Vol 16 No 4, April 2007
1009-1963/2007/16(04)/0928-05

Chinese Physics

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Mean first-passage time of an asymmetric bistable system driven by colour-correlated noise *

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(Received 5 June 2006; revised manuscript received 27 July 2006)

In this paper, the effect of every parameter (including p, q, r, λ, τ) on the mean first-passage time (MFPT) is investigated in an asymmetric bistable system driven by colour-correlated noise. The expression of MFPT has been obtained by applying the steepest-descent approximation. Numerical results show that (1) the intensity of multiplicative noise p and the intensity of additive noise q play different roles in the MFPT of the system, (2) suppression appears on the curve of the MFPT with small λ (e.g. $\lambda < 0.5$) but there is a peak on the curve of the MFPT when λ is big (e.g. $\lambda > 0.5$), and (3) with different values of r (e.g. $r = 0.1$, 0.5, 1.5), the effort of τ on the MFPT is diverse.

Keywords: mean first- passage time, asymmetric bistable system, colour-correlated noise
PACC: 0540, 0250

1. Introduction

In recent years, the mean first-passage time (MFPT) has been the significant topic in the nonlinear stochastic dynamical systems.[1−7] The MFPT is introduced to depict the transient properties of the systems. Wang *et al*[4] have shown the existence of cross-correlation noise which leads to symmetry-breaking in the MFPT. Xie and Mei [5] studied the MFPT of a bistable kinetic model driven by correlated multiplicative coloured noise and additive white noise. They found that the MFPT corresponding to weakly and strongly correlated noises exhibits a very different dependence on the intensities of multiplicative noise even though the correlation time of the noises is zero.

Recently some researchers are interested in the characters of the asymmetric bistable system, because the asymmetric bistable system also exists in the actual physics system.[6−10] Li[9] studied the effort of the asymmetry of the potential on stochastic resonance and he has shown that the asymmetry of the bistable potential can weaken the phenomenon of stochastic resonance. These papers discuss the transients of the system which is driven by additive noise, but most previous studies indicate that the systems will present some peculiar characteristics if it is driven by colour noise or colour correlation noise.[3,5,11−15] So in this paper, we investigate the MFPT of an asymmetric bistable system driven by colour-correlated noise, and discuss the effects of the bias of bistable system and the colour-correlation noise on the MFPT.

2. The MFPT of an asymmetric bistable system

Consider an asymmetric bistable kinetic system described by the following stochastic differential equation with coloured cross-correlated multiplicative and additive white noise:

$$\dot{x} = -U'(x) + r + x\xi(t) + \eta(t), \tag{1}$$

$$U(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4, \tag{2}$$

where $U(x)$ is a symmetry bistable potential function, r denotes the bias of bistable system. $\xi(t)$ and $\eta(t)$ are both Gauss white noises with the following statistical properties:

$$\begin{aligned} &\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0, \\ &\langle \xi(t)\xi(t') \rangle = 2p\delta(t-t'), \\ &\langle \eta(t)\eta(t') \rangle = 2q\delta(t-t'), \\ &\langle \xi(t)\eta(t') \rangle = \langle \eta(t)\xi(t') \rangle = \frac{\lambda\sqrt{pq}}{\tau} \exp\left[-\frac{1}{\tau}|t-t'|\right]. \end{aligned} \tag{3}$$

where p and q describe the intensity of multiplicative and additive noise respectively. The parameter

*Project supported by the National Natural Science Foundation of China (Grants Nos 10472091, 10332030 and 10502042) and the Graduate Starting Seed Fund of Northwestern Polytechnical University (Grant No Z200655).

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$\lambda(0 \le \lambda < 1)$ measures the strength of correlation between multiplicative and additive noise terms, and τ is the correlation time of the correlation between the noises.

According to Eq.(2), the potential $U(x)$ has two stable states $x_1 = -1$, $x_2 = 1$ and an unstable state $x_0 = 0$.

By virtue of the Novikov theorem,[16] Fox's approach[17] and the unified coloured-noise approximation,[18] we can get the Fokker–Planck equation (FPE) from Eqs.(1)–(3):

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}A(x)P(x,t) + \frac{\partial^2}{\partial x^2}B(x)P(x,t),$$

where

$$A(x) = x - x^3 + r + \frac{1}{2}B'(x),$$
$$B(x) = px^2 + \frac{2\lambda\sqrt{pq}}{1+2\tau}x + q. \quad (4)$$

The stationary probability function can be attained from Eq.(4):

$$P(x,t) = \frac{N}{\sqrt{px^2 + \dfrac{2\lambda\sqrt{pq}}{1+2\tau}x + q}} \exp\left[-\frac{\tilde{U}(x)}{p}\right], \quad (5)$$

where N is the normalization constant of Eq.(5), the generalized potential $\tilde{U}(x)$ is given by

$$\tilde{U}(x) = -\int \frac{x - x^3 + r}{x^2 + \dfrac{2\lambda\sqrt{R}}{1+2\tau}x + R}\mathrm{d}x, \quad (6)$$

where $R = q/p$ denotes the ratio of additive noise intensity to multiplicative one.

