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Topological aspect of Chern–Simons p-branes*

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By generalizing the topological current of Abelian Chern–Simons (CS) vortices, we present a topological tensor current of CS p-branes based on the ϕ-mapping topological current theory. It is revealed that CS p-branes are located at the isolated zeros of the vector field $\boldsymbol{\phi}(x)$, and the topological structure of CS p-branes is characterized by the winding number of the ϕ-mappings. Furthermore, the Nambu–Goto action and the equation of motion for multi CS p-branes are obtained.

Keywords: topological tensor current, winding number, Chern–Simons p-branes, Nambu–Goto action

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1. Introduction

Over the last decade a great deal of work on the (2+1)-dimensional Abelian Chern–Simons (CS) theory has been done.[1−4] In the Abelian CS theory, the CS action is often expressed as

$$S = \frac{\kappa}{2}\int \epsilon^{\mu\nu\lambda} A_\mu F_{\nu\lambda} \mathrm{d}^3x. \tag{1}$$

More recently, there has been a wide interest in the higher dimensional CS theories.[5−7] They are the theories in $(2N+1)$ (odd) spacetime dimensions constructed from characteristic classes in $(2N+2)$ dimensions in exactly the same way as their (2+1)-dimensional analogues. Higher dimensional CS theories have been shown recently to have generically the non-vanishing number of degrees of freedom,[5] the gauge theory of CS p-branes in the phase-space is constructed in a Hamilton gauge similar to the ordinary W-gravity,[6] and the global CS $(2N-1)$-form leads to the conservation law of a modified Noether current.[7] Of particular interest are the higher dimensional CS theories with gauge group $SO(2N+1,1)$ or $SO(2N,2)$ in $(2N+1)$ dimensions because they determine gravitational theories.[8]

As is well known, CS p-branes[6] are the extended objects with p spatial dimensions in the higher dimensional CS theories, which play an essential role in revealing the nonperturbative structure of the superstring theories and M-theories.[9] For the CS p-branes, as an analogue of Eq.(1), the well-known Nambu–Goto action is defined as

$$S = \kappa \int_M \mathrm{d}\tau d^p\sigma \sqrt{-g}, \tag{2}$$

which is the basis of CS p-brane theory and will be obtained directly by the ϕ-mapping topological current theory.[10] The work mentioned above has covered many research fields of higher dimensional CS theories. But for lack of powerful methods, the topological theory describing the topological defects in the CS theory has not been established yet.

The starting point of this paper is to find a topological theory foundation of 'electric' CS p-branes. For this purpose, in a similar way to that in Duan *et al*'s work on 'magnetic' $\tilde{p}$-branes,[11] we construct a topological tensor current of 'electric' CS p-branes in the $SO(N)$ non-Abelian gauge field theory, which is different from the usual generalized Kalb–Ramond gauge field theory.[12] It is found that the CS p-branes may be considered as a kind of topological defect and a straightforward generalization of the (2+1)-dimensional Abelian CS vortices.

2. Topological tensor current of CS p-branes

Let X be a D (odd)-dimensional smooth manifold with a metric tensor $g_{\mu\nu}$ and local coordinates

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x^μ $(\mu, \nu = 0, \ldots, D-1)$ with $x^0 = t$ being time, and R^d be an Euclidean space of dimension $d < D$. Consider a smooth map $\phi : X \to R^d$, which gives a d-dimensional smooth vector field on X

$$\phi^a = \phi^a(x), \quad a = 1, 2, \ldots, d. \tag{3}$$

The direction unit field of $\boldsymbol{\phi}(x)$ can be expressed as

$$n^a = \frac{\phi^a}{||\phi||}, \quad ||\phi|| = \sqrt{\phi^a \phi^a}, \quad n^a(x) n^a(x) = 1. \tag{4}$$

