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Adaptive synchronization of hyperchaotic Lü system with uncertainty*

Gao Bing-Jian(高秉建)[a)b)†] and Lu Jun-An(陆君安)[a)]

[a)] *School of Mathematics & Statistics, Wuhan University, Wuhan* 430072, *China*

[b)] *School of Science, Wuhan University of Technology, Wuhan* 430070, *China*

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This paper presents a novel adaptive control scheme for synchronization of the latest hyperchaotic Lü system. Based on the Lyapunov stability theory, a feedback controller and a parameter update law are designed for the synchronization of hyperchaotic Lü systems with uncertainty. Numerical simulations are given to demonstrate the validity of the synchronization technique.

Keywords: synchronization, parameter identification, hyperchaotic Lü system
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1. Introduction

Since synchronization of chaotic systems was first introduced by Pecora and Carroll,[1,2] the chaos synchronization has been an active research topic in nonlinear science, and studied extensively. Its possible applications have attracted a great deal of attention e.g. in control theory, telecommunications, and encoding message.[3−5] Recently, a wide variety of approaches have been proposed to the synchronization of chaotic systems, which include manifold-based method,[6] feedback method,[7−9] impulsive control method,[10] active control,[11] adaptive control,[12−16] and time-delay feedback approach,[17] etc.

However, few synchronization methods are concerned with the synchronization of hyperchaotic system characterized by more than one positive Lyapunov exponent, which has more complex dynamical behaviours. Moreover, most researches are based on knowing exactly the system parameters. But in many practical situations, the values of some systems' parameters are unavailable and the synchronization will be destroyed with the effects of those uncertainties.[13,18,19]

Luckily, adaptive control scheme is an effective method of synchronizing chaotic systems. We can utilize adaptive controllers to compensate the effects of uncertainty.

Very recently, in Ref.[20], Chen *et al* have developed the latest hyperchaotic Lü system via state feedback control, which has a more complicated topological structure and richer dynamical behaviours. It is believed that the adaptation of hyperchaotic system can enhance the security of communication scheme.

In this paper, a novel adaptive control scheme is proposed for the synchronization of the latest hyperchaotic Lü system with uncertainty. Based on the Lyapunov stability theory,[21] a feedback controller and a parameter update law are designed for the synchronization of two uncertain hyperchaotic Lü systems.

2. Hyperchaotic Lü system

The Lü system is a typical transition system, which connects the well-known Lorenz and Chen attractors and represents the transition from one to another.[22,23] The Lü system is described by

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = -xz + cy, \\ \dot{z} = xy - bz. \end{cases}$$

When $a = 36, b = 3$, and $c = 20$, Lü system has a critical chaotic attractor with Lyapunov exponents $\lambda_1 = 1.5046, \lambda_2 = 0$, and $\lambda_3 = -22.5044$ and Lyapunov dimension $\lambda_L = 2.0669$. Lü chaotic attractor is shown in Fig.1.

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†E-mail: bjgao@mail.whut.edu.cn

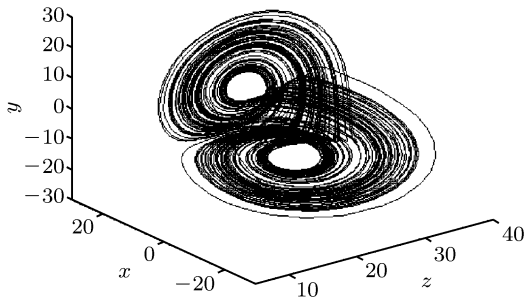


Fig.1. Lü chaotic attractor.

The hyperchaotic system has more than one positive Lyapunov exponent, that is, it is unstable in more than one direction.

Very recently, Chen *et al*[20] have initiated the latest hyperchaotic Lü system via state feedback control.

