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Enhancing the capability of controlling quantum systems via ancillary systems*

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This paper explores the potential of controlling quantum systems by introducing ancillary systems and then performing unitary operation on the resulting composite systems. It generalizes the concept of pure state controllability for quantum systems and establishes the link between the operator controllability of the composite system and the generalized pure state controllability of its subsystem. It is constructively demonstrated that if a composite quantum system can be transferred between any pair of orthonormal pure vectors, then its subsystem is generalized pure-state controllable. Furthermore, the unitary operation and the coherent control can be concretely given to transfer the system from an initial state to the target state. Therefore, these properties may be potentially applied in quantum information, such as manipulating multiple quantum bits and creating entangled pure states. A concrete example has been given to illustrate that a maximally entangled pure state of a quantum system can be generated by introducing an ancillary system and performing open-loop coherent control on the resulting composite system.

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1. Introduction

Dating from the birth of quantum theory, control of a quantum system has been an important issue.[1] Quantum system control theory has developed since the 1980s.[2−4] Controllability conditions of quantum systems[2,5−19] have been discussed. Although the concepts of controllability for quantum systems are mostly discussed in the sense of open-loop coherent control,[2,5−18] some efforts[19] have been made to generalize the open-loop controllability, and gradually make it clear that the concept of controllability for quantum systems does not need to be limited to the narrow sense of open-loop coherent control. It can be concluded from recent researches[17,19,20] that the ability of controlling a quantum system will be enhanced if more resources are permitted to be put into use. However, the potential of introducing ancillary system has not been fully explored yet.

In this paper, we explore the potential of controlling quantum systems by introducing ancillary systems and then performing unitary operation on the resulting composite systems. By this way, we can generalize the concept of controllability for quantum systems. We constructively demonstrate that if a composite quantum system is operator controllable,[5,16] then its subsystem is generalized pure-state controllable. That is, if a composite quantum system S^{AB} consisting of two subsystems S^A and S^B is operator controllable, then any initial pure state of either subsystem can be transferred to any target pure state. Subsequently, it is also demonstrated that if a composite quantum system can be transferred between any pair orthonormal pure vectors, then its subsystem is generalized pure-state controllable. Furthermore, the unitary operation can be constructively designed by this method. Therefore, these results can be potentially applied in quantum information, such as manipulating multiple quantum bits[21] and creating pure entangled state.[22−24] It is easy to conclude that if $N+1$ quantum bits are operator controllable, then N quantum bits can be transferred from any initial state $|\psi_0\rangle$ to any target state $|\psi_s\rangle$ just by performing a single unitary operation on the $N+1$ quantum bits. Now entangled pure states play an important role in quantum computation and quantum communication;[21,25] a concrete example is given to illustrate that the maximally entangled pure state of two 2-level particles can

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be generated by introducing an ancillary 2-level particle and performing open-loop coherent control H_c on the three 2-level particles.

This paper is organized as follows. In Section 2, we briefly review and generalize the controllability of quantum systems. The basic lemma for any pair of orthonormal vectors is derived in this section. Section 3 illustrates the advantage of controlling quantum systems by introducing ancillary systems. In Section 4, we discuss the implication of introducing ancillary system in quantum computation. Section 5 gives a concrete example of creating a maximally entangled pure state by introducing an ancillary system. The paper concludes with Section 6.

2. Preliminaries

In this section, we first briefly review the pure state controllability of quantum systems. Consider an n-dimensional closed quantum dynamical system in its Hilbert space H

$$|\dot{\psi}(t)\rangle = \frac{\mathrm{i}}{\hbar}\left(H_0 + \sum_{i=1}^{L} u_i(t)H_i\right)|\psi(t)\rangle, \qquad (1)$$

it is pure state controllable if, for each pair pure state $|\psi_1\rangle$ and $|\psi_2\rangle$ in H, there exist controls $u_1, u_2, \ldots, u_l$ and a time $t > 0$ such that $|\psi_2\rangle = X(t)|\psi_1\rangle$, where $X(t)$ is the solution of the following equation:

$$\dot{X}(t) = \frac{\mathrm{i}}{\hbar}\left(H_0 + \sum_{i=1}^{L} u_i(t)H_i\right)X(t) \qquad (2)$$

with initial condition $X(0) = I_n$.

