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Entanglement swapping without joint measurement via a Λ-type atom interacting with bimodal cavity field*

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This paper proposes a scheme for realizing entanglement swapping in cavity QED. The scheme is based on the resonant interaction of a two-mode cavity field with a Λ-type three-level atom. In contrast with the previously proposed schemes, the present scheme is ascendant, since the fidelity is 1.0 and the joint measurement isn't needed. And the scheme is experimentally feasible based on the current cavity QED technique.

Keywords: entanglement swapping, Λ-type atom, two-mode cavity field, joint measurement
PACC: 0367, 0365, 4250

Entanglement plays an important role in quantum information processing. Several schemes have been proposed to generate entangled states, such as the nonlinear interaction between optical pulse and nonlinear crystal,[1] the interaction between different particles, and so on.[2,3] After the entanglement generation, the entangled particles must be distributed among distant users for quantum communication purpose. During the distribution process, the entanglement of the particles will inevitably decrease. The longer the distance, the bigger the decrease. To avoid this problem, an alternative method has been proposed to generate entanglement between two distant particles which have never interacted before. This is the so-called entanglement swapping.[4] In this method, there are usually three spatially separate users, and two of them have shared one pair of entangled particles with the third. Then the third user will operate a joint measurement (such as Bell-state measurement) on the two particles he possesses. Corresponding to the measurement result, the two particles possessed by the two spatially separate users will collapse into an entangled state without any entanglement before the joint measurement. In addition to the entanglement generation, entanglement purification is another application of entanglement swapping.[5] So the realization of entanglement swapping is very important for the quantum communication. In the method,[4] the realization of the Bell-state measurement or other types of joint measurement is important for the entanglement swapping. However, it is still difficult for us to discriminate the four Bell states totally and conclusively in experiment, especially for the atomic Bell states.

In the context of cavity QED, few schemes have been proposed for entanglement swapping with joint measurement.[6−9] Recently, Yang *et al*[10] have proposed a novel scheme for entanglement swapping by using concepts of cavity QED, the distinct feature is that it only needs single measurement rather than joint measurement. In the scheme, the success probability is $|b|^2(1-|b|^2)$ and the fidelity is about $|b|^2/[|b|^2+(1-|b|^2)(0.079)^2]$. The fidelity is bigger than 0.9 unless $b < 0.25$. In this paper, we present an alternative scheme for entanglement swapping without joint measurement, which is based on the resonant interaction of a two-mode cavity field with a Λ-type three-level atom. For entanglement swapping, the fidelity of our scheme is 1.0 and the total time required to complete the entanglement swapping process is shortened, which is prior to the scheme in Ref.[10].

We first consider a three-level atom in the Λ configuration interacting with a two-mode cavity field (see Fig.1). The modes with annihilation operators a_1 and a_2 interact with the $|e\rangle \leftrightarrow |g\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ transitions respectively. The Hamiltonian under the rotating wave approximation can be written as

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$$H=\omega_1 a_1^+ a_1+\omega_2 a_2^+ a_2+\omega_e|e\rangle\langle e|+\omega_g|g\rangle\langle g|+\omega_f|f\rangle\langle f|+g_1(a_1^+|g\rangle\langle e|+a_1|e\rangle\langle g|)+g_2(a_2^+|f\rangle\langle e|+a_2|e\rangle\langle f|), \quad (1)$$

where $\omega_e, \omega_g, \omega_f$ are the energy levels for the atomic levels $|e\rangle$, $|g\rangle$ and $|f\rangle$ respectively. $\omega_i (i \in 1, 2)$ is the frequency of the cavity modes 1 and 2, and g_i is the atom–cavity coupling constant. We assume g_i to be real.