The nonlinear differential equations that describe the hyperchaotic Lü system are

$$\begin{cases} \dot{x} = a(y - x) + u, \\ \dot{y} = -xz + cy, \\ \dot{z} = xy - bz, \\ \dot{u} = xz + du. \end{cases} \tag{1}$$

When $a = 36, b = 3, c = 20$, and $-0.35 \leq d \leq 1.3$ the system has two positive Lyapunov exponents. So it is hyperchaotic. The hyperchaotic Lü system has a more complicated topological structure and richer dynamical behaviours. The hyperchaotic Lü system's attractor and circuit implementation are shown in Fig.2. It is believed that the adaptation of hyperchaotic Lü system can enhance the security of communication scheme.

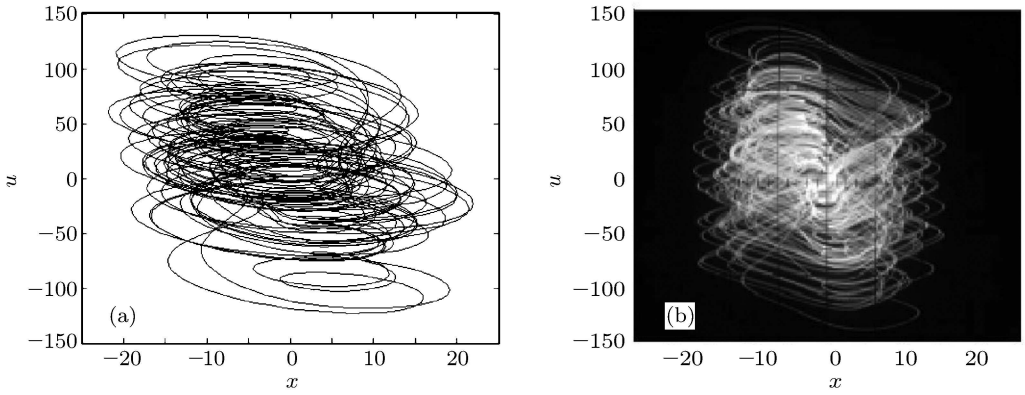


Fig.2. Hyperchaotic Lü attractor's (a) phase portrait in x–u projective plane and (b) experimental observation in x–u projective plane.

3. Adaptive synchronization of uncertain hyperchaotic Lü systems

In order to observe the synchronization behaviours of hyperchaotic Lü system, we define the master and slave systems as follows:

$$\begin{cases} \dot{x}_{\mathrm{m}} = a(y_{\mathrm{m}} - x_{\mathrm{m}}) + u_{\mathrm{m}}, \\ \dot{y}_{\mathrm{m}} = -x_{\mathrm{m}} y_{\mathrm{m}} + c y_{\mathrm{m}}, \\ \dot{z}_{\mathrm{m}} = x_{\mathrm{m}} y_{\mathrm{m}} - b z_{\mathrm{m}}, \\ \dot{u}_{\mathrm{m}} = x_{\mathrm{m}} z_{\mathrm{m}} + d u_{\mathrm{m}}, \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{x}_{\mathrm{s}} = a_1(y_{\mathrm{s}} - x_{\mathrm{s}}) + u_{\mathrm{s}} + u_1, \\ \dot{y}_{\mathrm{s}} = -x_{\mathrm{s}} z_{\mathrm{s}} + c_1 y_{\mathrm{s}} + u_2, \\ \dot{z}_{\mathrm{s}} = x_{\mathrm{s}} y_{\mathrm{s}} - b_1 z_{\mathrm{s}} + u_3, \\ \dot{u}_{\mathrm{s}} = x_{\mathrm{s}} z_{\mathrm{s}} + d_1 u_{\mathrm{s}} + u_4, \end{cases} \tag{3}$$

where $\alpha = (a, b, c, d)^{\mathrm{T}}$ is a given parameter vector, while the parameter vector $\alpha_1 = (a_1, b_1, c_1, d_1)^{\mathrm{T}}$ is uncertain.