According to the definition of the MFPT and the steepest-descent approximation,[19] we can get the expression of the MFPT for a particle to reach the point x_2, from the point x_1:

$$T = \sqrt{2}\pi \left|1 + \frac{2\lambda}{(1+2\tau)\sqrt{R}} + \frac{1}{R}\right|^{-\varphi_1/p} \exp\left[-\frac{1}{2p} + \frac{2\lambda\sqrt{R}}{(1+2\tau)p} + \frac{\varphi_2}{p\sqrt{\varphi_3}}\left(\arctan\frac{\lambda}{\sqrt{\varphi_3}} - \arctan\frac{\lambda + 1/\sqrt{R}}{\sqrt{\varphi_3}}\right)\right], \quad (0 \le \lambda < 1). \quad (7)$$

Where

$$\varphi_1 = \frac{1}{2}\left(\frac{4\lambda^2 R}{(1+2\tau)^2} - R - 1\right),$$
$$\varphi_2 = \frac{\lambda R}{1+2\tau}\left(3 - \frac{4\lambda^2}{(1+2\tau)^2}\right) + \frac{\lambda}{1+2\tau} - \frac{r}{\sqrt{R}}, \quad (8)$$
$$\varphi_3 = 1 - \frac{\lambda^2}{(1+2\tau)^2},$$
$$R = q/p.$$

3. Discussion

In conformity with Eq.(7) of the MFPT, the intensity of multiplicative and additive noise, the strength of correlations between multiplicative and additive noise, the correlation time between the noises and the bias of bistable system on the MFPT can be discussed by numerical calculation.

From Fig.1, it is easy to find that the MFPT for $R > 1$ is the largest among the three cases $R > 1$, $R = 1$, $R < 1$, and the MFPT increases as R increases. Simultaneously the MFPT as a function of λ is nonmonotonic; the curves indicate that the MFPT has a valley with small λ, and a peak with big λ. Namely there is a suppression on the MFPT with small $\lambda(\lambda < 0.5)$ and the resonance appears on the curves of the MFPT when the λ increases further $(\lambda > 0.5)$.

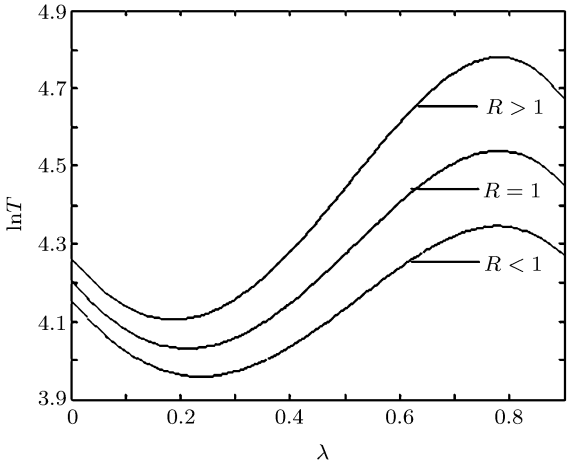


Fig.1. The MFPT as a function of λ for $q = 0.1$, $\tau = 0.1$, $r = 0.1$ with $p = 0.09(R > 1)$, $0.1(R = 1)$, and 0.11 $(R < 1)$.

The curve of the MFPT versus the correlation time between the noises τ with different R (the ratio of additive noise intensity to multiplicative one) is plotted in Fig.2. The plot shows that the MFPT for $R > 1$ is the largest among the three curves. With the increasing correlation time between the noises, the curve of MFPT presents a maximum, i.e. it is a single resonant peak. And the position of the resonant peak is shifted slightly to small values of τ with the increase of R.

