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Phase control in an open Λ-type system with spontaneously generated coherence*

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This paper investigates the control role of the relative phase between the probe and driving fields on the gain, dispersion and populations in an open Λ system with spontaneously generated coherence (SGC). It shows that by adjusting the value of the relative phase, a change from lasing with inversion to lasing without inversion can be realized; the values and frequency spectrum regions of the inversionless gain and dispersion can be obviously varied; high refractive index with zero absorption and electromagnetically induced transparency can be achieved. It is also found that when the driving field is resonant, the shapes of the dispersion and the gain curves versus the probe detuning are very similar if the relative phase of the dispersion lags $\pi/2$ than that of the gain, however for the off-resonant driving field the similarity will disappear; the gain, dispersion and populations are periodical functions of the relative phase, the modulation period is always 2π; the contribution of SGC to the inversionless gain and dispersion is much larger than that of the dynamically induced coherence.

Keywords: spontaneously generated coherence, phase control, electromagnetically induced transparency, lasing without inversion
PACC: 4250

1. Introduction

Recently, the study of spontaneously generated coherence (SGC) created in the systems with near-degenerate levels has been an interesting subject.[1−10] The existence of the SGC effects depends on the nonorthogonality of dipole matrix elements. It has been shown that atomic systems with SGC are sensitive to the relative phase of the applied fields.[11−18] The quantum coherence and atomic interference have led to a lot of interesting optical phenomena, such as lasing without inversion (LWI),[19,20] enhancement of the refractive index[21,22] and electromagnetically induced transparency (EIT).[23] In the studies relative to SGC, the closed atomic systems are usually used. In this paper, we will explore the effect of the relative phase between the probe and driving fields on the gain, dispersion and populations in an open Λ type atomic system with SGC. We find that LWI, EIT and high refractive index with zero absorption can be realized just by adjusting this relative phase. This paper is organized as follows: in Section 2, we present the density matrix equations of motion of an open Λ system with SGC, and give the analytical solution of the equations under the condition of the steady state; in Section 3, we discuss the control role of the relative phase on the gain and dispersion by using the numerical results from the analytical solution obtained in Section 2; finally in Section 4, we give a brief summary.

2. Model and analytical solution

Consider an open Λ three-level atomic system with an upper state $|1\rangle$ and two lower closely states $|2\rangle$ and $|3\rangle$, as illustrated in Fig.1. Transition between $|1\rangle$ and $|2\rangle$ is driven by a strong coherent driving field with the amplitude $\boldsymbol{E}_{\rm c}$, frequency $\omega_{\rm c}$ and Rabi frequency $G_{\rm c}$. An incoherent pump field with a pumping rate 2Λ and a weak coherent probe field with the amplitude $\boldsymbol{E}_{\rm p}$, frequency $\omega_{\rm p}$ and Rabi frequency $G_{\rm p}$ are applied between levels $|1\rangle$ and $|3\rangle$. $2\gamma_1$ and $2\gamma_2$ denote

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the spontaneous emission rates from level $|1\rangle$ to levels $|3\rangle$ and $|2\rangle$ respectively. J_2 and J_3 are the atomic injection rates for level $|2\rangle$ and $|3\rangle$ respectively, γ_0 denotes the atomic exit rate from the cavity.

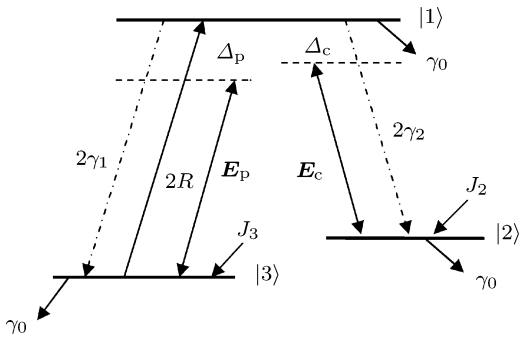


Fig.1. An open Λ-type three-level atomic system.