In a practical situation, the output signals of the master system (2) can be received by the slave system (3), but the parameter vector of the master system (2) may not be known, and is to be identified. Therefore, the goal of control is to design and implement an appropriate controller $U = [u_1, u_2, u_3, u_4]^{\mathrm{T}}$ for the slave system and a parameter estimation update law

of α_1, such that the controlled slave system (3) could be synchronous with the master system (2), and the parameter α_1 approaches α, i.e.

$$\lim_{t\to+\infty}||X_{\rm s}-X_{\rm m}||=0 \text{ and } \lim_{t\to+\infty}(\alpha_1-\alpha)=0,$$

where $X_{\rm m}=(x_{\rm m},y_{\rm m},z_{\rm m},u_{\rm m})^{\rm T}$ and $X_{\rm s}=(x_{\rm s},y_{\rm s},z_{\rm s},u_{\rm s})^{\rm T}$; $||\cdot||$represents the Euclidean norm, and $(\cdot)^{\rm T}$ denotes transposition of $(\cdot)$.

Lemma Let ϕ: $R\to R$ be a twice continuously differentiable function in $[0,+\infty)$. Suppose that $\lim_{t\to+\infty}\phi(t)$ exists and $\phi''(t)$ is bounded in $[0,+\infty)$, then,

$$\phi'(t)\to 0,\ \text{as } t\to+\infty.$$

In many practical applications, the synchronization technique based on a scalar output signal is easy to implement. By virtue of the above uncertain problem, we have the following theorem.

Theorem With the controller

$$\begin{cases} u_1=-ae_2-e_4,\\ u_2=z_{\rm s}e_1-2ce_2,\\ u_3=-y_{\rm s}e_1-x_{\rm m}e_4,\\ u_4=-z_{\rm s}e_1-2de_4, \end{cases} \tag{4}$$

and the parameter update law of $\alpha_1=(a_1,b_1,c_1,d_1)^{\rm T}$

$$\begin{cases} \dot a_1=(x_{\rm s}-y_{\rm s})e_1,\\ \dot b_1=z_{\rm s}e_3,\\ \dot c_1=-y_{\rm s}e_2,\\ \dot d_1=-u_{\rm s}e_4, \end{cases} \tag{5}$$

the controlled uncertain hyperchaotic Lü system (3) is synchronous with the master system (2), and the parameter α_1 approaches α under any initial condition $(x_{\rm m}(0),y_{\rm m}(0),z_{\rm m}(0),u_{\rm m}(0))$ and $(x_{\rm s}(0),y_{\rm s}(0),z_{\rm s}(0),u_{\rm s}(0))$, i.e.

$\lim_{t\to+\infty}||e(t)||=0$, and $\lim_{t\to+\infty}(a_1-a)=\lim_{t\to+\infty}(b_1-b)=\lim_{t\to+\infty}(c_1-c)=\lim_{t\to+\infty}(d_1-d)=0$, where $e=(e_1,e_2,e_3,e_4)^{\rm T}=(x_{\rm s}-x_{\rm m},y_{\rm s}-y_{\rm m},z_{\rm s}-z_{\rm m},u_{\rm s}-u_{\rm m})^{\rm T}$.

Proof Subtracting Eq.(2) from Eq.(3) yields the error dynamical system

$$\begin{cases} \dot e_1=a_1(y_{\rm s}-x_{\rm s})-a(y_{\rm m}-x_{\rm m})+e_4+u_1,\\ \dot e_2=-z_{\rm s}e_1-x_{\rm m}e_3+c_1y_{\rm s}-cy_{\rm m}+u_2,\\ \dot e_3=y_{\rm s}e_1+x_{\rm m}e_2-b_1z_{\rm s}+bz_{\rm m}+u_3,\\ \dot e_4=z_{\rm s}e_1+x_{\rm m}e_3+d_1u_{\rm s}-du_{\rm m}+u_4. \end{cases} \tag{6}$$

Choose Lyapunov function as follows:

$$V(e,e_a,e_b,e_c,e_d)=\frac{1}{2}(e_1^2+e_2^2+e_3^2+e_4^2+e_a^2+e_b^2+e_c^2+e_d^2),$$

where parameter errors are $e_a=a_1-a, e_b=b_1-b, e_c=c_1-c$, and $e_d=d_1-d$.