We work under the condition that

$$\omega_1=\omega_e-\omega_g, \quad \omega_2=\omega_e-\omega_f, \quad g_1=g_2=g. \quad (2)$$

Thus in the interaction picture, we derive an effective Hamiltonian describing the system of Fig.1,

$$H_{\text{eff}}=g(a_1^+|g\rangle\langle e|+a_2^+|f\rangle\langle e|)+\text{H.c.} \quad (3)$$

Such an effective Hamiltonian leads to the following evolutions (with the evolution time t):

$$\begin{aligned}
|e\rangle|m,n\rangle \to{}& \cos(gt\sqrt{m+n+2})|e\rangle|m,n\rangle-\frac{\mathrm{i}\sin(gt\sqrt{m+n+2})}{\sqrt{m+n+2}} \\
&\times[\sqrt{m+1}|g\rangle|m+1,n\rangle+\sqrt{n+1}|f\rangle|m,n+1\rangle], \quad (4a)\\
|g\rangle|m+1,n\rangle \to{}& \frac{1}{m+n+2}[(m+1)\cos(gt\sqrt{m+n+2})+n+1]|g\rangle|m+1,n\rangle \\
&+\frac{\sqrt{(m+1)(n+1)}}{m+n+2}[\cos(gt\sqrt{m+n+2})-1]|f\rangle|m,n+1\rangle \\
&-\mathrm{i}\frac{\sqrt{m+1}}{\sqrt{m+n+2}}\sin(gt\sqrt{m+n+2})|e\rangle|m,n\rangle, \quad (4b)\\
|f\rangle|m,n+1\rangle \to{}& \frac{1}{m+n+2}[(n+1)\cos(gt\sqrt{m+n+2})+m+1]|f\rangle|m,n+1\rangle \\
&+\frac{\sqrt{(m+1)(n+1)}}{m+n+2}[\cos(gt\sqrt{m+n+2})-1]|g\rangle|m+1,n\rangle \\
&-\mathrm{i}\frac{\sqrt{n+1}}{\sqrt{m+n+2}}\sin(gt\sqrt{m+n+2})|e\rangle|m,n\rangle, \quad (4c)
\end{aligned}$$

where m and n represent the photon number of the cavity mode 1 and 2 respectively.

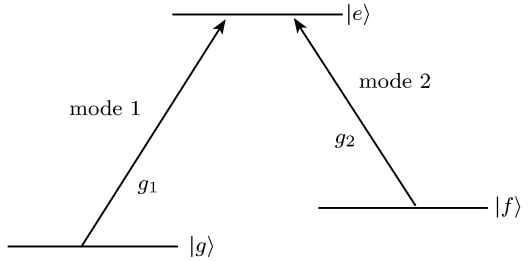


Fig.1. Schematic diagram of a Λ-type three-level atom interacting with a two-mode cavity field. Here g_1 and g_2 are the atom–cavity field coupling constants of the mode 1 and mode 2 with the corresponding transitions respectively.

Now we use the above system to implement entanglement swapping. Suppose there are three spatially separate users Alice, Bob, and Clare. Alice and Clare have shared a pair of atoms (1, 2) with atom 1 belonging to Alice and atom 2 belonging to Clare. These two atoms have been previously prepared in the following entangled state:

$$|\Phi\rangle_{12}=a|g\rangle_1|g\rangle_2+b|f\rangle_1|f\rangle_2, \quad (5)$$

where a and b are the normalization coefficients.

In addition to atom 2, Clare also possesses one two-mode cavity 3, which is entangled with another two-mode cavity 4, and cavity 4 belongs to Bob. Similarly, we also suppose that the two cavities have been prepared in the following entangled state:

$$|\Phi\rangle_{34}=a|0,1\rangle_3|1,0\rangle_4+b|1,0\rangle_3|0,1\rangle_4, \quad (6)$$

where the normalization coefficients are all the same to the atomic entangled state in Eq.(5). From the above two entangled states, we conclude that there is no correlation between the atom 1 and cavity 4 at this moment. After entanglement swapping, the atom

1 and cavity 4 will be left in a maximally entangled state.