In our previous work,[10,13] the topological current of pointlike defects and stringlike defects has been discussed systematically via the ϕ-mapping topological current theory. In order to study the topological properties of higher dimensional CS p-branes, we extend the concept to present a higher dimensional generalized topological tensor current. In the context of the $SO(N)$ non-Abelian CS theory, one can construct a rank-$(D-d)$ topological tensor current for CS p-branes from the 'electric' source as follows:

$$j^{\mu_1 \cdots \mu_{D-d}} = \frac{(-1)^{d/2}}{2^d A(S^{d-1})(d-1)!} \frac{\epsilon^{\mu_1 \cdots \mu_{D-d} \mu_{D-d+1} \cdots \mu_D}}{\sqrt{-g_x}} \cdot \epsilon_{a_1 a_2 \ldots a_{d-1} a_d} F^{a_1 a_2}_{\mu_{D-d+1} \mu_{D-d+2}} \cdots F^{a_{d-1} a_d}_{\mu_{D-1} \mu_D}, \tag{5}$$

where $A(S^{d-1}) = 2\pi^{N/2}/(\Gamma/2)$ is the surface area of the unit sphere S^{d-1} and $g_x = \det(g_{\mu\nu})$ is the determinant of the metric tensor $g_{\mu\nu}$. From Eq.(5), one can see that the rank-$(D-d)$ topological tensor current is just the generalization of the topological current for CS vortices in (2+1)-dimensional spacetime[13]

$$j^\lambda = \frac{1}{8\pi} \frac{\epsilon^{\lambda\mu\nu}}{\sqrt{-g_x}} \epsilon_{ab} F^{ab}_{\mu\nu}. \tag{6}$$

Now we will prove that the topological tensor current $j^{\mu_1 \cdots \mu_{D-d}}$ is identically conserved. Since the Levi-Civita symbol $\epsilon^{\mu_1 \cdots \mu_{D-d} \mu_{D-d+1} \cdots \mu_D}$ is totally anti-symmetric, the Bianchi identity

$$\begin{aligned} D_\mu F^{a_1 a_2}_{\nu\lambda} \quad &+ D_\nu F^{a_1 a_2}_{\lambda\mu} + D_\lambda F^{a_1 a_2}_{\mu\nu} = 0, \\ &\mu, \nu, \lambda = \mu_1, \ldots, \mu_D \end{aligned} \tag{7}$$

can be written as

$$\epsilon^{\mu_1 \ldots \mu_{i-1} \mu_i \mu_{i+1} \cdots \mu_D} D_{\mu_{i-1}} F^{a_1 a_2}_{\mu_i \mu_{i+1}} = 0. \tag{8}$$

Noting that $j^{\mu_1 \cdots \mu_{D-d}}$ does not have the indices $a_1, \ldots, a_d$, one deduces that

$$D_{\mu_i}(\sqrt{-g_x} j^{\mu_1 \cdots \mu_{D-d}}) = \partial_{\mu_i}(\sqrt{-g_x} j^{\mu_1 \cdots \mu_{D-d}}) = 0. \tag{9}$$

Therefore, this tensor current behaves as a conserved quantity

$$\nabla_{\mu_i} j^{\mu_1 \cdots \mu_{D-d}} = \frac{1}{\sqrt{-g_x}} \partial_{\mu_i}(\sqrt{-g_x} j^{\mu_1 \cdots \mu_{D-d}}) = 0. \tag{10}$$

Furthermore, corresponding to the rank-$(D-d)$ topological tensor current $j^{\mu_1 \cdots \mu_{D-d}}$, the $(D-d+1)$-form strength F satisfies the field equation

$$\nabla_\mu F^{\mu\mu_1\mu_2 \cdots \mu_{D-d}} = j^{\mu_1 \cdots \mu_{D-d}}. \tag{11}$$

And the dual field strength ${}^*F$ satisfies

$$\nabla^*_\mu F^{\mu\mu_1\mu_2 \cdots \mu_{D-d}} = \tilde{j}^{\mu_1 \cdots \mu_{D-d}}, \tag{12}$$

in which $\tilde{j}^{\mu_1 \cdots \mu_{D-d}}$ is a rank-$(D-d)$ topological tensor current corresponding to a 'magnetic' source. It is known that the $(D-d+1)$-form field strength F can be expressed as

$$F = dA, \tag{13}$$

where A is a $(D-d)$-form as the tensor gauge potential. Just as the Maxwell's equations, Eqs.(11), (12) and (13) imply that the presence of topological tensor current in Eq.(5) results from an 'electric' source, but not a 'magnetic' source. On the other hand, the counterparts of the 'electric' p-branes, namely, the 'magnetic' $\tilde{p}$-branes have been formally discussed elsewhere.[11]