The time derivative of V along the trajectory of the error dynamical system (7) is given by

$$\begin{aligned} \dot V= &\ e_1\dot e_1+e_2\dot e_2+e_3\dot e_3+e_4\dot e_4+(a_1-a)\dot a_1\\ &+(b_1-b)\dot b_1+(c_1-c)\dot c_1+(d_1-d)\dot d_1\\ = &\ (a_1-a)y_{\rm s}e_1-(a_1-a)x_{\rm s}e_1+ae_1e_2-ae_1^2\\ &+e_1e_4+e_1u_1-z_{\rm s}e_1e_2-x_{\rm m}e_2e_3+(c_1-c)y_{\rm s}e_2\\ &+ce_2^2+e_2u_2+y_{\rm s}e_1e_3+x_{\rm m}e_2e_3-(b_1-b)z_{\rm s}e_3\\ &-be_3^2+e_3u_3+z_{\rm s}e_1e_4+x_{\rm m}e_3e_4+(d_1-d)u_{\rm s}e_4\\ &+de_4^2+e_4u_4+(a_1-a)\dot a_1+(b_1-b)\dot b_1\\ &+(c_1-c)\dot c_1+(d_1-d)\dot d_1. \end{aligned} \tag{7}$$

Substituting Eq.(5) into Eq.(7) give

$$\begin{aligned} \dot V= &\ ae_1e_2-ae_1^2+e_1e_4+e_1u_1+ce_2^2-z_{\rm s}e_1e_2\\ &+e_2u_2+y_{\rm s}e_1e_3-be_3^2+e_3u_3+z_{\rm s}e_1e_4\\ &+x_{\rm m}e_3e_4+de_4^2+e_4u_4. \end{aligned} \tag{8}$$

Again, substituting Eq.(4) into Eq.(8), we have

$$\dot V=-ae_1^2-be_2^2-ce_3^2-de_4^2\le -ke^{\rm T}e,$$

where $k=\min\{a,b,c,d\}>0$. Since $\dot V$ is negative semidefinite, we cannot immediately conclude that the origin of error system (6) is asymptotically stable. In fact, as $\dot V\le 0$, then $e_1,e_2,e_3,e_4\in L_\infty$ and $e_a,e_b,e_c,e_d\in L_\infty$. From the error system (6), we have $\dot e_1,\dot e_2,\dot e_3,\dot e_4\in L_\infty$. Since $\dot V\le -ke^{\rm T}e$, we have

$$\begin{aligned} \int_0^t||e||^2{\rm d}t &= \int_0^t e^{\rm T}e{\rm d}t\le -\frac{1}{k}\int_0^t\dot V{\rm d}t\\ &=\frac{1}{k}[V(0)-V(t)]\le\frac{1}{k}V(0). \end{aligned}$$

Thus $e_1,e_2,e_3,e_4\in L_2$. According to the Barbalat's lemma, we have

$$e_1(t),e_2(t),e_3(t),e_4(t)\to 0 \text{ as } t\to\infty, \text{i.e.}\ \lim_{t\to+\infty}||e(t)||=0.$$

On the one hand, the state variables of hyperchaotic Lü system are bounded. From error system (6), we see that $\ddot e(t)$ is bounded for all $t\ge 0$. By the above introduced Lemma, we have $\lim_{t\to+\infty}\dot e=0$. On the other hand, from Eq.(5), we obtain

$$\lim_{t\to+\infty}\dot a_1=\lim_{t\to+\infty}\dot b_1=\lim_{t\to+\infty}\dot c_1=\lim_{t\to+\infty}\dot d_1=0,$$

$$\lim_{t\to+\infty}(a_1-a)=k_1,\ \lim_{t\to+\infty}(b_1-b)=k_2,$$

$$\lim_{t\to+\infty}(c_1-c)=k_3, \text{and } \lim_{t\to+\infty}(d_1-d)=k_4,$$

where k_1, k_2, k_3, and k_4 are constants.