In the above definition, only open-loop coherent controls are permitted to be performed on the quantum systems. In the following, we will generalize the pure state controllability.

Definition 1 (Generalized Reachability of Pure State) For a quantum system in Hilbert space H, $|\psi_2\rangle$ in H is generalized pure state reachable from $|\psi_1\rangle$ if there exist physically implementable methods so that the quantum system can be transferred from $|\psi_1\rangle$ to $|\psi_2\rangle$.

Definition 2 (Generalized Pure State Controllability of Quantum System) A quantum system in Hilbert space H is generalized pure state controllable if for each pair of pure state $|\psi_1\rangle$ and $|\psi_2\rangle$ in H, there exist implementable physical methods so that the quantum system can be transferred from $|\psi_1\rangle$ to $|\psi_2\rangle$.

For example, we can control a quantum system by introducing an ancillary system and then performing open-loop coherent control on the resulting composite system which consists of the controlled system and the ancillary system. In this sense, the concept of pure state controllability is generalized.

Next, in order to facilitate further discussions, we will demonstrate the basic lemma for any pair of orthonormal vectors.

Lemma For any pair of orthonormal vectors $|\psi_1\rangle$ and $|\psi_2\rangle$ in n-dimensional Hilbert space H, the Hermitian unitary operator U can be constructed so that $U^+ = U$, $U^+U = UU^+ = U^2 = I$, $|\psi_2\rangle = U|\psi_1\rangle$ and $|\psi_1\rangle = U|\psi_2\rangle$.

Proof Denote

$$U = I + |\psi_2\rangle\langle\psi_1| + |\psi_1\rangle\langle\psi_2| - |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2|. \qquad (3)$$

It is obvious that $U^+ = U$. Now since $\langle\psi_1|\psi_2\rangle = \langle\psi_2|\psi_1\rangle = 0$, we have $U|\psi_1\rangle = |\psi_1\rangle + |\psi_2\rangle\langle\psi_1|\psi_1\rangle + |\psi_1\rangle\langle\psi_2|\psi_1\rangle - |\psi_1\rangle\langle\psi_1|\psi_1\rangle - |\psi_2\rangle\langle\psi_2|\psi_1\rangle = |\psi_2\rangle$ and $U|\psi_2\rangle = |\psi_2\rangle + |\psi_2\rangle\langle\psi_1|\psi_2\rangle + |\psi_1\rangle\langle\psi_2|\psi_2\rangle - |\psi_1\rangle\langle\psi_1|\psi_2\rangle - |\psi_2\rangle\langle\psi_2|\psi_2\rangle = |\psi_1\rangle$. To complete the proof, we just need to demonstrate $U^+U = UU^+ = U^2 = I$. Since $(I - |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2|)^2 = I - |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2|$, $(I - |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2|)(|\psi_2\rangle\langle\psi_1| + |\psi_1\rangle\langle\psi_2|) = 0$ and $(|\psi_2\rangle\langle\psi_1| + |\psi_1\rangle\langle\psi_2|)^2 = |\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|$, we have $U^2 = I - |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| = I$. This completes the proof.

3. Enhance the controllability of quantum systems by introducing ancillary quantum systems

Suppose a finite-dimensional quantum system S^{AB} is composed of two subsystems S^A and S^B. The finite-dimensional Hilbert space H_{AB} of S^{AB} is given by the tensor product of the individual Hilbert spaces $H_A \bigotimes H_B$. Let $n := \dim H_A$ and $m := \dim H_B$. We have the following result.

For a pair of pure vectors $|\psi_{A1}\rangle$ and $|\psi_{A2}\rangle$ in an n-dimensional Hilbert space H_A and any pair of orthonormal pure vectors $|\psi_{B1}\rangle$ and $|\psi_{B2}\rangle$ in an m-dimensional Hilbert space H_B, the pure states $|\psi_{A1}\rangle|\psi_{B1}\rangle$ and $|\psi_{A2}\rangle|\psi_{B2}\rangle$ are a pair of orthonormal pure vectors in Hilbert space H_{AB}.

Moreover, we can easily derive the following results from the above lemma.