In the interaction representation, under the rotating wave and the electric dipole approximations, the semiclassical density matrix equations of motion of this system can be written as

$$\dot{\rho}_{11} = -2(\gamma_1+\gamma_2)\rho_{11} + \mathrm{i}G_\mathrm{p}\rho_{31} - \mathrm{i}G_\mathrm{p}^*\rho_{13} + 2\Lambda\rho_{33} + \mathrm{i}G_\mathrm{c}\rho_{21} - \mathrm{i}G_\mathrm{c}^*\rho_{12} - \gamma_0\rho_{11}\,, \tag{1a}$$

$$\dot{\rho}_{22} = 2\gamma_2\rho_{11} + \mathrm{i}G_\mathrm{c}^*\rho_{12} - \mathrm{i}G_\mathrm{c}\rho_{21} + J_2 - \gamma_0\rho_{22}\,, \tag{1b}$$

$$\dot{\rho}_{33} = 2\gamma_1\rho_{11} - 2\Lambda\rho_{33} + \mathrm{i}G_\mathrm{p}^*\rho_{13} - \mathrm{i}G_\mathrm{p}\rho_{31} + J_3 - \gamma_0\rho_{33}, \tag{1c}$$

$$\dot{\rho}_{12} = -(\gamma_1+\gamma_2+\mathrm{i}\Delta_\mathrm{c})\rho_{12} + \mathrm{i}G_\mathrm{p}\rho_{32} - \mathrm{i}G_\mathrm{c}(\rho_{11}-\rho_{22}), \tag{1d}$$

$$\dot{\rho}_{13} = -(\gamma_1+\gamma_2+\Lambda+\mathrm{i}\Delta_\mathrm{p})\rho_{13} + \mathrm{i}G_\mathrm{c}\rho_{23} - \mathrm{i}G_\mathrm{p}(\rho_{11}-\rho_{33}), \tag{1e}$$

$$\dot{\rho}_{23} = -(\Lambda+\mathrm{i}\Delta_\mathrm{p}-\mathrm{i}\Delta_\mathrm{c})\rho_{23} + 2\sqrt{\gamma_1\gamma_2}\,\eta\rho_{11}\cos\theta + \mathrm{i}G_\mathrm{c}^*\rho_{13} - \mathrm{i}G_\mathrm{p}\rho_{21}. \tag{1f}$$

The above equations are constrained by $\rho_{ij} = \rho_{ji}^*$. Where $\rho_{ij}(i \neq j)$ is the atomic polarization between states $|i\rangle$ and $|j\rangle$, $\rho_{ii}(i = 1-3)$ is the population of the state $|i\rangle$, $\Delta_\mathrm{c} = \omega_{12} - \omega_\mathrm{c}$ and $\Delta_\mathrm{p} = \omega_{13} - \omega_\mathrm{p}$ are the detunings of the probe and driving fields from their relevant atomic transitions respectively. $G_\mathrm{c} = \boldsymbol{d}_{12}\cdot\boldsymbol{E}_\mathrm{c}/h = G_\mathrm{c}^0\sin\theta$ and $G_\mathrm{p} = \boldsymbol{d}_{13}\cdot\boldsymbol{E}_\mathrm{c}/h = G_\mathrm{p}^0\sin\theta$ are the Rabi frequencies for the probe and driving fields respectively. $2\sqrt{\gamma_1\gamma_2}\eta\rho_{11}\cos\theta$ represents the quantum interference effect, i.e., the SGC effect, resulting from the cross coupling between the spontaneous emission $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$. The dipole moments $\boldsymbol{d}_{12}$ and $\boldsymbol{d}_{13}$ are not orthogonal, which is necessary for the existence of the SGC effect, θ represents the angle between the two dipole moments. If levels $|2\rangle$ and $|3\rangle$ lie so closely that the SGC effect has to be taken into account, then $\eta = 1$, otherwise $\eta = 0$.

Because of the existence of the SGC effect, i.e., the existence of $2\sqrt{\gamma_1\gamma_2}\eta\rho_{11}\cos\theta$, the properties of the open system do not only depend on the amplitudes and detunings of the applied fields but also on their phases, so Rabi frequencies can not be treated as real parameters. If we define ϕ_p and ϕ_c as phases of the probe and driving fields respectively, then G_p and G_c can be written as $G_\mathrm{p} = g_\mathrm{p}\exp(-\mathrm{i}\phi_\mathrm{p})$ and $G_\mathrm{c} = g_\mathrm{c}\exp(-\mathrm{i}\phi_\mathrm{c})$ respectively, g_p and g_c are real. Redefining system variables in Eqs.(1) as $\tilde{\rho}_{ii} = \rho_{ii}$, $\tilde{\rho}_{12} = \rho_{12}\exp(\mathrm{i}\phi_\mathrm{c})$, $\tilde{\rho}_{13} = \rho_{13}\exp(\mathrm{i}\phi_\mathrm{p})$, and $\tilde{\rho}_{23} = \rho_{23}\mathrm{e}^{\mathrm{i}\,\Phi}$, where $\Phi = \phi_\mathrm{p} - \phi_\mathrm{c}$, we get the equations of motion for the redefined density matrix elements $\tilde{\rho}_{ij}$, which are found to be identical to Eqs.(1) except that η, G_p and G_c are replaced by $\eta_\Phi = \eta\mathrm{e}^{\mathrm{i}\,\Phi}$, g_p and g_c respectively. In the following, we define $p = 2\sqrt{\gamma_1\gamma_2}\eta_\Phi\cos\theta$.

