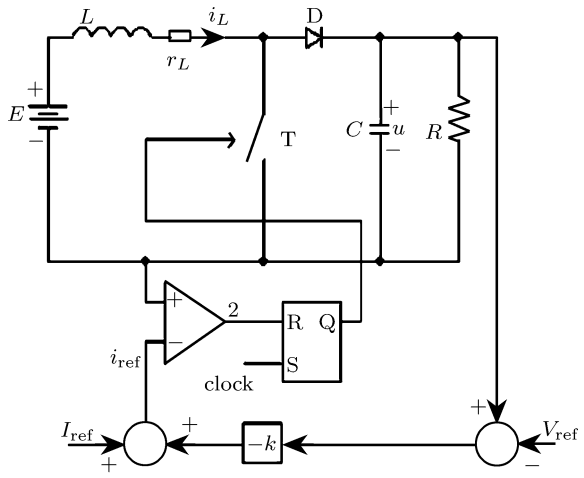


Fig.1. Schematic diagram of close-loop current-programmed boost converter.

According to the circuit topple structure in Fig.1 and using KCL and KVL laws, we find that the state equations are derived as follows:

$$\dot{x} = A_1 x + B_1 E, \quad \mathrm{T}: on \quad \mathrm{D}: off, \tag{1}$$

$$\dot{x} = A_2 x + B_2 E, \quad \mathrm{T}: off \quad \mathrm{D}: on, \tag{2}$$

where x denotes the state vector of the circuit, i.e.

$$x = [x_1, x_2]^{\mathrm{T}} = [u, i_L]^{\mathrm{T}},$$

and

$$A_1 = \begin{bmatrix} -1/RC & 0 \\ 0 & -r_L/L \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1/RC & 1/C \\ -1/L & -r_L/L \end{bmatrix},$$

$$B_1 = B_2 = B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}.$$

Furthermore, $A_1^{-1} = \begin{bmatrix} -RC & 0 \\ 0 & -1/r_L \end{bmatrix}$ and $A_2^{-1} = \dfrac{1}{R+r_L}\begin{bmatrix} -r_L RC & -RL \\ RC & -L \end{bmatrix}$ can be derived. The control law of on or off of switch T is described as follows. Switch T is set on or off by comparing the inductance current i_L with the reference current i_{ref}. When clock impulse starts, switch T is turned on and i_L increases. As i_L climbs up to the reference current i_{ref}, flip-flop RS is reset and switch T is turned off. In this duration, the coming clock impulse doesn't work. Then the resonance of inductor L and output resistance R and capacity C causes a decrease of i_L. The switch T is turned on until the next clock impulse comes. From Fig.1, one can write

$$i_{\mathrm{ref}} = I_{\mathrm{ref}} - k(u - V_{\mathrm{ref}}), \tag{3}$$

where I_{ref} is the steady-state reference current, k is feedback gain factor, and V_{ref} is output reference voltage. For Fig.1, the component values are shown in Table 1 and k is chosen as 0.21. Taking the bifurcation parameter as I_{ref} and applying the stroboscopic map, i.e. sampling value of i_L and duty cycle D at integrated period T, we can obtain bifurcation diagrams shown by Figs.2 and 3.

Table 1. Component values of the circuit.

Circuit components	values
Switching period T/μs	100
Input voltage E/V	5.0
Inductance L/mH	1.5
Capacitance $C/\mu F$	47
Load R/Ω	40
Reference voltage V_{ref}/V	7.83
Steady-state reference current I_{ref}/A	0.3528
Equivalent series Resistance r_L/Ω	0.02

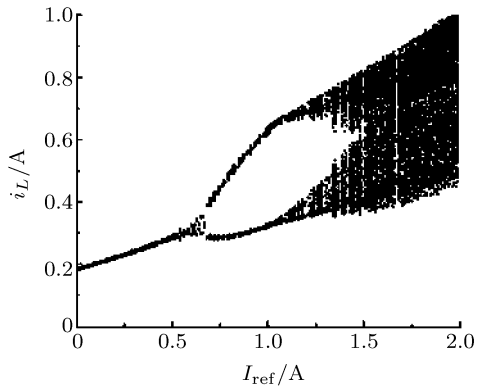


Fig.2. Bifurcation diagram of the inductance i_L.