Result 1 Suppose that $|\psi_{A1}\rangle$ and $|\psi_{A2}\rangle$ are a pair of pure vectors in n-dimensional Hilbert

space H_A. $|\psi_{A2}\rangle$ is generalized reachable from $|\psi_{A1}\rangle$ if there exists a pair of orthonormal pure vectors $|\psi_{B1}\rangle$ and $|\psi_{B2}\rangle$ so that the unitary operation $U = I + |\psi_{A2}\rangle\langle\psi_{A1}| \bigotimes |\psi_{B2}\rangle\langle\psi_{B1}| + |\psi_{A1}\rangle\langle\psi_{A2}| \bigotimes |\psi_{B1}\rangle\langle\psi_{B2}| - |\psi_{A1}\rangle\langle\psi_{A1}| \bigotimes |\psi_{B1}\rangle\langle\psi_{B1}| - |\psi_{A2}\rangle\langle\psi_{A2}| \bigotimes |\psi_{B2}\rangle\langle\psi_{B2}|$ is implementable.

Furthermore, we can also establish the link between the operator controllability[5,16] of the composite system and the generalized pure state controllability of its subsystem.

Result 2 The system S^A is generalized pure state controllable if the composite quantum system S^{AB} is operator controllable.

Proof To prove that the system S^A is generalized pure state controllable, we need to demonstrate that for any pair of pure states $|\psi_{A1}\rangle$ and $|\psi_{A2}\rangle$ in the Hilbert space H_A, the system S^A can be transferred from $|\psi_{A1}\rangle$ to $|\psi_{A2}\rangle$.

Let $|\psi_{B1}\rangle$ and $|\psi_{B2}\rangle$ be a pair of orthonormal pure vectors in m-dimensional Hilbert space H_B. We can conclude that the pure states $|\phi_1\rangle = |\psi_{A1}\rangle|\psi_{B1}\rangle$ and $|\phi_2\rangle = |\psi_{A2}\rangle|\psi_{B2}\rangle$ are a pair of orthonormal vectors in Hilbert space H_{AB}.

Therefore, from the aforementioned lemma, we can perform a unitary operator

$$U = I + |\phi_2\rangle\langle\phi_1| + |\phi_1\rangle\langle\phi_2| - |\phi_1\rangle\langle\phi_1| - |\phi_2\rangle\langle\phi_2| \quad (4)$$

on the composite quantum system S^{AB} to transfer the system from $|\phi_1\rangle = |\psi_{A1}\rangle|\psi_{B1}\rangle$ to $|\phi_2\rangle = |\psi_{A2}\rangle|\psi_{B2}\rangle$. Since S^{AB} is operator controllable, there exists open-loop coherent control so that unitary operator is implementable, that is, for any pair of pure states $|\psi_{A1}\rangle$ and $|\psi_{A2}\rangle$ in the Hilbert space H_A, the system S^A can be transferred from $|\psi_{A1}\rangle$ to $|\psi_{A2}\rangle$. This completes the proof.

Remark Result 2 not only establishes the link between the operator controllability of the composite system and the generalized pure state controllability of its subsystem, but also gives the constructive methods of designing the unitary operator. Furthermore, the corresponding coherent control can be given by means of unitary operator.

In fact, the sufficient conditions for generalized pure state controllability of subsystems can be relaxed.

Theorem If for any pair of orthonormal pure states of $|\phi_1\rangle$ and $|\phi_2\rangle$ in Hilbert space H_{AB}, the composite quantum system S^{AB} can be transferred from $|\phi_1\rangle$ to $|\phi_2\rangle$, then the system S^A is generalized pure state controllable.

Proof To prove this theorem, we need to show that for any pair of pure states $|\psi_{A1}\rangle$ and $|\psi_{A2}\rangle$ in the Hilbert space H_A, the system S^A can be transferred from $|\psi_{A1}\rangle$ to $|\psi_{A2}\rangle$.

Let $|\psi_{B1}\rangle$ and $|\psi_{B2}\rangle$ be a pair of orthonormal pure vectors in the Hilbert space H_B. It is obvious that the pure states $|\phi_1\rangle = |\psi_{A1}\rangle|\psi_{B1}\rangle$ and $|\phi_2\rangle = |\psi_{A2}\rangle|\psi_{B2}\rangle$ are a pair of orthonormal vectors in Hilbert space H_{AB}.