In Ref.[14], it has been proved that the topological current (5) can be formulated in terms of the unit vector field n^a clearly as

$$j^{\mu_1 \cdots \mu_{D-d}} = \frac{1}{A(S^{d-1})(d-1)!(d-2)} \frac{\epsilon_{\mu_1 \ldots \mu_{D-d} \mu_{D-d+1} \cdots \mu_D}}{\sqrt{-g_x}} \cdot \epsilon_{a_1 a_2 \ldots a_{d-1} a_d} \partial_{\mu_{D-d+1}} n^{a_1} \ldots \partial_{\mu_D} n^{a_d}. \tag{14}$$

From Eq.(4), we have

$$\partial_\mu n^a = \frac{1}{||\phi||}\partial_\mu \phi^a + \phi^a \partial_\mu (\frac{1}{||\phi||}), \quad \frac{\partial}{\partial \phi^a}(\frac{1}{||\phi||}) = -\frac{\phi^a}{||\phi||^3}. \tag{15}$$

Using the above expressions, the general tensor current can be rewritten as

$$\begin{aligned} j^{\mu_1 \cdots \mu_{D-d}} = & \frac{1}{A(S^{d-1})(d-1)!(d-2)}(\frac{1}{\sqrt{-g_x}})\epsilon_{\ a_1 \ldots a_d} \cdot \epsilon^{\mu_1 \cdots \mu_{D-d}\mu_{D-d+1}\cdots\mu_D} \cdot \partial_{\mu_{(D-d+1)}}\phi^a \\ & \cdot \partial_{\mu_{(D-d+2)}}\phi^{a_2} \ldots \partial_{\mu_D}\phi^{a_d} \cdot \frac{\partial}{\partial \phi^a}\frac{\partial}{\partial \phi^{a_1}}(\frac{1}{||\phi||^{d-2}}). \end{aligned} \tag{16}$$

By defining a generalized Jacobian tensor

$$\begin{aligned} \epsilon^{a_1 \cdots a_d} J^{\mu_1 \cdots \mu_{D-d}}\left(\frac{\phi}{x}\right) = & \ \epsilon^{\mu_1 \cdots \mu_{D-d}\mu_{D-d+1}\mu_{D-d+2}\cdots\mu_D} \\ & \cdot \partial_{\mu_{(D-d+1)}}\phi^{a_1}\partial_{\mu_{(D-d-2)}}\phi^{a_2} \ldots \partial_{\mu_D}\phi^{a_d}, \end{aligned} \tag{17}$$

and making use of the generalized Laplacian Green function relation in ϕ-space

$$\frac{\partial}{\partial \phi^a}\frac{\partial}{\partial \phi^a}\left(\frac{1}{||\phi||^{d-2}}\right) = \frac{4\pi^{d/2}}{\Gamma(d/2-1)}\delta(\boldsymbol{\phi}), \tag{18}$$

we obtain a δ-function-like tensor current

$$j^{\mu_1 \cdots \mu_{D-d}} = \left(\frac{1}{\sqrt{-g_x}}\right)\delta(\boldsymbol{\phi}) J^{\mu_1 \cdots \mu_{D-d}}\left(\frac{\phi}{x}\right). \tag{19}$$

From Eq.(19), one can see that $j^{\mu_1 \cdots \mu_{D-d}} \neq 0$ only when $\boldsymbol{\phi} = 0$, which is just the singularity of $j^{\mu_1 \cdots \mu_{D-d}}$. In detail, the kernel of the ϕ-mapping is the singularity of the topological tensor current $j^{\mu_1 \cdots \mu_{D-d}}$ in the odd-dimensional smooth manifold X, i.e. the inner structure of the topological tensor current is labelled by the zeros of ϕ-mapping. We think that this is the essence of the topological tensor current theory and ϕ-mapping method is the key to the study of this theory. From the above formula we see that the topological current does not vanish only at the zero points of the vector field $\boldsymbol{\phi}(x)$. Therefore, it is essential to discuss the solution of the equations $\phi^a(x) = 0$ $(a = 1, \ldots, d)$.