Before swapping, the state of the total system is

$$|\Psi\rangle_{1234} = (a|g\rangle_1|g\rangle_2 + b|\rangle_1|f\rangle_2) \otimes (a|0,1\rangle_3|1,0\rangle_4 + b|1,0\rangle_3|0,1\rangle_4). \quad (7)$$

To realize the entanglement swapping, Clare sends the atom 2 through the bimodal cavity 3. After the interaction time t, the state of the total system evolves into the following state:

$$\begin{aligned}|\Psi\rangle_{1234} = {} & a^2|g\rangle_1|1,0\rangle_4|g\rangle_2|0,1\rangle_3 + b^2|f\rangle_1|0,1\rangle_4|f\rangle_2|1,0\rangle_3 + ab|g\rangle_1|0,1\rangle_4 \\ & \times \left\{\frac{1}{2}[\cos(\sqrt{2}gt)+1]|g\rangle_2|1,0\rangle_3 + \frac{1}{2}[\cos(\sqrt{2}gt)-1]|f\rangle_2|0,1\rangle_3 \right. \\ & \left. -\frac{\mathrm{i}}{\sqrt{2}}\sin(\sqrt{2}gt)|e\rangle_2|0,0\rangle_3\right\} + ab|f\rangle_1|1,0\rangle_4\left\{\frac{1}{2}[\cos(\sqrt{2}gt)+1]|f\rangle_2|0,1\rangle_3 \right. \\ & \left. +\frac{1}{2}[\cos(\sqrt{2}gt)-1]|g\rangle_2|1,0\rangle_3 - \frac{\mathrm{i}}{\sqrt{2}}\sin(\sqrt{2}gt)|e\rangle_2|0,0\rangle_3\right\}. \end{aligned} \quad (8)$$

We adjust the atomic velocity carefully to make the interaction time in the way that $\sqrt{2}gt = \frac{\pi}{2}$. Then we have

$$\begin{aligned}|\Psi\rangle_{1234} = {} & a^2|g\rangle_1|1,0\rangle_4|g\rangle_2|0,1\rangle_3 + b^2|f\rangle_1|0,1\rangle_4|f\rangle_2|1,0\rangle_3 \\ & +ab|g\rangle_1|0,1\rangle_4\left(\frac{1}{2}|g\rangle_2|1,0\rangle_3 - \frac{1}{2}|f\rangle_2|0,1\rangle_3 - \frac{\mathrm{i}}{\sqrt{2}}|e\rangle_2|0,0\rangle_3\right) \\ & +ab|f\rangle_1|1,0\rangle_4\left(\frac{1}{2}|f\rangle_2|0,1\rangle_3 - \frac{1}{2}|g\rangle_2|1,0\rangle_3 - \frac{\mathrm{i}}{\sqrt{2}}|e\rangle_2|0,0\rangle_3\right). \end{aligned} \quad (9)$$

After the atom flying out of the cavity, Clare detects atom 2. If atom 2 is detected in the state $|e\rangle_2$, the atom 1 and cavity 4 collapse approximately into a maximally entangled state without detection on cavity 3:

$$|\Psi\rangle_{14} = \frac{1}{\sqrt{2}}(|g\rangle_1|0,1\rangle_4 + |f\rangle_1|1,0\rangle_4) \quad (10)$$

with probability $P = |b|^2(1-|b|^2)$. The fidelity of the output state relative to a perfect maximally entangled state is $F = 1.0$. The biggest successful probability can reach 0.25. After swapping, Bob can send a Λ-type three-level atom initially in state $|g\rangle$ or $|f\rangle$ through cavity 4. If Bob sets the interaction time appropriately, the two atoms, which belong to Alice and Bob and have never interacted before, are in a maximally entangled state.