For the steady state and in the limit of a weak probe field, we obtain the analytical representatives of the populations $\tilde{\rho}_{ii}(i = 1-3)$ and polarizations $\tilde{\rho}_{ij}(i \neq j)$ to the zero order of g_p and all orders of g_c as follows:

$$\tilde{\rho}_{11}^{(0)} = \frac{2g_\mathrm{c}^2(\gamma_1+\gamma_2)[(2\Lambda+\gamma_0)J_2 + 2\Lambda J_3] + 2\Lambda J_3\gamma_0[(\gamma_1+\gamma_2)^2+\Delta_\mathrm{c}^2]}{S}\,, \tag{2a}$$

$$\tilde{\rho}_{22}^{(0)} = \frac{2g_\mathrm{c}^2(\gamma_1+\gamma_2)[(2\Lambda+\gamma_0)J_2+2\Lambda J_3] + [4\Lambda\gamma_2 J_3 + 4\Lambda\gamma_2 J_2 + 2\gamma_0(\gamma_1+\gamma_2)J_2 + 2\Lambda J_2\gamma_0 + \gamma_0^2 J_2][(\gamma_1+\gamma_2)^2+\Delta_\mathrm{c}^2]}{S}, \tag{2b}$$

$$\tilde{\rho}_{33}^{(0)} = \frac{2\gamma_1\tilde{\rho}_{11}^{(0)} + J_3}{2\Lambda+\gamma_0}\,, \tag{2c}$$

$$\tilde{\rho}_{12}^{(0)} = -\frac{\mathrm{i}g_{\mathrm{c}}}{\gamma_1+\gamma_2+\mathrm{i}\Delta_{\mathrm{c}}}(\tilde{\rho}_{11}^{(0)}-\tilde{\rho}_{22}^{(0)})\,, \tag{2d}$$

$$\tilde{\rho}_{13}^{(0)} = \frac{\mathrm{i}pg_{\mathrm{c}}\tilde{\rho}_{11}^{(0)}}{(\gamma_1+\gamma_2+\Lambda)(\Lambda-\mathrm{i}\Delta_{\mathrm{c}})+g_{\mathrm{c}}^2}\,, \tag{2e}$$

$$\tilde{\rho}_{23}^{(0)} = \frac{p(\gamma_1+\gamma_2+\Lambda)\tilde{\rho}_{11}^{(0)}}{(\gamma_1+\gamma_2+\Lambda)(\Lambda-\mathrm{i}\Delta_{\mathrm{c}})+g_{\mathrm{c}}^2}\,, \tag{2f}$$

where

$$S = 4g_{\mathrm{c}}^2(\gamma_1+\gamma_2)(2\Lambda+\gamma_0+\gamma_1)\gamma_0+[4\Lambda\gamma_2+2\gamma_0(\gamma_1+\gamma_2)+2\Lambda\gamma_0+\gamma_0^2][(\gamma_1+\gamma_2)^2+\Delta_{\mathrm{c}}^2]\gamma_0. \tag{2g}$$

Using Eqs.(2), we have the population difference $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}$ and the polarization $\tilde{\rho}_{31}^{(1)}$ to the first order of g_{p}, while to all orders of g_{c}:

$$\tilde{\rho}_{11}^{(1)} = \frac{2g_{\mathrm{c}}^2(\gamma_1+\gamma_2)[(2\Lambda+\gamma_0)Q_2+2\Lambda Q_3]+2\Lambda Q_3\gamma_0[(\gamma_1+\gamma_2)^2+\Delta_{\mathrm{c}}^2]}{S}\,, \tag{3a}$$

$$\tilde{\rho}_{33}^{(1)} = \frac{2\gamma_1\rho_{11}^{(1)}+J_3+B}{2\Lambda+\gamma_0}\,, \tag{3b}$$

$$\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)} = \frac{(2\Lambda-2\gamma_1+\gamma_0)\tilde{\rho}_{11}^{(1)}-J_3-B}{2\Lambda+\gamma_0}\,, \tag{3c}$$