Suppose that the vector field $\boldsymbol{\phi}(x)$ possesses ℓ isolated zeros, according to the implicit function theorem,[15] when the zeros are regular points of ϕ-mapping, i.e. the rank of the Jacobian matrix $[\partial_\mu \phi^a]$ is d, the solutions of $\boldsymbol{\phi}(x) = 0$ can be parameterized by

$$x^\mu = z_i^\mu\left(u^1, u^2, \ldots, u^{D-d}\right), \quad i = 1, \ldots, \ell. \tag{20}$$

Here the subscript i represents the ith solution and the parameters u^I $(I = 1, \ldots, D-d)$ span a $(D-d)$-dimensional submanifold, with the metric tensor $g_{IJ} = g_{\mu\nu}(\partial x^\mu/\partial u^I)(\partial x^\nu/\partial u^J)$ denoted by N_i, which corresponds to a CS p-brane $(p = D-d-1)$ with spatial p-dimension and N_i is its world volume.

It is easy to see that the tensor current $j^{\mu_1 \cdots \mu_{D-d}}$ does not vanish only on the world volume manifolds N_i $(i = 1, \ldots, l)$, each of which corresponds to an 'electric' CS p-brane, that is, every isolated zero of $\boldsymbol{\phi}(x)$ on X corresponds to an 'electric' CS p-brane. Thus, we point out that the CS p-branes, sometimes, may be considered as topological defects.[16] In this case for our theory the vector field $\phi^a(x)$ $(a = 1, \ldots, d)$ may be looked upon as the generalized order parameter for CS p-branes.

For each singular manifold N_i, one can define a normal submanifold M_i in X which is spanned by the parameters v^A with the parameter equation

$$x^\mu = x^\mu(v^1, \ldots, v^d), \quad \mu = 1, \ldots, D, \tag{21}$$

and the metric tensor $g_{AB} = g_{\mu\nu}(\partial x^\mu/\partial v^A)(\partial x^\nu/\partial v^B)$ $(A, B = 1, \ldots, d)$. The intersection point of M_i and N_i is denoted by p_i which can be expressed parametrically by $v^A = p_i^A$. In the words of differential topology, M_i is transversal to N_i at the point p_i with the relation

$$g_{\mu\nu}\left.\frac{\partial x^\mu}{\partial u^I}\frac{\partial x^v}{\partial v^A}\right|_{p_i} = 0, \quad I = 1, \ldots, D-d, \quad A = 1, \ldots, d. \tag{22}$$

3. Inner topological structure of CS p-branes

In the following, the inner topological structure of CS p-branes will be investigated in detail. As pointed out in Ref.[17], the unit vector field defined in Eq.(4) gives a Gauss map $n : \partial M_i \to S^{d-1}$, and the generalized winding number can be given by this Gauss map[18]

$$W_i = \frac{1}{A(S^{d-1})(d-1)!}\int_{\partial M_i} n^*(\epsilon_{a_1\cdots a_d} n^{a_1}\mathrm{d}n^{a_2} \wedge\cdots\wedge \mathrm{d}n^{a_d}), \tag{23}$$

where ∂M_i is the boundary of the neighbourhood M_i of p_i on M with $p_i \notin \partial M_i$, $M_i \cap M_j = \emptyset$ and n^* is the pull-back of n. The generalized winding number is a topological invariant and also called the degree of Gauss map.[18] It means that, when the point v^A covers ∂M_i once, the unit vector n^a will cover a region $n[\partial M_i]$, whose area is $A(S^{d-1})$, W_i times: that is to say, the unit vector n^a will cover the unit sphere S^{d-1} W_i times. Using the Stokes theorem, the generalized winding number can also be written in terms of the parameters v^A of M_i

$$W_i = \frac{1}{A(S^{d-1})(d-1)!}\int_{n[\partial M_i]} \epsilon_{a_1\cdots a_d} n^{a_1}\mathrm{d}n^{a_2}\wedge\cdots\wedge \mathrm{d}n^{a_d} = \frac{1}{A(S^{d-1})(d-1)!}\int_{M_i} \epsilon^{A_1\cdots A_d} \cdot\epsilon_{a_1\cdots a_d}\partial_{A_1} n^{a_1}\ldots\partial_{A_d} n^{a_d} d^d v. \tag{24}$$