Therefore, from the condition of this theorem, the composite quantum system S^{AB} can be transferred from $|\phi_1\rangle = |\psi_{A1}\rangle|\psi_{B1}\rangle$ to $|\phi_2\rangle = |\psi_{A2}\rangle|\psi_{B2}\rangle$. That is, for any pair of pure states $|\psi_{A1}\rangle$ and $|\psi_{A2}\rangle$ in the Hilbert space H_A, the system S^A can be transferred from $|\psi_{A1}\rangle$ to $|\psi_{A2}\rangle$. This completes the proof.

So far, it has been demonstrated that introducing ancillary systems will enhance the ability of controlling the quantum systems.

4. Implication of introducing ancillary systems in quantum computation

In this section, we will discuss the implication of introducing ancillary systems for manipulating multiple qubits.

Let S^{AB} be $N+1$ quantum bits, S^A is N quantum bits and S^B is a quantum bit. We can get the following results easily.

Corollary 1 If $(N+1)$ quantum bits can be transferred between any pair orthonormal pure states by physically implementable methods, then the N quantum bits can be transferred from any initial pure state to any target pure state.

Remark This corollary emphasizes that if the orthonormal pure states of $N+1$ quantum bits can be manipulated by physical methods, then any pure state of N quantum bits can be manipulated by introducing an ancillary quantum bit.

It is also easy to gain the following corollary from Result 2.

Corollary 2 If $N+1$ quantum bits are operator controllable, then N quantum bits can be transferred from any initial state $|\psi_0\rangle$ to any target state $|\psi_s\rangle$ just by introducing an ancillary quantum bit and then performing a single unitary operation on the $N+1$ quantum bits.

Proof Suppose $|0\rangle$ and $|1\rangle$ are orthonormal pure

states of an ancillary quantum bit. Denote $|\phi_0\rangle = |\psi_0\rangle|0\rangle$ and $|\phi_1\rangle = |\psi_1\rangle|1\rangle$, we can construct the unitary operator

$$U = I + |\phi_0\rangle\langle\psi_1| + |\phi_1\rangle\langle\phi_0| - |\phi_0\rangle\langle\phi_0| - |\phi_1\rangle\langle\phi_1| \quad (5)$$

so that $N+1$ quantum bits can be transferred from $|\phi_0\rangle = |\psi_0\rangle|0\rangle$ to $|\phi_1\rangle = |\psi_1\rangle|1\rangle$. Therefore, N quantum bits can be transferred from $|\psi_0\rangle$ to $|\psi_1\rangle|$. This completes the proof.

Because any unitary operation on $N+1$ qubits may be implemented exactly by composing single qubit and controlled-NOT gates,[21] the N quantum bits may be transferred from any initial pure state to any target pure state by introducing an ancillary qubit. Therefore, even if the target pure state is not unitarily equivalent to the initial pure state, we can still transfer the initial pure state to the target pure state by physical means in principle. In this sense, we can enhance the ability of manipulating multiple qubits by introducing an ancillary qubit.

5. Generation of a maximally entangled pure state by introducing an ancillary quantum system

In this section, the generation of a maximally entangled pure state by introducing an ancillary quantum system will be discussed.

From Result 2, we can easily have the following corollary.

Corollary 3 If the composite quantum system S^{AB} of two subsystems S^A and S^B is operator controllable, any (maximally) entangled pure state of quantum system S^A can be created from any initial pure state by introducing ancillary system S^B and then just performing a single unitary operation on the composite quantum system.

Suppose that the initial state of S^A is $|\psi_{A0}\rangle$ and the initial state of ancillary system S^B is $|\psi_{B0}\rangle$. Let $|\psi_{AE}\rangle$ be the (maximally) entangled pure state to be generated. Select the target pure state of ancillary system $|\psi_{BS}\rangle$ so that $\langle\psi_{B0}|\psi_{BS}\rangle = 0$, and denote $|\phi_0\rangle = |\psi_{A0}\rangle|\psi_{B0}\rangle$ and $|\phi_S\rangle = |\psi_{AE}\rangle|\psi_{BS}\rangle$, it is easy to conclude that $\langle\phi_0|\phi_S\rangle = 0$. Therefore we can construct the following unitary operator:

$$U = I + |\phi_0\rangle\langle\phi_S| + |\phi_S\rangle\langle\phi_0| - |\phi_0\rangle\langle\phi_0| - |\phi_S\rangle\langle\phi_S| \quad (6)$$

so that the composite quantum system S^{AB} can be transferred from $|\phi_0\rangle = |\psi_{A0}\rangle|\psi_{B0}\rangle$ to $|\phi_S\rangle = |\psi_{AE}\rangle|\psi_{BS}\rangle$.