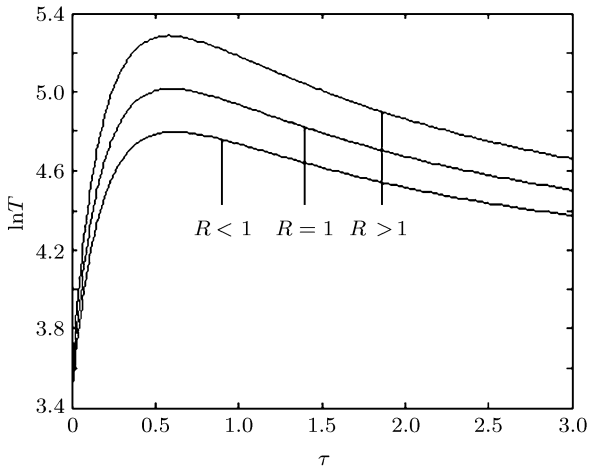


Fig.2. The MFPT as a function of τ for $q = 0.1$, $\lambda = 0.5$, $r = 0.1$ with $p = 0.09(R > 1)$, $0.1(R = 1)$, and $0.11(R < 1)$.

In Fig.3 we present the MFPT as the function of the intensity of multiplicative noise. When we fix the values of $q = 0.03$, $\tau = 0.1$, $\lambda = 0.7$, Fig.3 shows that the MFPT decreases monotonically as the p increases, and the MFPT increases as the r increases.

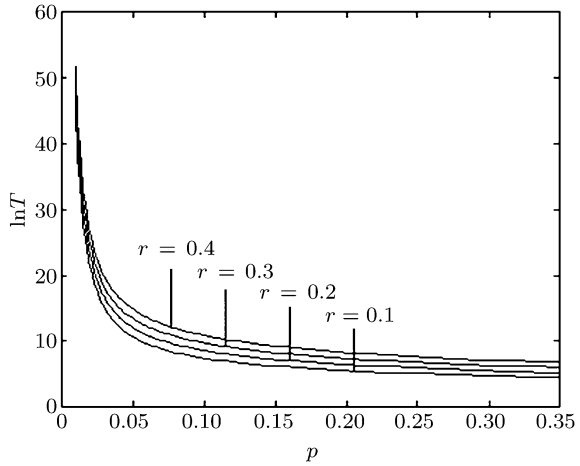


Fig.3. The MFPT as a function of p for $q = 0.03$, $\tau = 0.1$, $\lambda = 0.7$ with $r = 0.1$, 0.2, 0.3 and 0.4.

As shown in Fig.4, the MFPT behaves nonmonotonically as the intensity of additive noise q increases. The MFPT decreases at first and then increases as q increases, the intensity of additive noise causes the suppressive effort on the MFPT, whereas the MFPT increases as the correlation time between the noises τ increases. Comparing with Fig.3, we can see that p and q play different roles in the MFPT of the system.

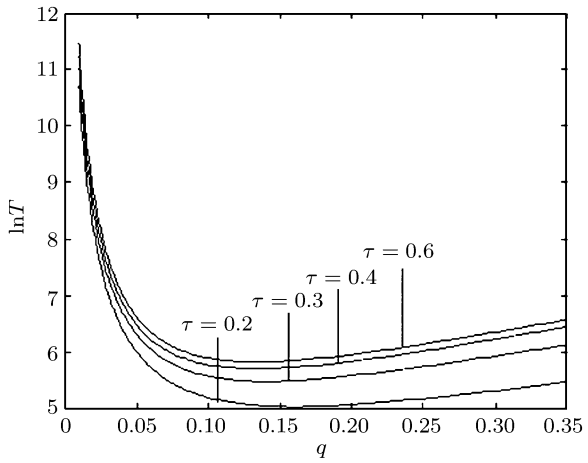


Fig.4. The MFPT as a function of q for $p = 0.1$, $r = 0.1$, $\lambda = 0.7$ with $\tau = 0.2$, 0.3, 0.4 and 0.6.

In Fig.5 we present the MFPT as the function of the bias of bistable system. When we fix the values of $q = 0.03$, $\tau = 0.1$, $p = 0.1$, Fig.5 shows that the MFPT increases as r increases and decreases as λ increases. With every fixed value of λ, the curves of the MFPT as the function of r are beelines. Namely the MFPT is a linear function of r if other parameters are fixed.

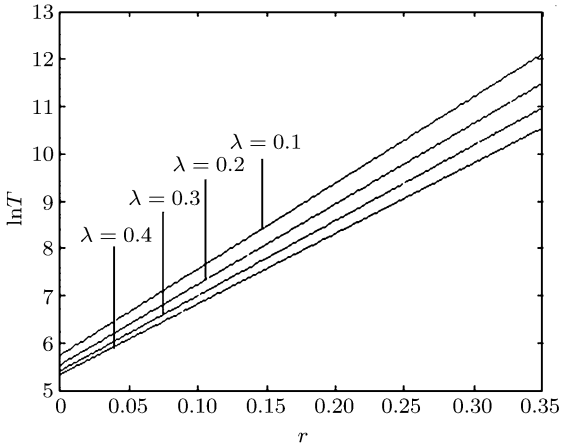


Fig.5. The MFPT as a function of r for $q = 0.03$, $\tau = 0.1$, $p = 0.1$ with $\lambda = 0.1$, 0.2, 0.3 and 0.4.