Substituting it into system(6), we have

$$\begin{cases} \lim\limits_{t\to+\infty}\dot{e}_1=k_1(y_{\rm s}-x_{\rm s}), \\ \lim\limits_{t\to+\infty}\dot{e}_2=k_3y_{\rm s}, \\ \lim\limits_{t\to+\infty}\dot{e}_3=-k_2z_{\rm s}, \\ \lim\limits_{t\to+\infty}\dot{e}_4=k_4u_{\rm s}. \end{cases}$$

Therefore, $k_1=k_2=k_3=k_4=0$. It follows from the above that the slave system (3) is synchronous with the master system (2) and the parameter α_1 approaches α under the feedback controller (4). The proof has been completed now. Thus, we achieve the synchronization of hyperchaotic Lü systems and identify the uncertain parameters simultaneously.

4. Numerical simulations

We choose hyperchaotic Lü system with parameters $a=36, b=3, c=20$, and $d=1.3$ as a master system. The initial values of state variables are taken as $X_{\rm m0}=(20,10,10,-15)^{\rm T}$ and $X_{\rm s0}=(5,17,19,-19)^{\rm T}$, and the initial values of slave system's parameters as $\alpha_{10}=(34,5,25,1)$. The dynamic synchronization errors for the master system (2) and the slave system (3) tend to zero asymptotically as shown in Fig.3. The errors of parameters evolving to zero asymptotically are shown in Fig.4.

Numerical simulations demonstrate that adaptive synchronization and parameter identification are achieved successfully.