$$\tilde{\rho}_{31}^{(1)} = \frac{\mathrm{i}g_{\mathrm{p}}[(\Lambda-\mathrm{i}\Delta_{\mathrm{p}}+\mathrm{i}\Delta_{\mathrm{c}})(\tilde{\rho}_{11}^{(0)}-\tilde{\rho}_{33}^{(0)})-\mathrm{i}g_{\mathrm{c}}\rho_{12}^{(0)}]-\mathrm{i}pg_{\mathrm{c}}\tilde{\rho}_{11}^{(1)}}{(\gamma_1+\gamma_2+\Lambda-\mathrm{i}\Delta_{\mathrm{p}})(\Lambda+\mathrm{i}\Delta_{\mathrm{c}}-\mathrm{i}\Delta_{\mathrm{p}})+g_{\mathrm{c}}^2}\,, \tag{3d}$$

where S is shown by Eq.(2g), and

$$Q_2 = J_2+A, \quad Q_3 = J_3-A-\frac{\gamma_0}{2\Lambda}(A+B)\,,$$

$$A = \mathrm{i}g_{\mathrm{p}}g_{\mathrm{c}}\frac{\mathrm{i}(\gamma_1+\gamma_2)(\tilde{\rho}_{32}^{(0)}+\tilde{\rho}_{23}^{(0)})+\Delta_{\mathrm{c}}(\tilde{\rho}_{32}^{(0)}-\tilde{\rho}_{23}^{(0)})}{[(\gamma_1+\gamma_2)^2+\Delta_{\mathrm{c}}^2]}, \quad B = \mathrm{i}g_{\mathrm{p}}(\tilde{\rho}_{13}^{(0)}-\tilde{\rho}_{31}^{(0)}).$$

The probe gain (absorption) and dispersion correspond to the imaginary and real parts of $\tilde{\rho}_{31}^{(1)}$ respectively. If $\mathrm{Im}\tilde{\rho}_{31}^{(1)}>0$, the system exhibits gain for the probe field; if $\mathrm{Im}\tilde{\rho}_{31}^{(1)}<0$, the probe field is attenuated. When $\mathrm{Im}\tilde{\rho}_{31}^{(1)}>0$ and $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}<0$, the lasing without inversion (LWI) can be realized; if both $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ and $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}$ are simultaneously positive, the lasing with inversion occurs.

From Eq.(3d), we can see that $\tilde{\rho}_{31}^{(1)}$ is contributed by three terms: the population difference term (proportional to $\tilde{\rho}_{11}^{(0)}-\tilde{\rho}_{33}^{(0)}$), the dynamically induced coherence term (proportional to $\tilde{\rho}_{12}^{(0)}$), and SGC term (proportional to $\tilde{\rho}_{11}^{(1)}$), an additional term compared with the first two terms which are usual in conventional systems for LWI.

In the following section, we will investigate the effects of the relative phase Φ on the gain, dispersion and populations by using the numerical results from the analytical solution, Eqs.(3), and the parameters, g_{c}, g_{p}, Δ_{c}, Δ_{p}, γ_0, J_2, J_3, Λ and γ_2, which are scaled by γ_1.

3. Phase control of gain and dispersion

Choosing the values of system parameters as $\eta=1$, $\Lambda=1.66$, $\theta=\pi/4$, $\Delta_{\mathrm{c}}=0$, $\Delta_{\mathrm{p}}=-8$, $\gamma_0=0.8$, $J_2=0.1$, $J_3=0.7$, $g_{\mathrm{c}}=10\sin\theta$, and $g_{\mathrm{p}}=0.1\sin\theta$, according to the numerical results from Eqs.(3), we plot the population difference $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}$ and the probe gain (absorption) $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ as functions of the relative phase Φ, as shown in Fig.2.

We can see from Fig.2 that, the curves of $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}$ and $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ are similar to the cosine and sine curves respectively, the period is 2π. With the increase of the value of Φ, the value of $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ is periodically the positive and negative, i.e., the probe field is periodically

amplified and attenuated; while the value of $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}$ is also periodically the positive and negative, i.e., the inversion and inversionless of the populations of the levels $|1\rangle$ and $|3\rangle$ occur periodically; hence a change from lasing with inversion to LWI takes place periodically. So by adjusting the value of the relative phase Φ, the transformation between LWI and lasing with inversion can be realized, the largest gain without inversion can be obtained. It should be pointed out that from Fig.3 on we do not give the curves of $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}$, but all the values of $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}$ are negative, so in these figures all the gains are inversionless gains.