Even if the composite quantum system S^{AB} is not operator controllable but the unitary operator Eq.(6) can be implemented, the (maximally) entangled pure state $|\psi_{AE}\rangle$ of quantum system S^A can still be generated from the initial pure state $|\psi_{A0}\rangle$ by introducing ancillary system S^B and then just performing a single unitary operation Eq.(6) on the composite quantum system.

Subsequently, a concrete example will be given to illustrate that a maximally entangled pure state of a quantum system can be generated by introducing an ancillary system and then performing open-loop coherent control on the composite system. Suppose that the dynamical equation of three 2−level particles is

$$|\dot{\psi}(t)\rangle = \frac{\mathrm{i}}{\hbar}(H_0 + H_c)|\psi(t)\rangle, \quad (7)$$

where H_0 is the internal Hamiltonian and H_c is the external control Hamiltonian. Let S^{AB} be three 2-level particles, S^A be two 2-level particles and S^B be one 2-level particle. Suppose that the initial states of S^A and S^B are $|\psi_{A0}\rangle = |0\rangle|0\rangle$ and $|\psi_{B0}\rangle = |0\rangle$ respectively. Let the (maximally) entangled pure state of quantum system S^A be

$$|\psi_{AE}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \quad (8)$$

and the target pure state $|\psi_{BS}\rangle = |1\rangle$. Denote $|\phi_0\rangle = |0\rangle|0\rangle|0\rangle$ and $|\phi_S\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle|1\rangle + |1\rangle|1\rangle|1\rangle)$. Then the unitary operator Eq.(6) can be constructed and performed on the three 2-level particles so that the (maximally) entangled pure state Eq.(8) of two 2-level particles is created by introducing an ancillary 2-level particle. Denote $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, the unitary operator Eq.(6) can be expressed in matrix form:

$$U = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}. \quad (9)$$

Furthermore the open-loop coherent control H_c can be constructively designed by means of U. Choose H_c so that $\frac{T_0}{\hbar}(H_0+H_c)=\frac{\ln U}{\mathrm{i}}$ equals the following matrix

$$\begin{pmatrix} 1.5708 & 0\,0\,0 & -1.1107 & 0\,0 & -1.1107 \\ 0 & 0\,0\,0 & 0 & 0\,0 & 0 \\ 0 & 0\,0\,0 & 0 & 0\,0 & 0 \\ 0 & 0\,0\,0 & 0 & 0\,0 & 0 \\ -1.1107 & 0\,0\,0 & 0.7854 & 0\,0 & 0.7854 \\ 0 & 0\,0\,0 & 0 & 0\,0 & 0 \\ 0 & 0\,0\,0 & 0 & 0\,1 & 0 \\ -1.1107 & 0\,0\,0 & 0.7854 & 0\,0 & 0.7854 \end{pmatrix}, \quad (10)$$

it is easy to conclude that

$$\begin{aligned} |\psi(T_0)\rangle &= \mathrm{e}^{\frac{\mathrm{i}T_0}{\hbar}(H_0+H_c)}|0\rangle|0\rangle|0\rangle = U|0\rangle|0\rangle|0\rangle \\ &= \frac{1}{\sqrt{2}}(|0\rangle|0\rangle+|1\rangle|1\rangle)|1\rangle \end{aligned}$$

where U is given by Eq.(9), i.e., the maximally entangled pure state of two 2-level particles can be generated by introducing an ancillary 2-level particle and performing open-loop coherent control H_c on the resulting three 2-level particles.

Remark Eq.(10) is gained by running Matlab function $\log m(U)$.

6. Conclusion

In this paper, we have explored the potential of controlling a quantum system by introducing an ancillary system and then performing unitary operation on the resulting composite system. By this way, we can generalize the concept of controllability for quantum systems. It has been demonstrated that the possibility of introducing ancillary systems will enhance the capability of controlling quantum systems. This method is advantageous for the reason that the unitary operator can be concretely obtained. Furthermore, the implication of introducing ancillary systems in quantum computation has been discussed. A concrete example has been given to illustrate that a maximally entangled pure state of a quantum system can be generated by introducing an ancillary system and then performing open-loop coherent control on the resulting composite system.

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