From Figs.6(a), (b), (c), we can see that the

MFPT has a maximum for small r (e.g. (a) $r = 0.1$, (b) $r = 0.5$) as τ increases, and the MFPT decreases monotonically for large r (e.g. $r = 1.5$) under the same τ. From the plots (b) and (c) we note that the MFPT decreases as the λ increases, whereas when r is the smallest of the three values (e.g. $r = 0.1$), with varied values of the bias of the bistable system, the effort of the correlation time between the noise τ on the MFPT is quite different. The MFPT as the function of λ is nonmonotonic.

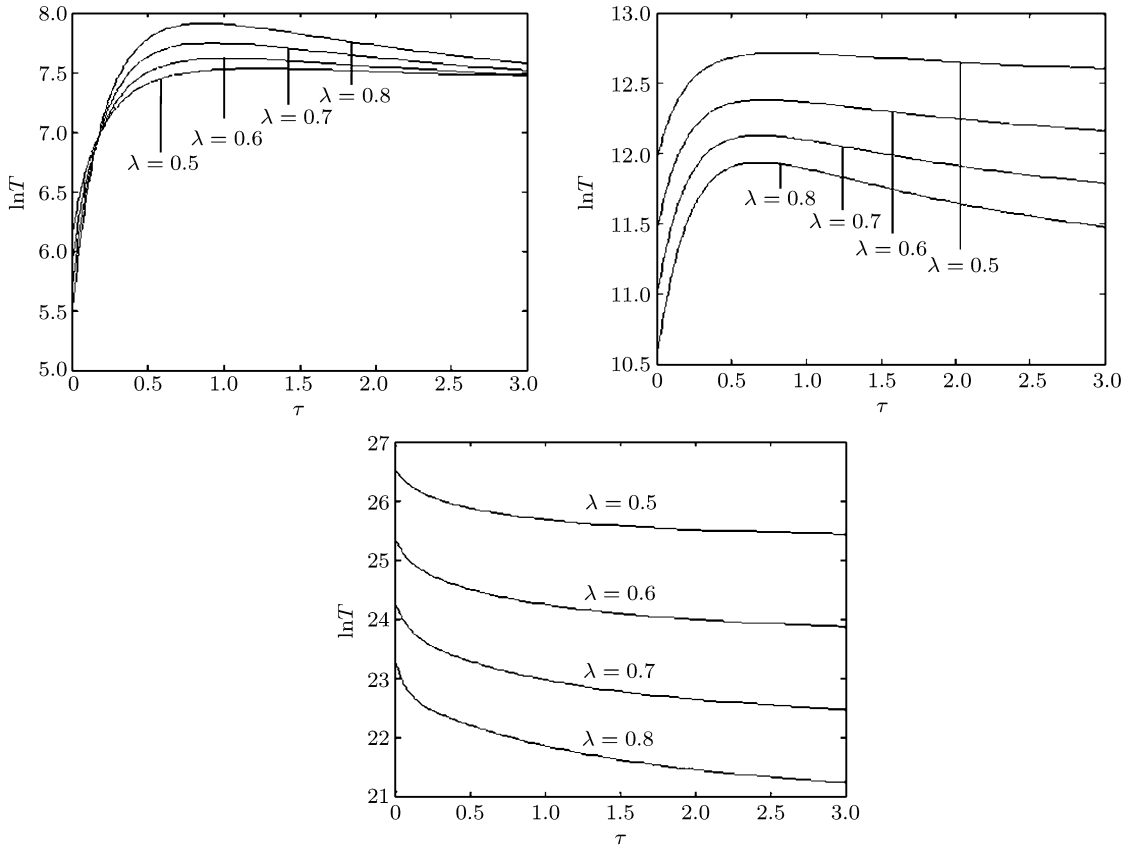


Fig.6. The MFPT as a function of τ for $r = 0.1$ (a), $r = 0.5$ (b), $r = 1.5$ (c), $q = 0.03$, $q = 0.1$, with $\lambda = 0.5$, 0.6, 0.7 and 0.8.

4. Conclusion

Based on the definition of the MFPT and the steepest-descent approximation, we get the approximate expression of the MFPT. In conclusion, the efforts of the intensity of multiplicative and additive noise on the MFPT are different. The strength of correlations between noises, λ, suppresses the MFPT as the λ is small and the λ enhances the MFPT as the λ is big. The effort of the correlation time between the noises on the MFPT is very interesting with the different bias of bistable system.

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