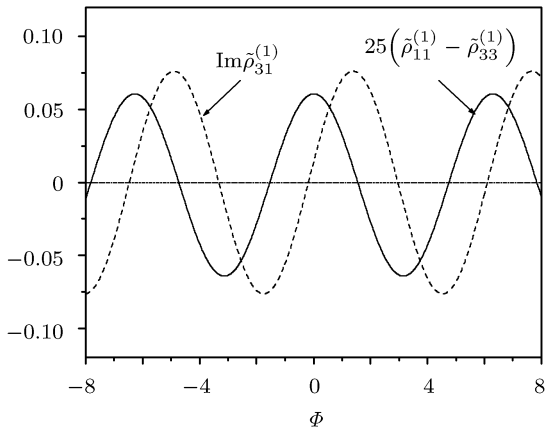


Fig.2. $\tilde{\rho}_{11}^{(1)}-\tilde{\rho}_{33}^{(1)}$ and $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ versus Φ with $\eta=1$, $\Lambda=1.66$, $\theta=\pi/4$, $\Delta_{\mathrm{c}}=0$, $\Delta_{\mathrm{p}}=-8$, $\gamma_0=0.8$, $J_2=0.1$, $J_3=0.7$, $\gamma_2=1$, $g_{\mathrm{c}}=10\sin\theta$, and $g_{\mathrm{p}}=0.1\sin\theta$. The parameters g_{c}, g_{p}, Δ_{c}, Δ_{p}, γ_0, J_2, J_3, Λ and γ_2 are scaled by γ_1.

Figure 3 illustrates the gain (absorption) $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$, dispersion $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ versus the probe detuning Δ_{p} for both cases $\eta=0$ and $\eta=1$ with the same parameter values as those in Fig.2 except $\Lambda=1.0$. In Figs.3(a) and 3(b), the mark $\eta=1$ means $\eta=1$ and $\Phi=0$. For clearness, Fig.3(e) gives an amplification of the region $-3\leq\Delta_{\mathrm{p}}\leq 3$ in Fig.3(a) for $\eta=0$. It is shows that when $\eta=0$, the probe field is amplified only when the values of Δ_{p} are small ($|\Delta_{\mathrm{p}}|\leq 1.8$), and the gain values are very small. However when $\eta=1$, the probe field can be amplified with a much larger gain, and the gain spectrum region is also much larger than that for $\eta=0$. The values of the dispersion for $\eta=1$ also are much larger than that for $\eta=0$. This shows that the contribution of SGC to the inversionless gain and dispersion is much larger than that of the dynamically induced coherence. In addition, when the SGC effect is considered ($\eta=1$), the probe gain and the dispersion are quite sensitive to the relative phase. The gain maxima corresponding to $\Phi=\pi/2$ and $\Phi=3\pi/2$ are much larger than that corresponding to $\Phi=0$ and $\Phi=\pi$; however the gain spectrum region for $\Phi=0$ and $\Phi=\pi$ are obviously wider than that for $\Phi=\pi/2$ and $\Phi=3\pi/2$; for the dispersion the situation is just the opposite.

Comparing the curves of the gain (absorption) and the dispersion, we find that the shape of the dispersion curve with $\Phi=\alpha+\pi/2$ is very similar to the gain (absorption) curve with $\Phi=\alpha$. It should be pointed out that here we consider the dispersion curve of $\Phi=0$ as the dispersion curve of $\Phi=2\pi$ and compare with the gain (absorption) curve of $\Phi=3\pi/2$. In order to check the generality of this similarity conclusion, we have used another several sets of parameter values corresponding to $\Delta_{\mathrm{c}}=0$ (i.e., the resonant driving field) to give corresponding gain (absorption) and dispersion curves for different values of Φ, the conclusion is the same.

From Figs.3(a) and 3(b) we can see that when $\eta=1$ and $\Phi=0,\pi$ the value of the dispersion $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ at about $\Lambda_{\mathrm{p}}=\pm 10$ is very large while the value of $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ is zero: i.e., we can get a large refractive index with zero absorption. Also Figs.3(a) and 3(b) show that when $\eta=0$, both $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ and $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ equal zero simultaneously at $\Delta_{\mathrm{p}}=0$: i.e., EIT occurs; however, when $\eta=1$, both $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ and $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ cannot simultaneously equal zero for any value of Φ including $\Phi=0$: i.e., EIT cannot arise. Hence, we can say that the presence of SGC destroys the original condition for the production of EIT. However, when SGC exists, we can still get EIT in the new condition as shown in Fig.4.