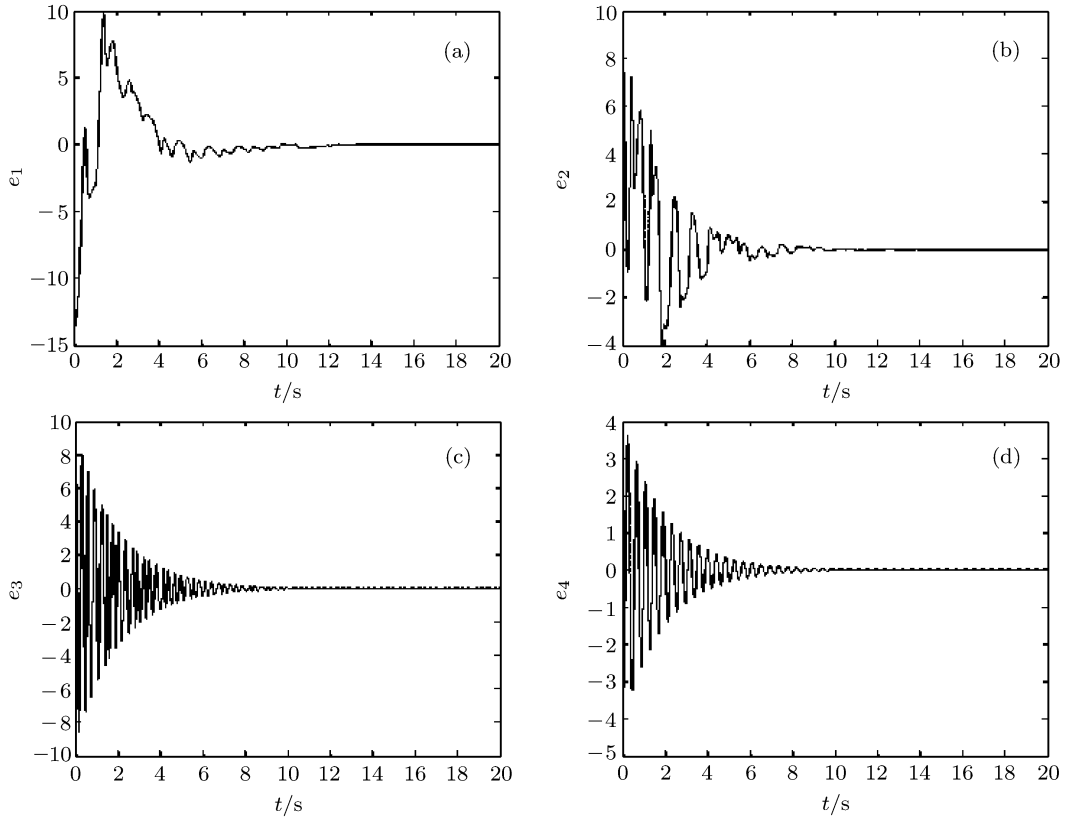


Fig.3. The errors signals between the two hyperchaotic Lü systems: $e_1(t)$, $e_2(t)$, $e_3(t)$, $e_4(t)$.

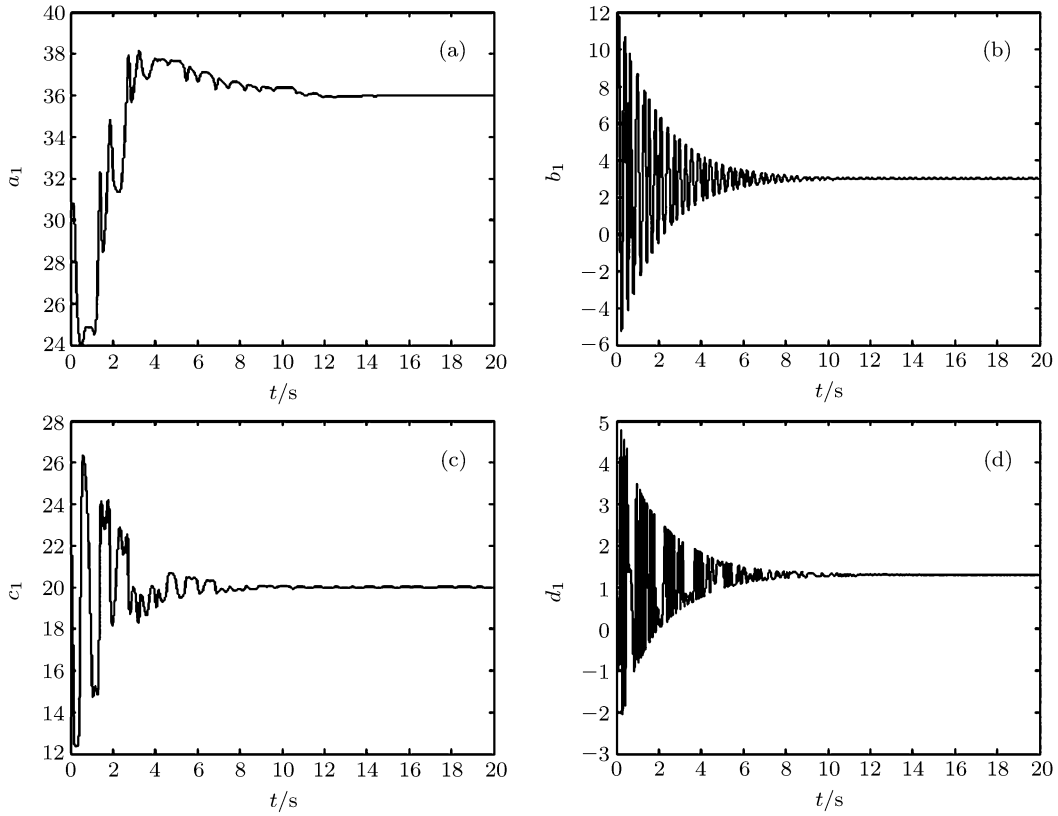


Fig.4. Variation of parameters a_1, b_1, c_1, d_1 of slave system with time t.

5. Conclusions

This paper addresses a novel adaptive control scheme for the synchronization of the latest hyperchaotic Lü system. Based on the Lyapunov stability theory, a feedback controller and a parameter update law are designed for the synchronization of hyperchaotic Lü systems with uncertainty. Numerical simulations are shown to verify the synchronization technique.

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