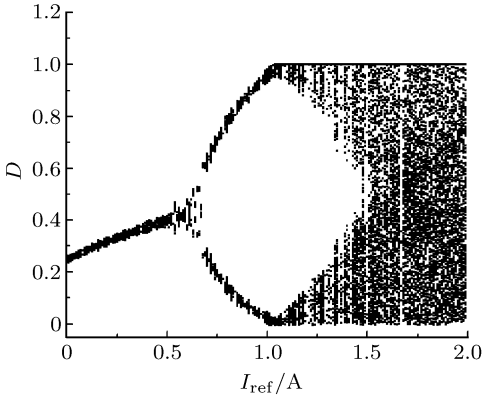


Fig.3. Bifurcation diagram of duty cycle D.

By comparing Fig.2 with Fig.3, it is found that the parameter space of stable duty cycle D and the inductance i_L are correspondingly matched whenever the circuit operates at steady periodic state or bifurcation. This also indicates that the stability of current-programmed boost converter relates intimately to the duty cycle. However, with this traditional numerical method, only numerical solution of D is obtained and the precision is not high. In addition, its analytical solution is written with difficulty and the operating physical meaning of the converter is indefinite. So, in the following text, we firstly establish the discrete map model for boost converter, then by inversive solution that the converter is supposed to operate at periodic state and the duty cycle is taken as independent variable, obtain the range of current reference $I_{\rm ref}$ working in periodic state.

3. Discrete map for boost converter

Due to its operating principle, the current-programmed boost converter has such characteristics as switching of mode occurring at most once during a switching cycle; switching from mode 2 to mode 1 is only at the instant of integrated period (synchronous switching). A discrete map model for boost converter can be established by stroboscopic map method. Let $x_n = x(nT)$ $x_{\rm n+1} = x((n+1)T)$, which are sampling value of x at the instant nT and $(n+1)T$ respectively, and d_n is defined as duty cycle from the nth period to $(n+1)$th period, then the state equations (1) and (2) can be rewritten as

$$\dot{x} = A_1 x + B_1 E, \quad nT \le t \le nT + d_n T, \tag{4}$$

$$\dot{x} = A_2 x + B_2 E, \quad nt + d_n T \le t \le (n+1)T. \tag{5}$$

Using the method to solve a state differential equation in linear system theory, we can obtain the stroboscopic map model for the converter as

$$x(nT + d_n T) = N_1(d_n T)x_n + [N_1(d_n T) - I]A_1^{-1}BE, \tag{6}$$

$$\begin{aligned} x_{n+1} = {} & N_2[(1-d_n)T]N_1(d_n T)x_n + N_2[(1-d_n)T][N_1(d_n T) - I]A_1^{-1}BE \\ & + [N_2(T - d_n T) - I]A_2^{-1}BE, \end{aligned} \tag{7}$$

where $I \in R^{2\times 2}$ is unit matrix and $N_j(\xi) = I + \sum_{k=1}^{\infty} \frac{1}{k!} A_j^k \xi^k$ is state transmission matrix of A_j, $j = 1, 2$. From operation of the circuit, we know that when $i(nT + d_n T) = i_{\rm ref}$, the switch T is turned off, i.e. switches from mode 1 to mode 2. Define the switching function as

$$\begin{aligned} s(x_n, d_n) \overset{\rm def}{=} {} & i_{\rm ref} - i_L = I_{\rm ref} - k[u(nT + d_n T) - V_{\rm ref}] - i_L(nT + d_n T) \\ = {} & I_{\rm ref} + kV_{\rm ref} + g^{\rm T} x(nT + d_n T) \\ = {} & I_{\rm ref} + kV_{\rm ref} + g^{\rm T}[N_1(d_n T)x_n + (N_1(d_n T) - I)A_1^{-1}BE], \end{aligned} \tag{8}$$

where $g^{\rm T} = (-k \quad -1)$. The duty cycle d_n can be solved via $s(x_n, d_n) = 0$. Furthermore x_{n+1} can be solved by substituting d_n into Eq.(7).