The parameter values in Fig.4 are the same as those in Fig.3 but $\Delta_{\mathrm{c}}=80$, i.e., the off-resonant driving field. We can see that when $\Phi=\pi/2$ (Fig.4(a)) both $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ and $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ equal simultaneously to zero at $\Delta_{\mathrm{p}}\approx -23.1$, i.e., EIT occurs; when $\Phi=3\pi/2$ (Fig.4(b)) we can get more larger inversionless gain than those in Fig.3. For simplicity, here we do not give the figures for $\Phi=0$, $\Phi=\pi$, these figures show that we can also get high refractive index with zero absorption but for the off-resonant driving field the similarity between the curves of the gain (absorption) and the dispersion, which arises for the resonant driving field, is not preserved.

In order to get a deeper insight into the modulation effect of the relative phase, in Fig.5 we display the probe gain $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ and dispersion $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ as functions of the relative phase Φ for $\Delta_{\mathrm{p}}=0,\pm 5,7$, the

other parameter values are the same as those in Fig.3. Figure 5 shows that both the inversionless gain (absorption) and dispersion are periodical functions of Φ, the modulation period is always 2π regardless of the value of $\Delta_{\rm p}$, but the value variation of $\Delta_{\rm p}$ will affect size of inversionless gain (absorption) and dispersion, different values of $\Delta_{\rm p}$ correspond to different maxima of the inversionless gain (absorption) and dispersion with different values of Φ.