Then, by repeating the process of the topological current, one obtains the compact form of W_i

$$W_i = \int_{M_i} \delta(\boldsymbol{\phi}(v)) J\left(\frac{\phi}{v}\right)\mathrm{d}^d v, \tag{25}$$

where $J\left(\dfrac{\phi}{v}\right)$ is the usual Jacobian determinant of $\boldsymbol{\phi}$ with respect to v

$$\epsilon^{a_1\cdots a_d} J\left(\frac{\phi}{v}\right) = \epsilon^{A_1\cdots A_d}\partial_{A_1} n^{a_1}\partial_{A_2} n^{a_2}\cdots \cdot\partial_{A_d} n^{a_d}. \tag{26}$$

According to the δ-function theory,[19] the δ-function $\delta(\boldsymbol{\phi})$ can be expanded by

$$\delta(\boldsymbol{\phi}) = \sum_{i=1}^{l} c_i \delta(N_i), \tag{27}$$

where the coefficient c_i must be positive, i.e. $c_i = |c_i|$, and $\delta(N_i)$ is the δ-function in spacetime X on a submanifold N_i[20]

$$\delta(N_i) = \int_{N_i} \frac{1}{\sqrt{-g_x}}\delta^n(\boldsymbol{x} - \boldsymbol{z}_i(u^1,\ldots,u^{D-d})) \cdot\sqrt{-g_u}\mathrm{d}^{D-d}u, \tag{28}$$

where $g_u = \det(g_{IJ})$. Substituting Eq.(28) into Eq.(25) gives

$$W_i = \int_{M_i} c_i\delta(N_i) J\left(\frac{\phi}{v}\right)\mathrm{d}^d v = \int_{M_i}\int_{N_i} c_i \frac{1}{\sqrt{-g_x}\sqrt{-g_v}}\delta^n(\boldsymbol{x} - \boldsymbol{z}_i(u^1,\ldots,u^{D-d})) \cdot J\left(\frac{\phi}{v}\right)\sqrt{-g_u}\mathrm{d}^{D-d}u\sqrt{-g_v}\mathrm{d}^d v \tag{29}$$

with $g_v = \det(g_{AB})$. Because $\sqrt{-g_u}\sqrt{-g_v}\mathrm{d}^{D-d}u\mathrm{d}^d v$ is the invariant volume element of the product manifold $M_i \times N_i$, it can be rewritten as $\sqrt{-g_x}\mathrm{d}^D x$. Thus, by calculating the integral, with positivity of c_i adopted, the coefficient c_i becomes

$$c_i = \frac{\beta_i\sqrt{-g_v}}{\left|J\left(\dfrac{\phi}{v}\right)_{p_i}\right|} = \frac{\beta_i\eta_i\sqrt{-g_v}}{J\left(\dfrac{\phi}{v}\right)_{p_i}}, \tag{30}$$

where $\beta_i = |W_i|$ is a positive integer called the Hopf index[18] of ϕ-mapping on M_i, and η_i=sgn $J\left(\dfrac{\phi}{v}\right)_{p_i}$ = ± 1 is the Brouwer degree of ϕ-mapping.[18] Moreover, with this expression of c_i, one can find the relation among the winding number W_i, the Hopf index β_i and the Brouwer degree η_i

$$W_i = \beta_i\eta_i. \tag{31}$$

And the expansion of the topological tensor current (19) of the CS p-branes is

$$j^{\mu_1\cdots\mu_D-d} = \frac{1}{\sqrt{-g_x}}\sum_{i=1}^{l}\frac{\beta_i\eta_i\sqrt{-g_v}}{J\left(\frac{\phi}{v}\right)\Big|_{p_i}}\delta(N_i) \cdot J^{\mu_1\cdots\mu_D-d}\left(\frac{\phi}{x}\right). \quad (32)$$

From the above equation, we arrive at the conclusion that the inner structure of $j^{\mu_1\cdots\mu_D-d}$ is labelled by the total expansion of $\delta(\boldsymbol{\phi})$, which includes the topological information about β_i and η_i. The inner structure of the generalized topological tensor current in Eq.(32) just represents ℓ CS p-branes with topological charges $g_i = W_i = \beta_i\eta_i$ moving in the D-dimensional spacetime manifold X. The $(D-d)$-dimensional singular submanifolds N_i $(i = 1, \ldots, \ell)$ are their world sheets in the spacetime.