4. Stability analysis

Suppose that the circuit operates at stable period-1 state, which also meets the requirement of a practical circuit. While the circuit is steady, it has $x_{n+1} = x_n = x_q$. Let $N_1 = N_1(d_n T)$ and $N_2 = N_2[(1-d_n)T]$. Combining with Eq.(7), we can derive the fixed point as

$$x_q = (I - N_2 N_1)^{-1}[N_2(N_1 - I)A_1^{-1}BE + (N_2 - I)A_2^{-1}BE]. \tag{9}$$

Equations (8) and (9) can be combined to obtain

$$\begin{aligned} s(x_q, d_n) = {} & I_{\rm ref} + kV_{\rm ref} + g^{\rm T}\{N_1(I - N_1 N_2)^{-1}[N_2(N_1 - I)A_1^{-1}BE \\ & + (N_2 - I)A_2^{-1}BE] + (N_1 - I)A_1^{-1}BE\}. \end{aligned} \tag{10}$$

Then let $s(x_q, d_n) = 0$, and the function of $I_{\rm ref}$ and d_n can be derived. The Jacobian matrix of fixed point x_q[15] is

$$\begin{aligned} J(x_q) = & \frac{\partial f}{\partial x_n} - \frac{\partial f}{\partial d_n}\left(\frac{\partial s}{\partial d_n}\right)^{-1}\left(\frac{\partial s}{\partial x_n}\right)\Big|_{x_n=x_q} \\ = & N_2N_1 - \frac{[TN_2(A_1-A_2)N_1x_q + TN_2(A_2-A_2N_1+N_1A_1)A_1^{-1}BE]\cdot g^{\rm T}N}{g^{\rm T}TN_1(A_1x_q+B_1E)}. \end{aligned} \tag{11}$$

By solving the following characteristic equation of matrix $J(x_q)$:

$$\det(\lambda I - J(x_q)) = 0, \tag{12}$$

one can get the function in relation to eigenvalues and duty cycle. In the presence of Eqs.(10), (11) and (12), the parameter range of $I_{\rm ref}$ for the circuit operating at stable period-1 can be derived. Table 1 shows the eigenvalues λ and the corresponding reference current $I_{\rm ref}$, here the duty cycle is taken as independent variable.

Table 2. Eigenvalues and state analysis for different duty cycle.

d_n	$\lambda_{1,2}$	$i_{\rm ref}$	Remarks
0.2500	0.4843,-0.4794	0.0060	Stable period-1
0.3000	0.5075,-0.5960	0.1438	Stable period-1
0.3500	0.5284, -7283	0.3044	Stable period-1
0.4300	0.5572,-0.9848	0.6256	Stable period-1
0.4310	0.5575,-0.9884	0.6303	Stable period-1
0.4330	0.5582,-0.9957	0.6396	Stable period-1
0.4350	0.5588,-1.0031	0.6490	Double bifurcation

As is shown in Table 2, when d_n increases to 0.4350, period-doubling occurs. The same result has been shown in bifurcation diagrams derived from numerical calculation. But a more precise result is found in Table 2, i.e. when $I_{\rm ref} < 0.6490$, the circuit operates at stable period-1 state.

5. Conclusion

DC–DC converters have many applied prospects in modern industry, communication and resident. Because the converters are high nonlinear system composed of piecewise linearity, it is necessary to adopt proper model to analyse their stability. In this paper, based on the operation characteristics of current-programmed boost converter, stroboscopic map model is firstly derived. Secondly in the presence of inversive solution, which supposes the converter operating at stably periodic state and takes the bifurcation parameter as duty cycle, the range of $I_{\rm ref}$ in stability is inversely derived. Through this method, solution of the duty cycle is avoided, and accuracy is increased. Furthermore, the form of analytical function of duty cycle and parameter $I_{\rm ref}$ is derived, which provides certain guides for practical application.

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