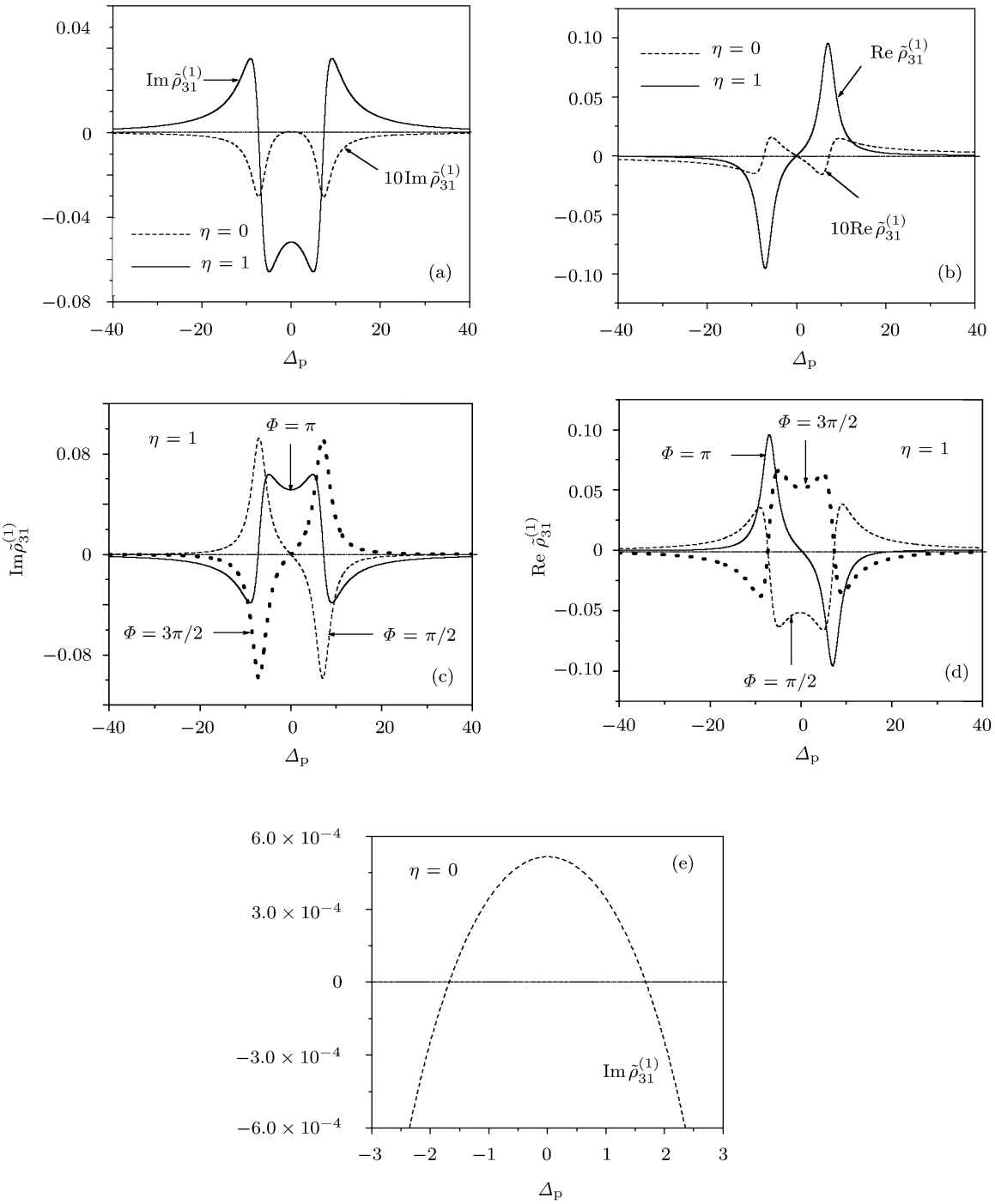


Fig.3. $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ and $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ vs $\Delta_{\rm p}$ for two cases $\eta = 0$ and $\eta = 1$ with the same parameters values as those in Fig.2 except $\Lambda = 1.0$.

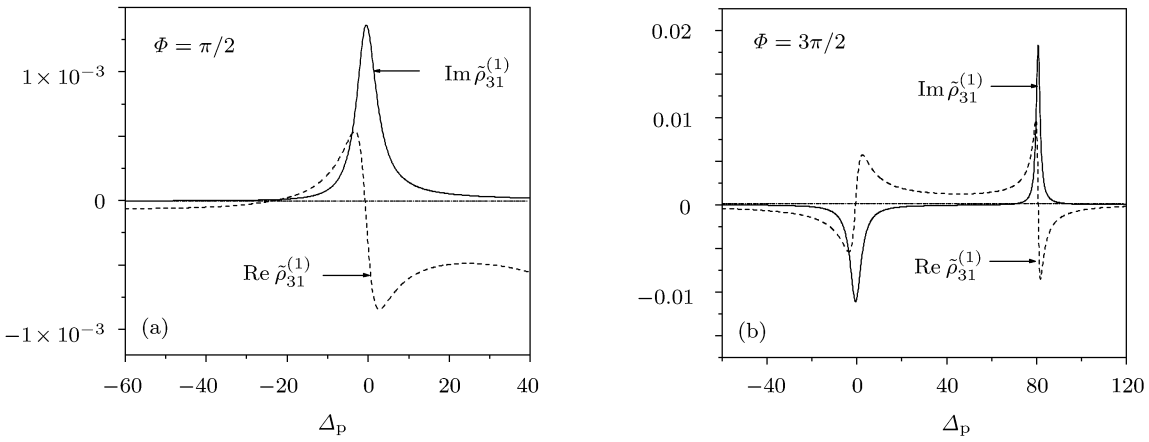


Fig.4. $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ and $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ vs Δ_p with $\eta = 1$, $\Delta_\mathrm{c} = 80$. The other parameters values are the same as those in Fig.3.

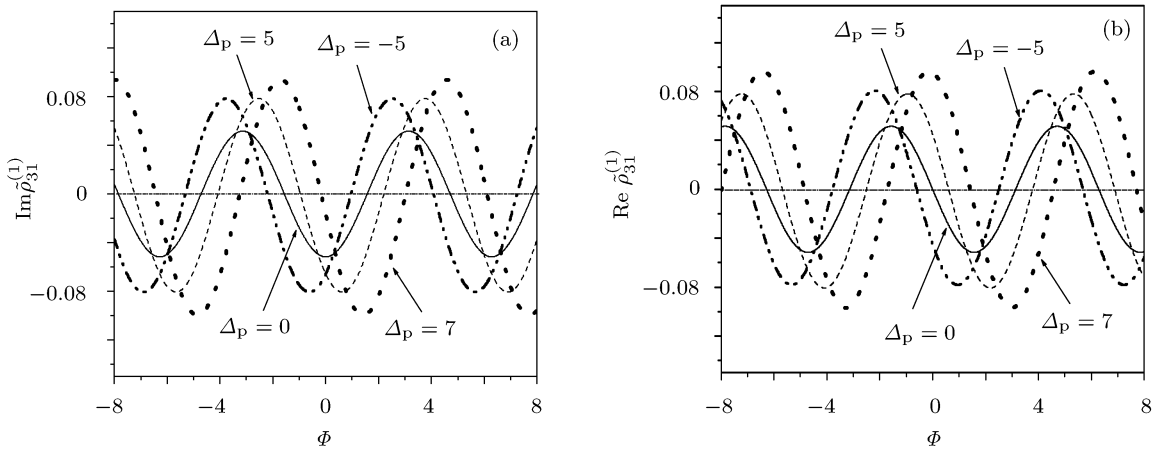


Fig.5. $\mathrm{Im}\tilde{\rho}_{31}^{(1)}$ and the dispersion $\mathrm{Re}\tilde{\rho}_{31}^{(1)}$ vs Φ with $\eta = 1$ and $\Delta_\mathrm{p} = 0, \pm5, 7$. The values of other parameters are the same as those in Fig.3.