Now define a Lagrangian for multi CS p-branes in terms of the topological tensor current

$$L = \kappa\sqrt{\frac{1}{(D-d)!}g_{\mu_1\nu_1}\cdots g_{\mu_{(D-d)}\nu_{(D-d)}}j^{\mu_1\ldots\mu_{D-d}}j^{\nu_1\ldots\nu_{D-d}}}, \quad (33)$$

which is the generalization of Nielsen's Lagrangian.[21] From the above deductions, one can prove that

$$L = \kappa\left(\frac{1}{\sqrt{-g_x}}\right)\delta(\boldsymbol{\phi}(x)). \quad (34)$$

Then, the action takes the form

$$S = \int_X L\sqrt{-g_x}\mathrm{d}^D x = \kappa\int_X \delta(\boldsymbol{\phi}(x))\mathrm{d}^D x, \quad (35)$$

which can be further expressed as

$$S = \kappa\int_X\sum_{i=1}^{l}\beta_i\eta_i\int_{N_i}\delta^D(x - z_i(u))\sqrt{-g_u}\mathrm{d}^{(D-d)}u\mathrm{d}^D x = \kappa\sum_{i=1}^{l}\beta_i\eta_i\int_{N_i}\sqrt{-g_u}\mathrm{d}^{(D-d)}u, \quad (36)$$

i.e.

$$S = \sum_{i=1}^{l}\beta_i\eta_i S_i. \quad (37)$$

Here $S_i = \kappa\int_{N_i}\sqrt{-g_u}\mathrm{d}^{(D-d)}u$ is just the Nambu–Goto action in Eq.(2) defined for CS p-branes ($p = D-d-1$). It must be pointed out here that the Nambu–Goto action, which is the basis of much work on defect theory, is obtained naturally by making use of the ϕ-mapping topological current theory. This action in Eq.(35) directly generalizes the Nambu action with $D-d=2$ for the string world-sheet action,[22] and is topologically quantized by the topological numbers: i.e. the Hopf indices and Brouwer degrees of ϕ-mapping.

From the principle of least action, one can obtain the equation of motion for CS p-branes

$$\frac{1}{\sqrt{g_u}}\frac{\partial}{\partial u^I}\left(\sqrt{g_u}g^{IJ}\frac{\partial x^\nu}{\partial u^J}\right) + g^{IJ}\Gamma^\nu_{\mu\lambda}\frac{\partial x^\mu}{\partial u^I}\frac{\partial x^\lambda}{\partial u^J} = 0, \qquad (I, J = 1, \ldots, D-d). \quad (38)$$

As a matter of fact, this is just the equation of harmonic map and has much to do with the Fock's coordinate in general relativity.

4. Conclusion

In this paper, the above formulation and rigorous proof indicate that the $SO(N)$ topological tensor current of 'electric' CS p-branes is an important development of the former $SO(2)$ topological current of CS vortices. Based on the ϕ-mapping topological current theory, it is shown that every isolated zero of the vector field $\boldsymbol{\phi}$ (i.e. the order parameters) is just corresponding to a CS p-brane ($p = D-d-1$). The inner topological structure of CS p-branes is determined by the winding numbers which can be represented by the Brouwer degrees and Hopf indices of ϕ-mapping. The Nambu–Goto action, which is defined for multi CS p-

branes in terms of the topological tensor current, can also be characterized by the Hopf indices and Brouwer degrees. Moreover, with the principle of least action, we obtain the equation of motion for CS p-branes.

Finally, we would like to point out that the results in this paper obtained without any particular hypothesis may be used in future work on the interaction between the extra-dimensional manifold and the $(3+1)$-dimensional spacetime, especially the new physics effect caused by extra-dimensional manifold.[23]

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