From the above process, we found that measurement on the atomic state is still needed in the above scheme. In fact, this scheme also can realize the entanglement swapping without measurement on the atoms. From the evolution result in Eq.(9), we found that Clare also can detect cavity 3 rather than atom 2 to complete the entanglement swapping process. Furthermore, Clare only needs an ordinary photon detector, which only distinguishes the vacuum and nonvacuum Fock states. If cavity 3 is detected in the vacuum state, atom 1 and cavity 4 will collapse approximately into the maximally entangled state expressed in Eq.(10) without detection on atom 2. The probability of obtaining this state is still $P' = |b|^2(1-|b|^2)$ and the fidelity of the output state relative to a perfect maximally entangled state is still $F' = 1.0$.

The coefficients in Eqs.(5) and (6) are chosen to be identical (in amplitude and phase) in this paper. If there exists an error between the coefficients in Eqs.(5) and (6), what will the result become? Through analysis, we find that the current scheme allows the existence of a small error between the coefficients in Eqs.(5) and (6). Suppose the states in Eqs.(5) and (6) can be reexpressed as

$$|\Phi\rangle_{12} = \sqrt{1-b^2}|g\rangle_1|g\rangle_2 + b|f\rangle_1|f\rangle_2, \quad (11)$$

$$|\Phi\rangle_{34} = \sqrt{1-b^2(1+k)^2}|0,1\rangle_3|1,0\rangle_4 + b(1+k)|1,0\rangle_3|0,1\rangle_4, \quad (12)$$

where k is the small error rate constant of the coefficient b, and k, b must satisfy $|b|<1$, $|b(1+k)|<1$. If the above mentioned error exists, the success proba-

bility and the fidelity of the output state will become

$$P_{\rm new} = \frac{1}{2}\{(1-b^2)b^2(1+k)^2 + b^2[1-b^2(1+k)^2]\}, \quad (13)$$

$$F_{\rm new} = \frac{\left[(1+k)\sqrt{1-b^2}+\sqrt{1-b^2(1+k^2)}\right]^2}{2\left[1+(1+k)^2-2b^2(1+k)^2\right]}. \quad (14)$$

If we let the error rate constant $k = 0.1$, $b = 0.6$, then $P_{\rm new} = 0.24098$, $F_{\rm new} = 0.99381$. That is to say, this kind of error will affect the probability of success and the fidelity very slightly.

Next, we will discuss the feasibility of the proposed scheme. From Refs.[11,12], we use the bimodal cavity like the one used by Haroche's group, the lifetime of the cavity field is $T_{\rm c} = 1.0 \times 10^{-3}\,{\rm s}$, and the three atomic levels $|e\rangle$, $|g\rangle$ and $|f\rangle$ are Rydberg levels as in Haroche's experiments, the radiative lifetime is $T_{\rm r} = 3 \times 10^{-2}\,{\rm s}$, the atom–cavity field coupling constant is about $g = 2\pi \times 25\,{\rm kHz}$. Then we get that the interaction time of atom and cavity is $\frac{\pi}{2\sqrt{2}g} = 7.1 \times 10^{-6}\,{\rm s}$, so we can evaluate that the total time for the whole scheme is about $T = 7.1 \times 10^{-5}\,{\rm s}$, which is much shorter than $T_{\rm r}$, $T_{\rm c}$. Hence, the current scheme might be realizable in the near future.

In summary, we have presented an alternative scheme for entanglement swapping based on the resonant interaction of a two-mode cavity field with a Λ-type three-level atom. The scheme does not need a joint measurement. In contrast with the scheme of entanglement swapping in Ref.[10], our scheme is more effective, since the fidelity (being 1) is higher than that in Ref.[10], which was based on a resonant interaction between an atom and a single mode cavity, and the total time of our scheme required to complete the process of entanglement swapping is greatly shortened (the total time of the scheme in Ref.[10] required to complete the same process is about $3.5 \times 10^{-4}\,{\rm s}$ with the same experimental parameters of Haroche's group), which is important in view of decoherence.

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