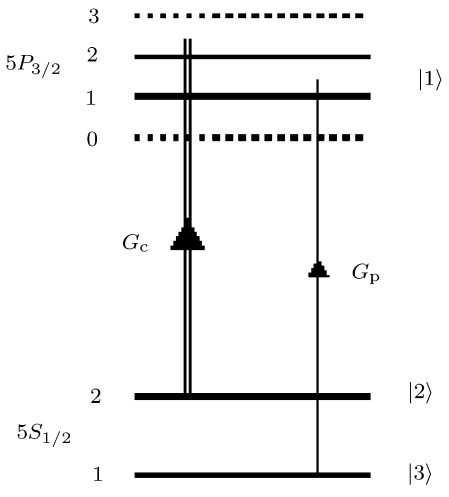


Fig.6. Energy-level diagram of atom ^{87}Rb. Double and single lines stand for the driving and probe laser fields respectively. Dashed lines depict uncoupled states in the energy diagram.

In order to examine the above theoretical result, one can perform experiment with the ^{87}Rb atomic vapour contained in a pyrex cell. The energy-level diagram of ^{87}Rb are illustrated in Fig.6. The extended cavity diode lasers are tuned to the D_2 line and form a Λ-type system. The probe (with Rabi frequency G_p) and driving (with Rabi frequency G_c) laser fields excite $5S_{1/2}(F = 1) \rightarrow 5P_{3/2}(F' = 0, 1, 2)$ and $5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F' = 1, 2, 3)$ transitions respectively, as shown in Fig.6. The double and single lines stand for the driving and probe laser fields respectively. The driving and probe fields cannot couple the ground states via the hyperfine levels $F' = 0$ and 3 of $5P_{3/2}$ state due to the selection rules, and these states are marked by the dashed lines. The spontaneous emission rates of the both transitions are $2\gamma_1 = 2\gamma_2 = 38.11$ MHz. The parameters g_c, g_p, Δ_c, Δ_p, γ_0, J_2, J_3, Λ and γ_2 are scaled by $\gamma_1 (= 19.055$ MHz). The amplitudes of both

the dipole moments $\boldsymbol{d}_{12}$ and $\boldsymbol{d}_{13}$ are $4.227(5)ea_0$, e and a_0 are elementary charge and Bohr radius respectively. The Rabi frequency amplitudes $g_{\rm p}$=$0.1\gamma_1$ and $g_{\rm c} = 10\gamma_1$ correspond to the probe and driving laser fields $E_{\rm p} = 0.2492$V/cm and $E_{\rm c} = 24.92$V/cm respectively. To be accurate, one can choose the driving frequency around $5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F' = 1)$ transition. The Rabi frequency $g_{\rm c}$ of the driving laser can be estimated from formula $g_{\rm c} = 2\pi\Gamma\sqrt{I/8}$, where I is the drive intensity (mW/cm^2), $\Gamma(\approx 6$MHz) is the natural width of the P-state of Rb atom.[24]

4. Conclusions

In this paper, at first we present the density-matrix equations of motion of an open Λ system with SGC and give the analytical solution of the equations under the condition of the steady state, then we discuss the control role of the relative phase between the probe and driving fields on the gain and dispersion by using the numerical results from the analytical solution. We find that by adjusting the value of the relative phase, a change from lasing with inversion to lasing without inversion can be realized; the value of the inversionless gain and dispersion, and the gain spectrum region can be obviously varied; high refractive index with zero absorption can be obtained; the gain (absorption), dispersion and populations are periodical functions of Φ, the modulation period is always 2π. It is also found that when the driving field is resonant and taking the probe detuning as variable, the shape of the dispersion curve versus with $\Phi = \alpha+\pi/2$ is very similar to the gain (absorption) curve with $\Phi = \alpha$, however for the off-resonant driving field the similarity will disappear; the contribution of SGC to the inversionless gain and dispersion is much larger than that of the dynamically induced coherence. Finally, we suggest to perform experiment with the ^{87}Rb atomic vapour contained in a pyrex cell to examine our theoretical results, and relative data of Rb atom are given.

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